Nuclear Science Computing Center at CCNU

Correlated Dirac eigenvalues around the transition temperature on $N_\tau = 8$ Lattices

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in collaboration with

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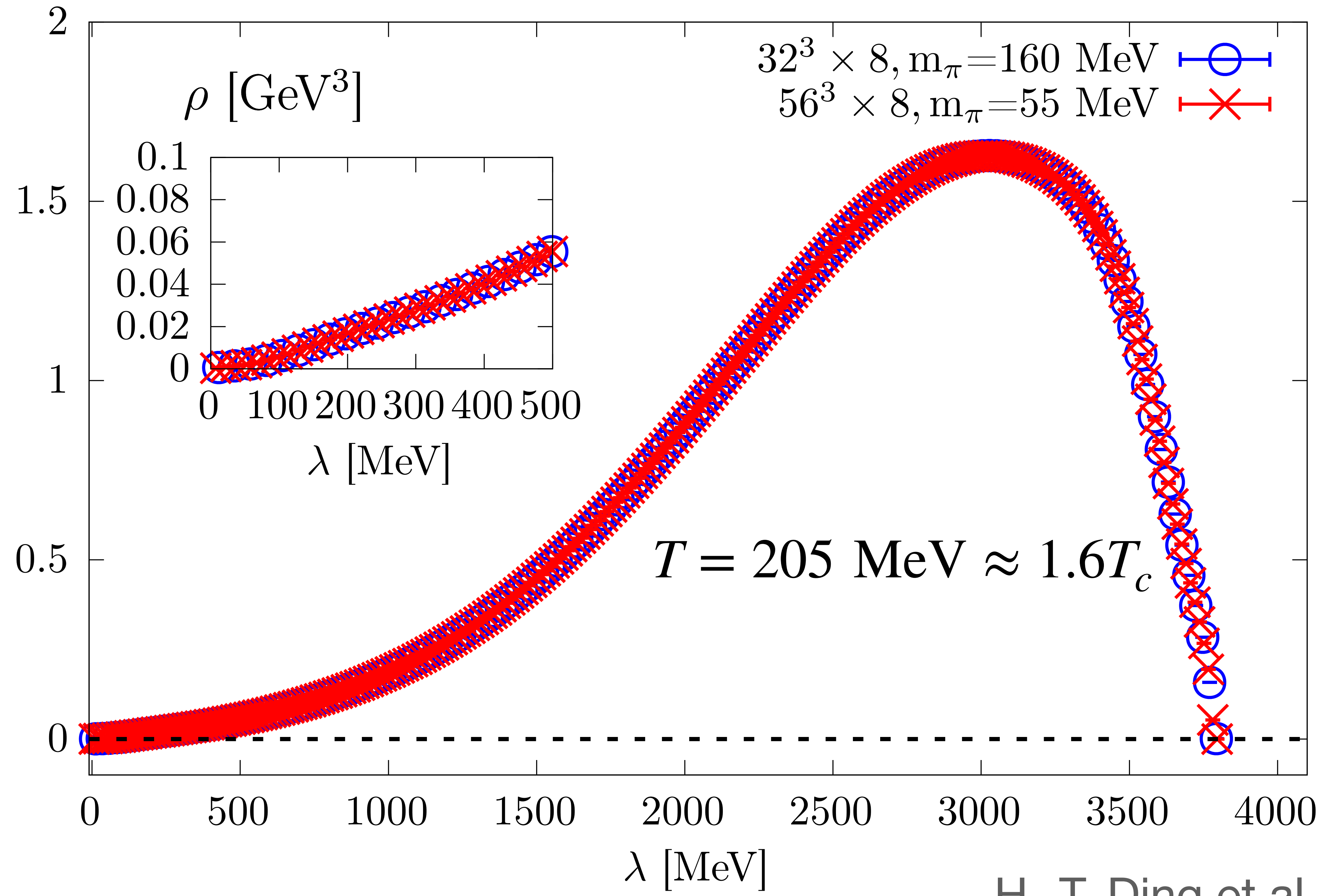
LATTICE 21

