The Chiral Phase Transition from Strong to Weak Coupling

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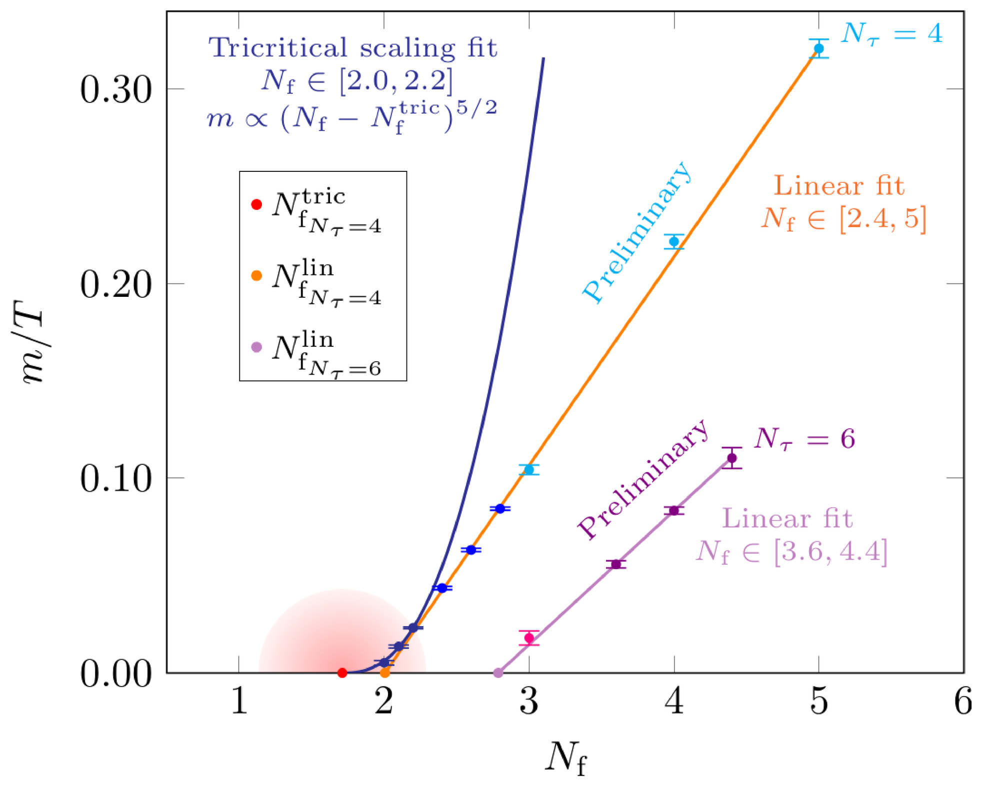
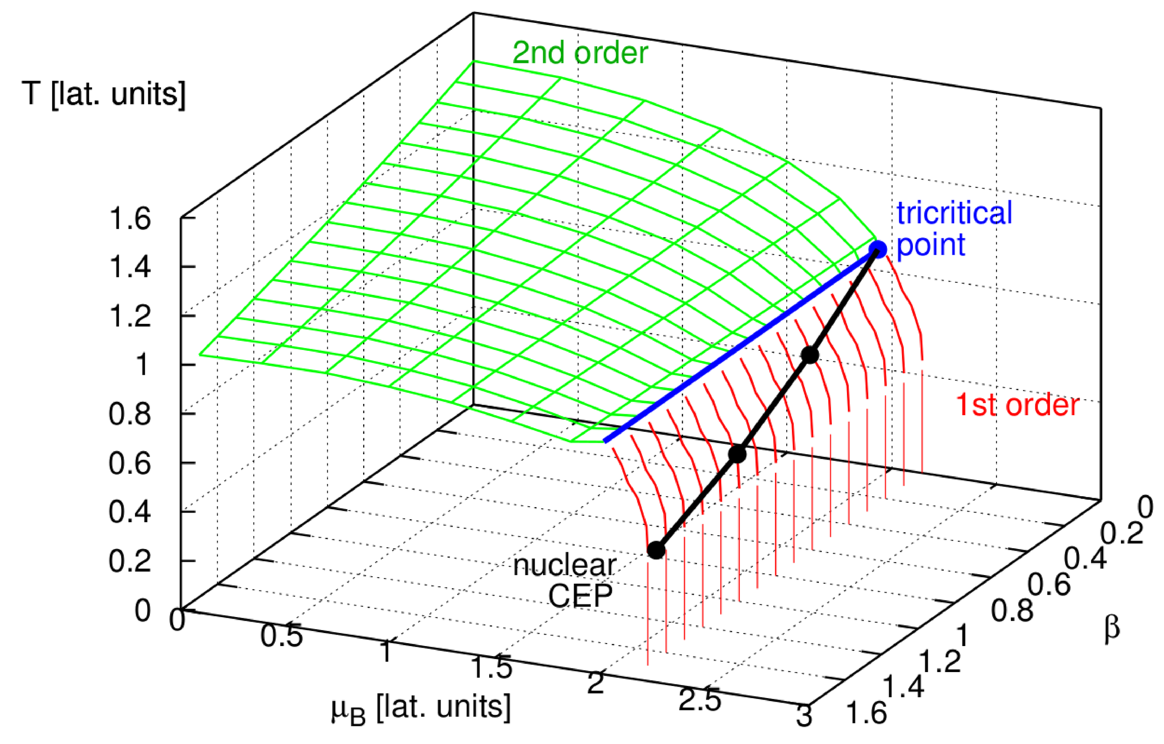
Introduction

In order to understand the nature of the chiral phase transition in the chiral limit, extrapolations are needed since it is not possible to simulate zero mass. In [1] and [2] N_f was suggested as extrapolation parameter, since it features a tricritical point in the chiral limit.

- Plot (a) shows the critical Z_2 mass as a function of the number of staggered quark flavours N_f for different lattice spacings (N_τ). The critical β values are in the range $[4.7, 5.2]$, and the cut-off dependence gets stronger for higher N_f.
- The Z_2 critical line separates the m/T vs N_f plot in a first-order region below and a crossover region above.
- As shown in plot (b), moving to the strong coupling regime for $N_f = 4$, at zero density, the phase transition is found to be second-order. Since smaller N_τ implies larger a and smaller β for a given temperature T, this means that the $N_f = 4$ first-order region must shrink as N_τ is reduced.



a) Weak coupling regime [2]



b) Strong coupling regime - $N_f = 4$ [3]

From strong to weak coupling

In this work we try to establish contact with the strong coupling region.

- The strategy we follow consists in performing new simulations on a coarser lattice at different N_f values and different degenerate quark masses.
- For lattice QCD the partition function $\mathcal{Z}$ is expressed as a path integral over the gluon fields U, including a determinant of the Dirac operator $D[U]$ for fermions. The expectation value of observable $\mathcal{O}$ reads

$$\langle \mathcal{O} \rangle = \mathcal{Z}^{-1} \int \mathcal{D}U \mathcal{O} \left(\det D[U]\right)^{N_f} \exp\{-S_g[U]\} ,$$

with gauge action $S_g[U]$. We use staggered fermions with parameters
  - lattice gauge coupling $\beta = 6/g^2$,
  - number of degenerate quarks flavours N_f,
  - quark mass m,
  - lattice temporal extent $N_\tau = (a(\beta)T)^{-1}$ =2.
- We set the chemical potential $\mu = 0$.
- As simulation program we employed our publicly available OpenCL [a]-based code CL²QCD. [b]

[a] See https://www.khronos.org/opencl/.
[b] See https://gitlab.itp.uni-frankfurt.de/lattice-qcd/ag-philipsen/cl2qcd/.

Order of phase transitions

- For fixed N_f and N_τ, the n-th standardized moment for a generic observable $\mathcal{O}$ is

$$B_n(\beta, m) = \frac{\langle (\mathcal{O} - \langle \mathcal{O} \rangle)^n \rangle}{\left\langle (\mathcal{O} - \langle \mathcal{O} \rangle)^2 \right\rangle^{n/2}}$$

- The order of the phase transition can be indentified using the analysis of the third and fourth standardized momenta of the distribution of the (approximate) order parameter, which in our simulations is the chiral condensate $\langle \bar{\psi}\psi \rangle$.
- The (pseudo-)critical β_c for the transition is given by $B_3(\beta_c, m) = 0$.
- We study the kurtosis $B_4(\beta_c(m), m)$ of the sampled distribution of the order parameter $\mathcal{O}$ for each investigation. The universal infinite volume values for the kurtosis and critical exponent ν are the following