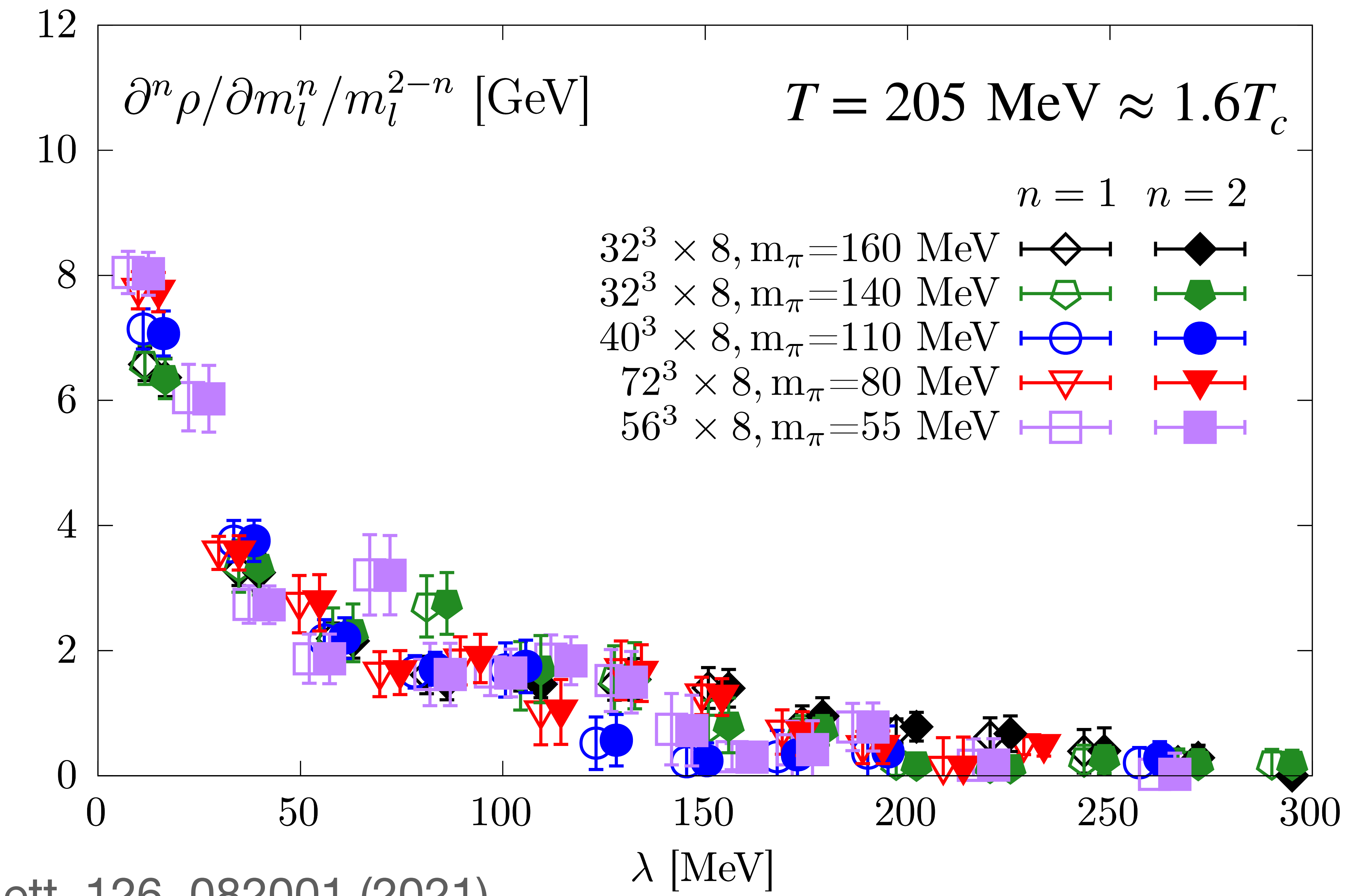
JULY 26-30 2021, ZOOM/GATHER@MIT

Dirac eigenvalue spectra and their correlations

Chebyshev filtering technique



A novel relation between $\partial^n \rho / \partial m^n$ and C_{n+1}



H.-T. Ding et al., Phys. Rev. Lett. 126, 082001 (2021)

See talk by Yu Zhang, Wed, 5:45 AM (EST)

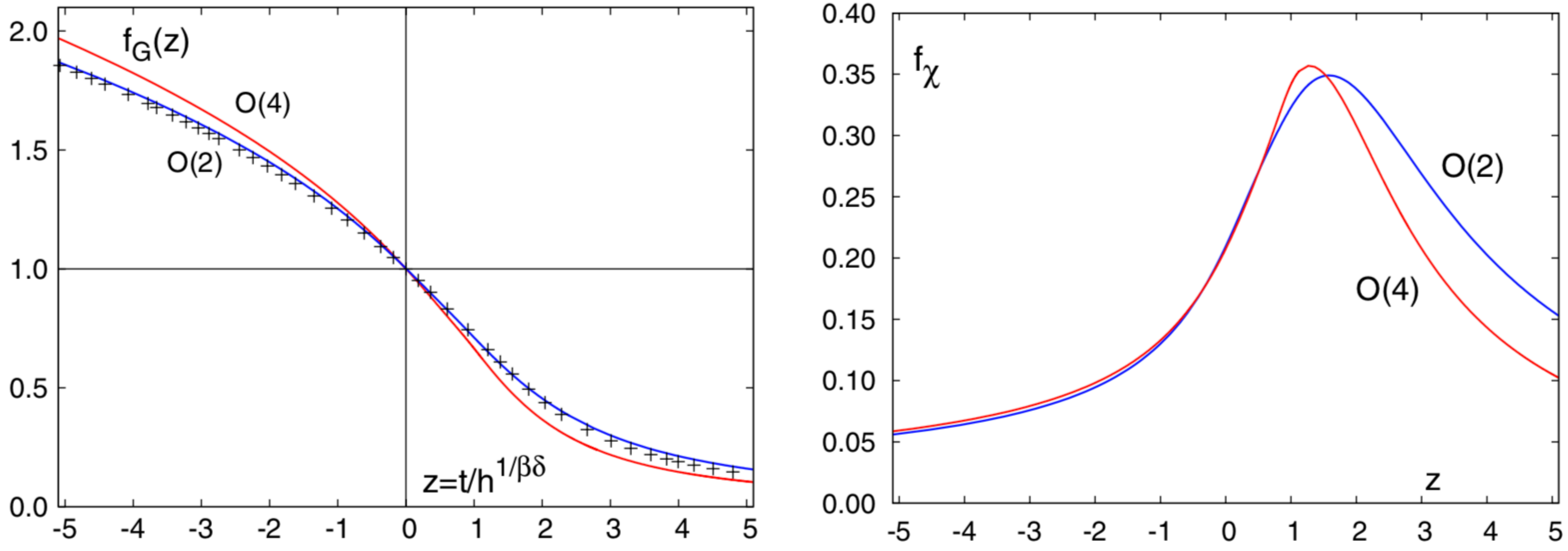
At $1.6T_c$, $m_l^{-1} \partial\rho/\partial m_l \approx \partial^2\rho/\partial m_l^2$ & $\partial^3\rho/\partial m_l^3 \approx 0 \implies \rho(\lambda \to 0, m_l) \propto m_l^2 \delta(\lambda)$

How about the case at $T \sim T_c$?

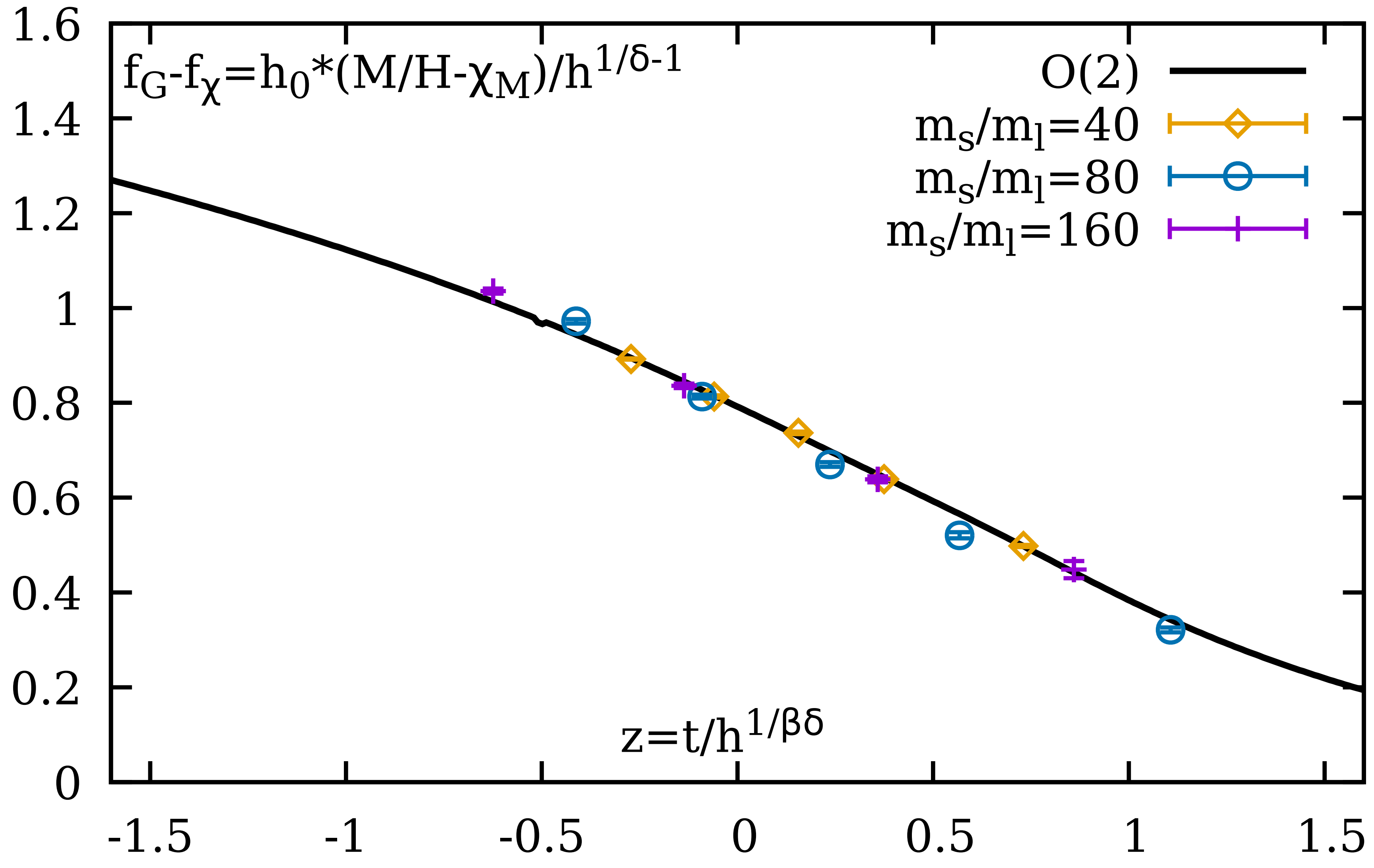
Critical behavior and Magnetic Equation of State