	Crossover	1^{st} order	$3D$ Ising
B_4	3	1	1.604
ν	–	1/3	0.6301(4)

- For N_σ^3 large enough and $\beta \sim \beta_c$, the kurtosis can be expanded in powers of the scaling variable $x = (X - X_c)N_\sigma^{1/\nu}$, and the expansion can be truncated after the linear term according to

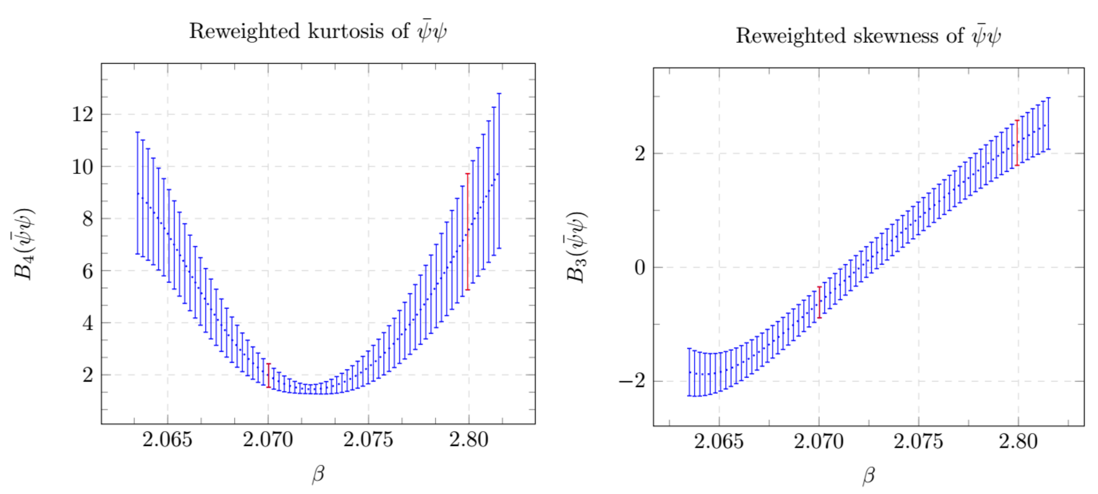
$$B_4(\beta_c, X, N_\sigma) = B_4(\beta_c, X_c, \infty) + c(X - X_c)N_\sigma^{1/\nu} + \dots \quad (1)$$

- In our analysis we fix $B_4(\beta_c, X_c, \infty) = 1.604$ and $\nu = 0.6301$.

Data analysis

For a given N_f value, many simulations can be performed by varying the mass parameter and the lattice spatial extent, as well as the β parameter.

- Studying B_3 and B_4 as a function of β, the zero crossing of the former and the minimum of the latter will fall at the same β value, which identifies the β_c.
- In order to avoid problems related to the resolution in β, we apply the Ferrenberg-Swendsen reweighting to the chiral condensate.
- As an example we show the reweighted kurtosis $B_4(\langle \bar{\psi}\psi \rangle)$ and skewness $B_3(\langle \bar{\psi}\psi \rangle)$ for $N_f = 8$, $m = 0.0010$, $N_\sigma = 16$, and $\beta \in [2.07, 2.10]$