$$M(t,h) = h^{1/\delta} f_G(z) + f_{\rm reg}(T,H) \qquad \chi_M(t,h) = \frac{\partial M}{\partial H} = h_0^{-1} h^{1/\delta-1} f_\chi(z) + f'_{\rm reg}$$

$N_\tau = 8$ HISQ data



S. Ejiri et al., Phys. Rev. D 80, 094505 (2009)



How critical behavior is manifested in ρ, $\partial^n\rho/\partial m_l^n$ around T_c ?

Lattice Setup

- Actions: Highly improved staggered quarks and tree-level Symanzik gauge action
- Lattices size: $40^3 \times 8$
- Quark mass: $m_s^{phy}/m_l = 40\ (m_\pi = 110\ \mathrm{MeV})$
- Temperatures: $T \in (137,176)\ \mathrm{MeV}$
- Use gauge configurations generated by HotQCD
- Measurements carried out on NSC^3 at CCNU

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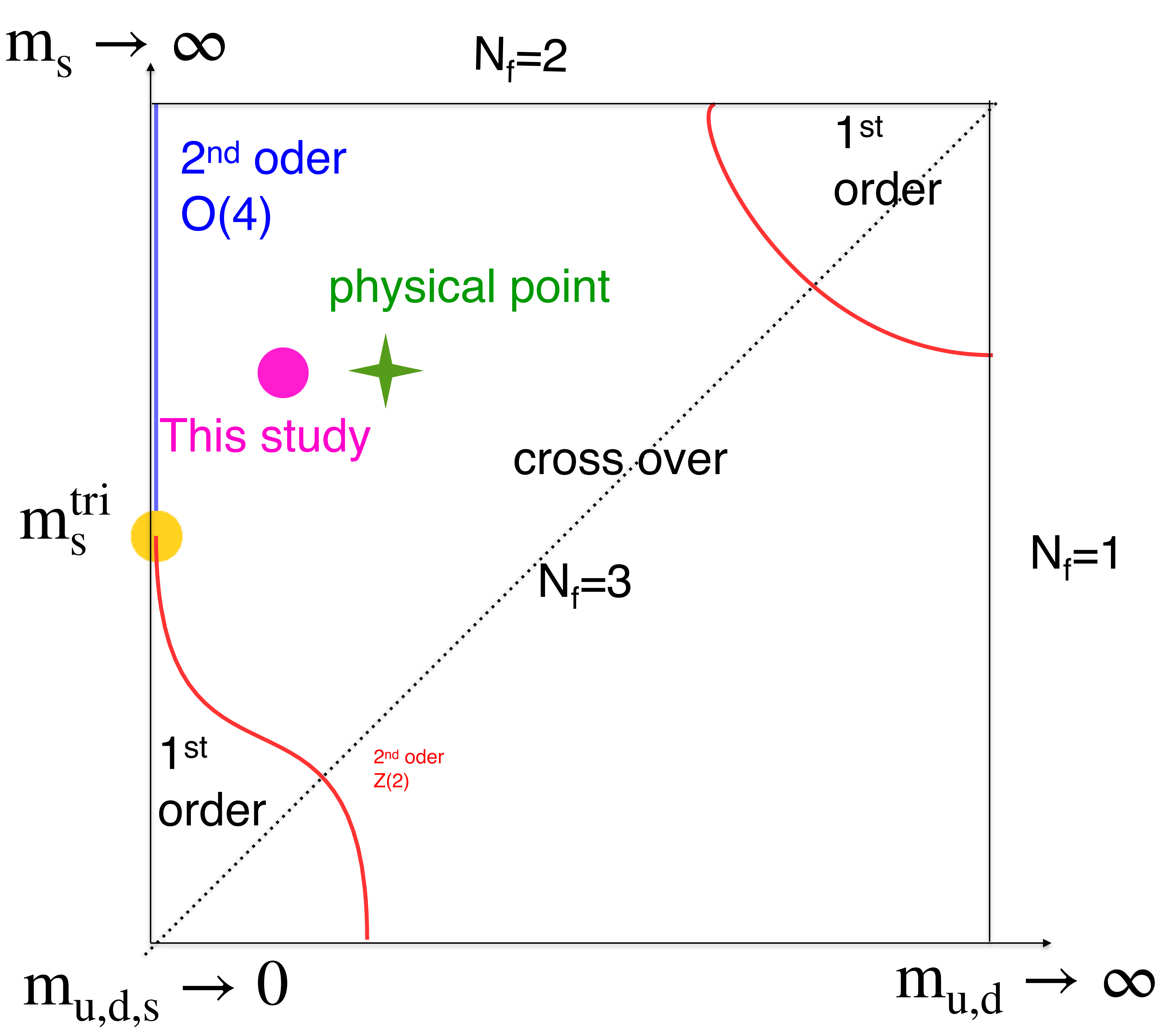
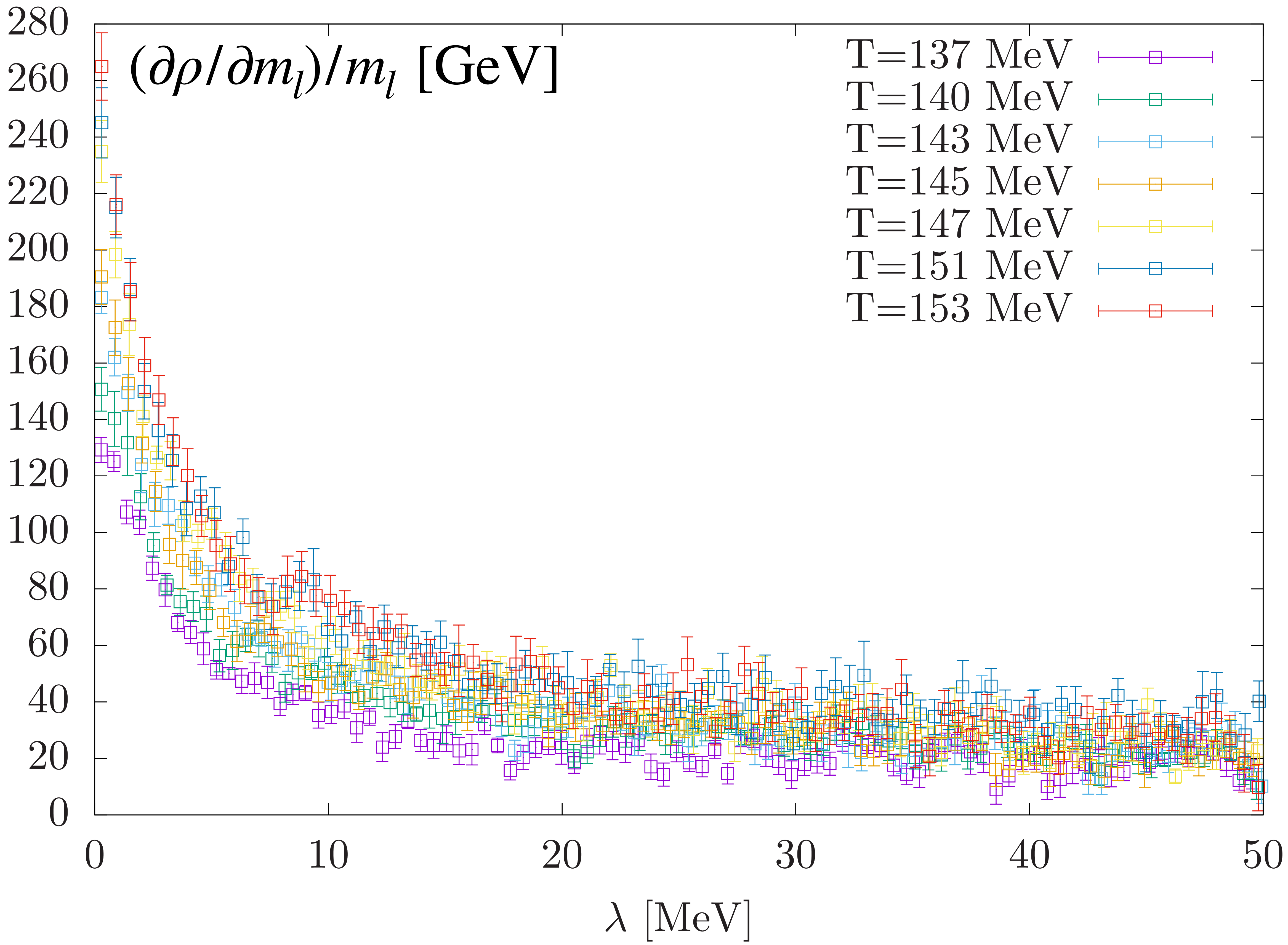


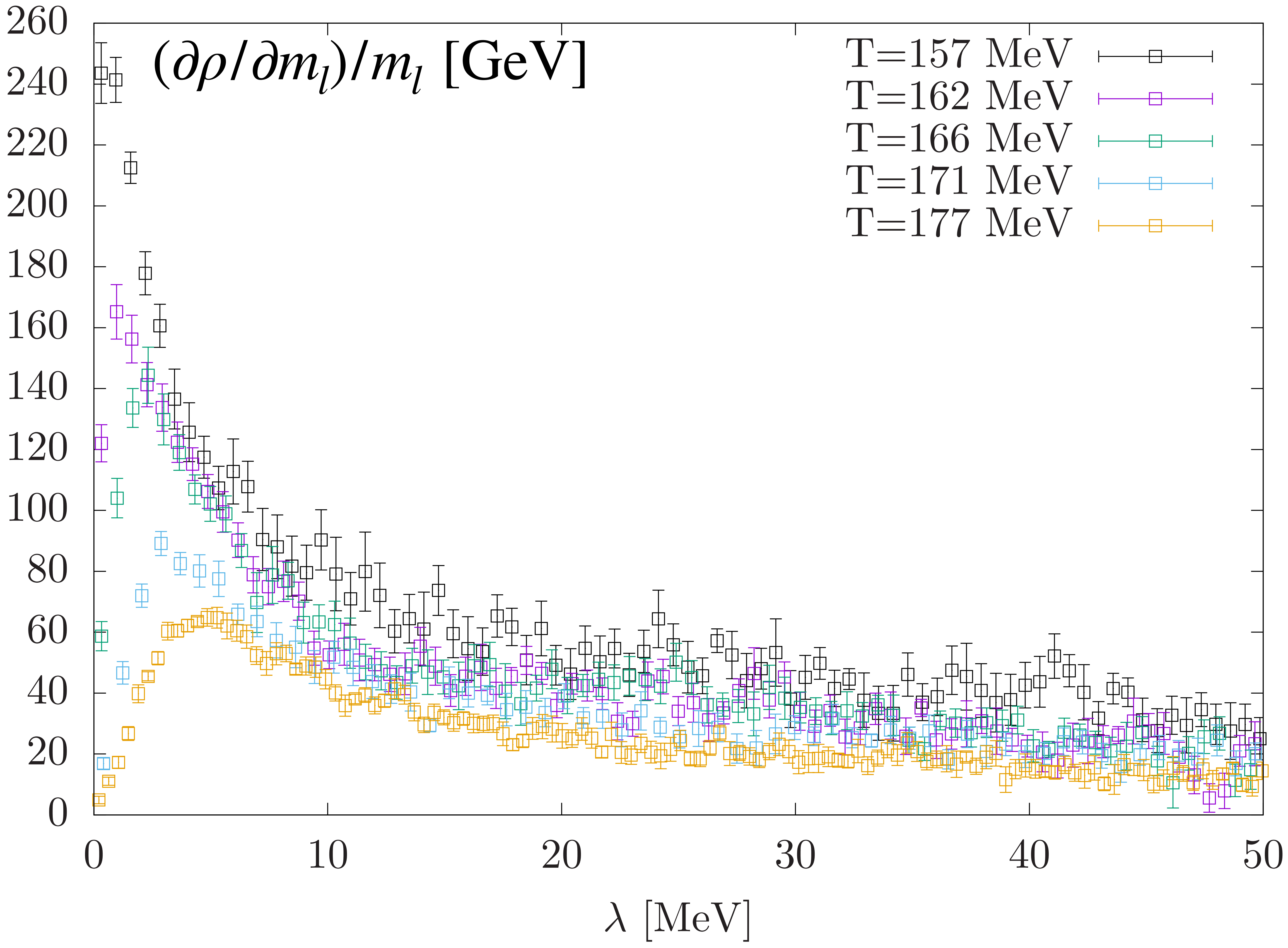
Reproduction of chiral observables via ρ and $\partial\rho/\partial m_l$

$$\langle\bar{\psi}\psi\rangle_l = \int_0^\infty \frac{4m_l \cdot \rho(\lambda, m_l)}{\lambda^2 + m_l^2}\, \mathrm{d}\lambda \qquad \chi_{\mathrm{disc}} = \int_0^\infty \mathrm{d}\lambda \frac{4m_l \cdot \partial\rho/\partial m_l}{\lambda^2 + m_l^2}$$