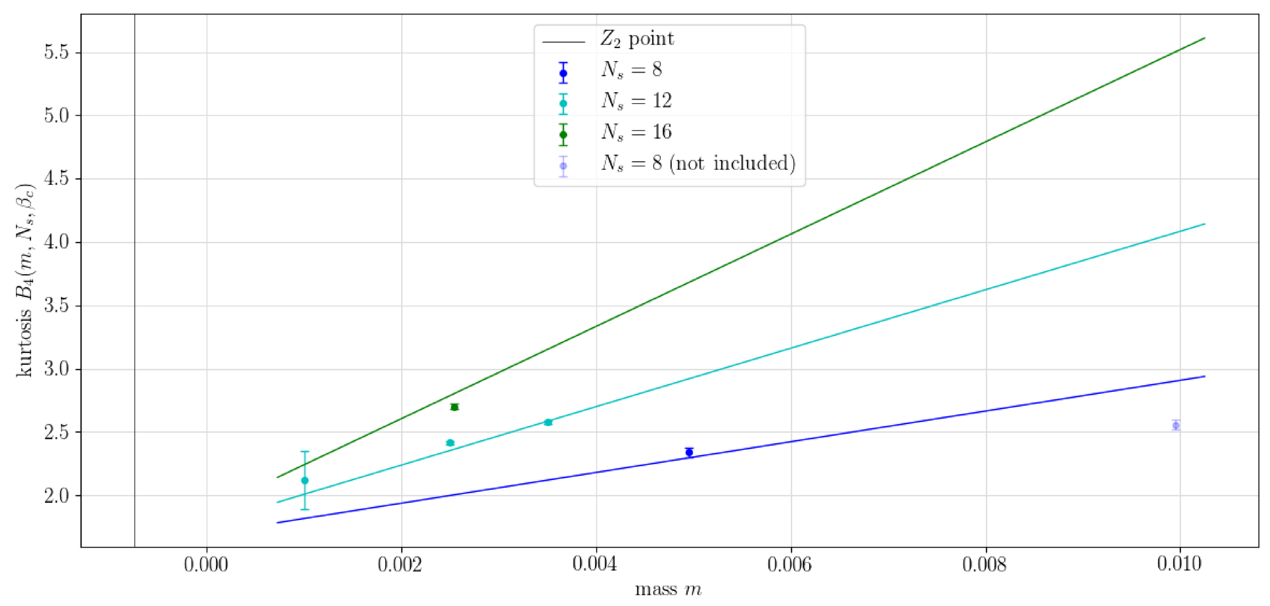


  - $\beta_c = 2.0725$, and $B_4(\beta_c) = 1.460(22)$. This makes this point a good candidate to the first-order region, being $B_4 < 1.604$.

- To investigate finite size scaling, we simulate on three $N_\sigma = 12, 16, 20$.
- Using eq.1 we fit the minimum of the kurtosis as a function of the quark masses. The critical mass m_{Z_2} is given by the mass corresponding to the intersection of the three fitted lines.