Direct measurement: calculations via inversions of the fermion matrix

1st order quark mass derivative of ρ



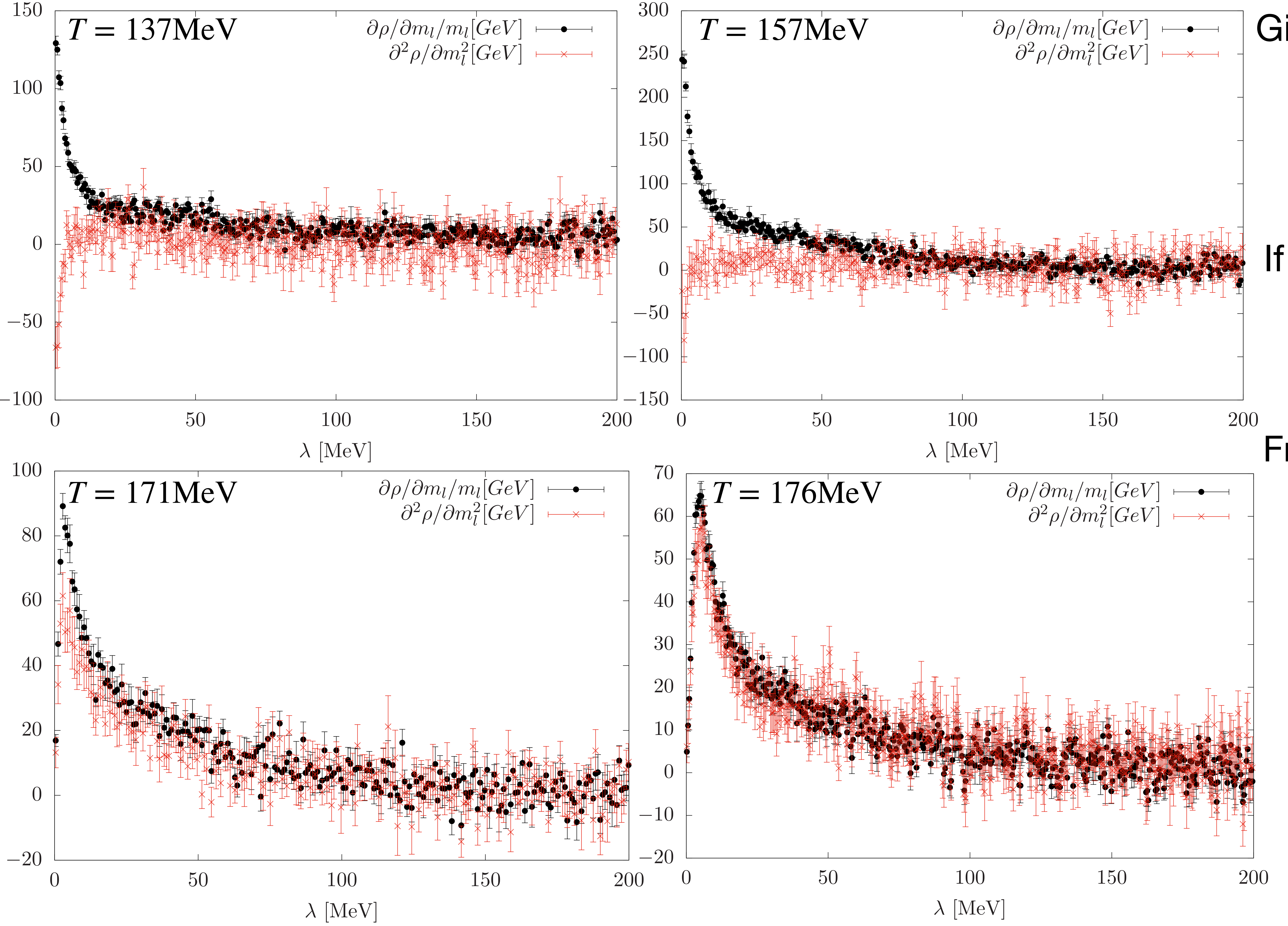


At $T \lesssim 153$ MeV , $T \uparrow \ \dfrac{\partial\rho}{\partial m_l} \uparrow$

At $T \gtrsim 157$ MeV , $T \uparrow \ \dfrac{\partial\rho}{\partial m_l} \downarrow$