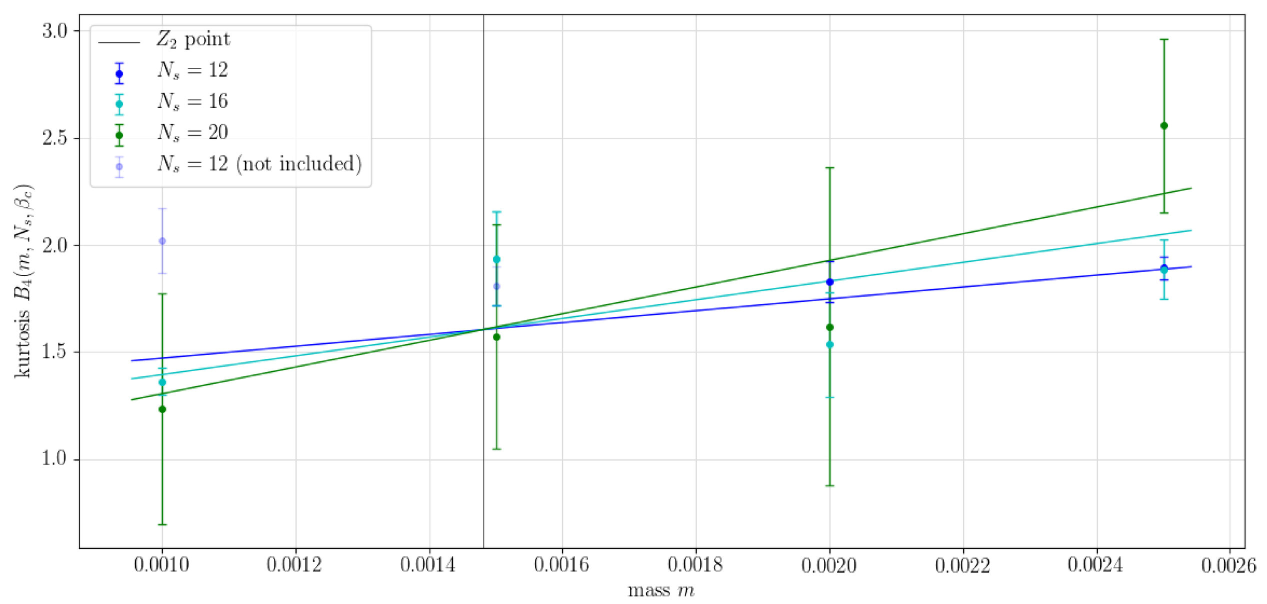
Results

Results for $N_f = 4$



- For all of the data points up to the lowest mass $m = 0.0010$, $B_4 > 2$.
- No critical m_{Z_2} can be detected, meaning that $m \geq 0.0010$ points belong to the crossover region.
- Compared to plot (b), this is not in the strong coupling region. All data points have $\beta_c \geq 3.45$.

Results for $N_f = 8$

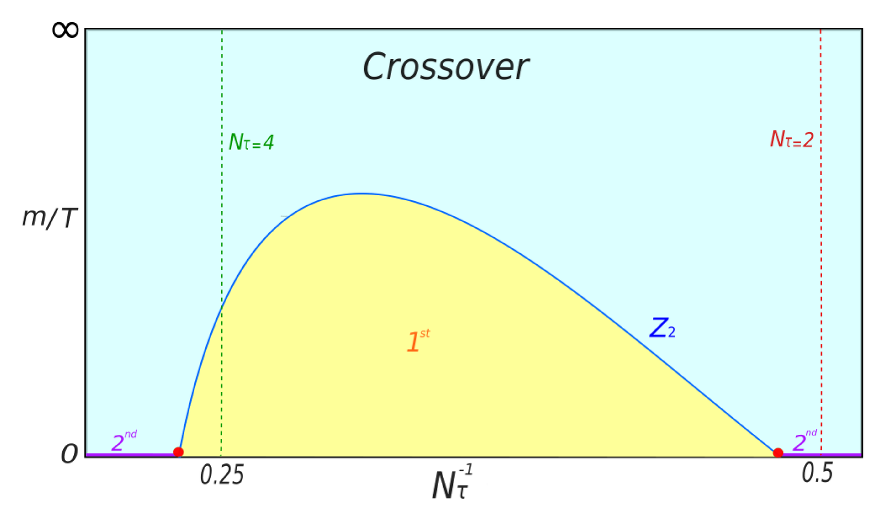


- For $m = 0.0010$ and $m = 0.0015$, the $N_\sigma = 12$ points are discarded for the purposes of the fit. Due to the presence of lattice finite size effects, the $N_\sigma = 12$ fitting line does not hit the two points within their error bars.
- Additional simulations at $N_\sigma = 24$ are then necessary.
- The error bars are still quite large and reflect the currently low amount of accumulated statistics.
- The critical mass m_{Z_2} from fit is expected be at $m = 0.00148(10)$.

Conclusions

In conclusion, using a coarser lattice is giving us more details about the shape of the critical Z_2 line when approaching the strong coupling regime.

- For $N_\tau = 2$, $N_f = 4$ the critical mass belonging to the critical line is too small to be detected.
- For $N_\tau = 2$, $N_f = 8$ a first estimate for the critical mass is $m_{Z_2} \approx 0.00148$, but still suffers from big uncertainity related to low statistics.



Sketch of the (m, N_τ)-Columbia plot for $N_f = 4$. Red dots represents the tricritical points, where the first-order $m = 0$ transition meets the second-order one (in purple).

References

[1] F. Cuteri, O. Philipsen and A. Sciarra, Phys. Rev. D **97** (2018) no.11, 114511 doi:10.1103/PhysRevD.97.114511 [arXiv:1711.05658 [hep-lat]].

[2] F. Cuteri, O. Philipsen and A. Sciarra, PoS **LATTICE2018** (2018), 170 doi:10.22323/1.334.0170 [arXiv:1811.03840 [hep-lat]].

[3] P. de Forcrand, O. Philipsen and W. Unger, PoS **CPOD2014** (2015), 073 doi:10.22323/1.217.0073 [arXiv:1503.08140 [hep-lat]].