T dependence of $\partial\rho/\partial m_l(\lambda \approx 0)$ is consistent with $\chi_{\rm disc}$

2nd order quark mass derivative of ρ



Given $\rho \propto m_l^c$,

$$\frac{\partial\rho/\partial m_l}{m_l} \propto c m_l^{c-2} \ , \quad \frac{\partial^2\rho}{\partial m_l^2} \propto c(c-1)m_l^{c-2}$$

If $c = 2$,

$$\frac{\partial\rho/\partial m_l}{m_l} \approx \frac{\partial^2\rho}{\partial m_l^2}$$

From data, for T around $T_c \approx 143$MeV,

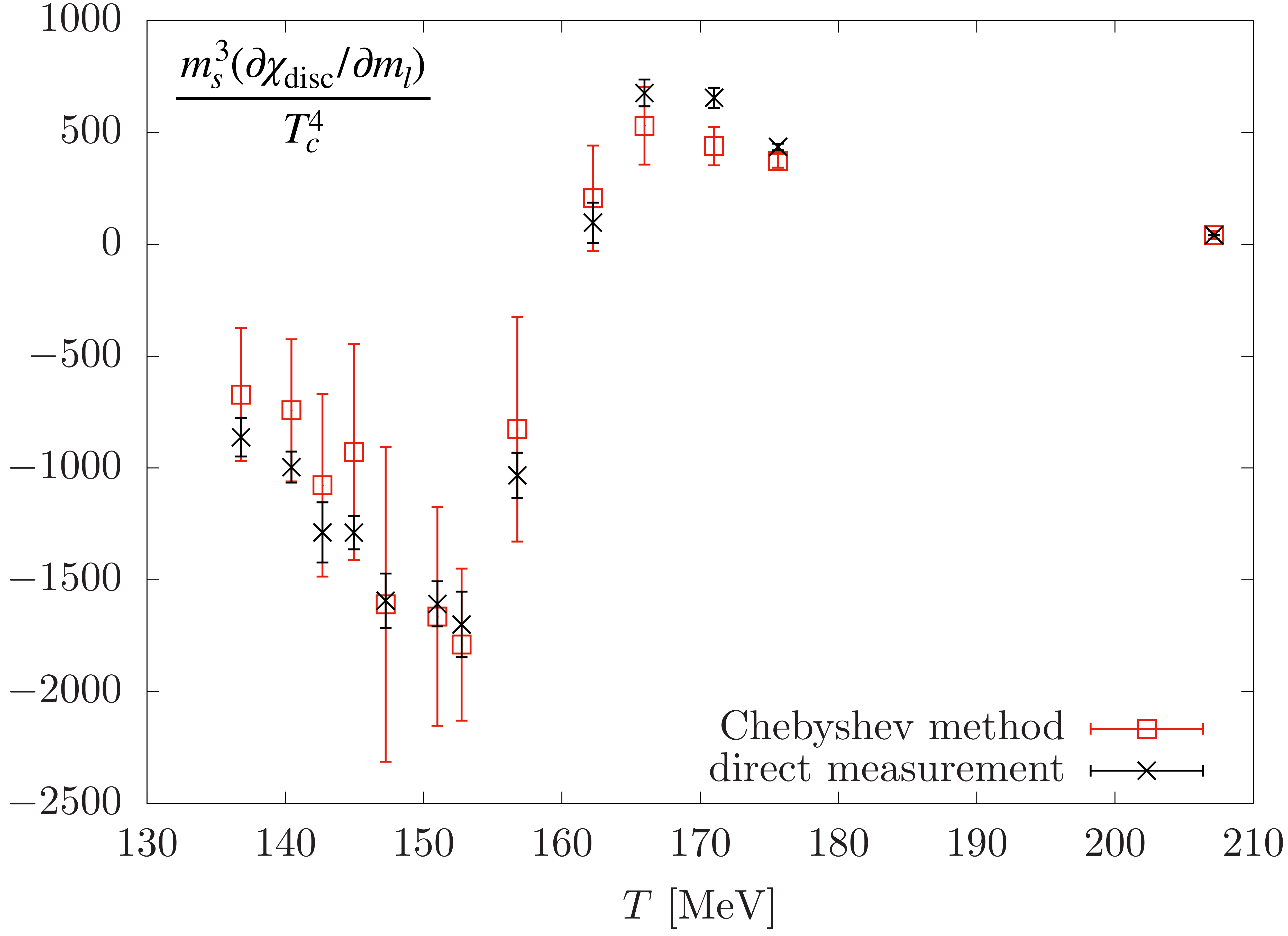
$$c \neq 2 \ \& \ c \in (0,1)$$

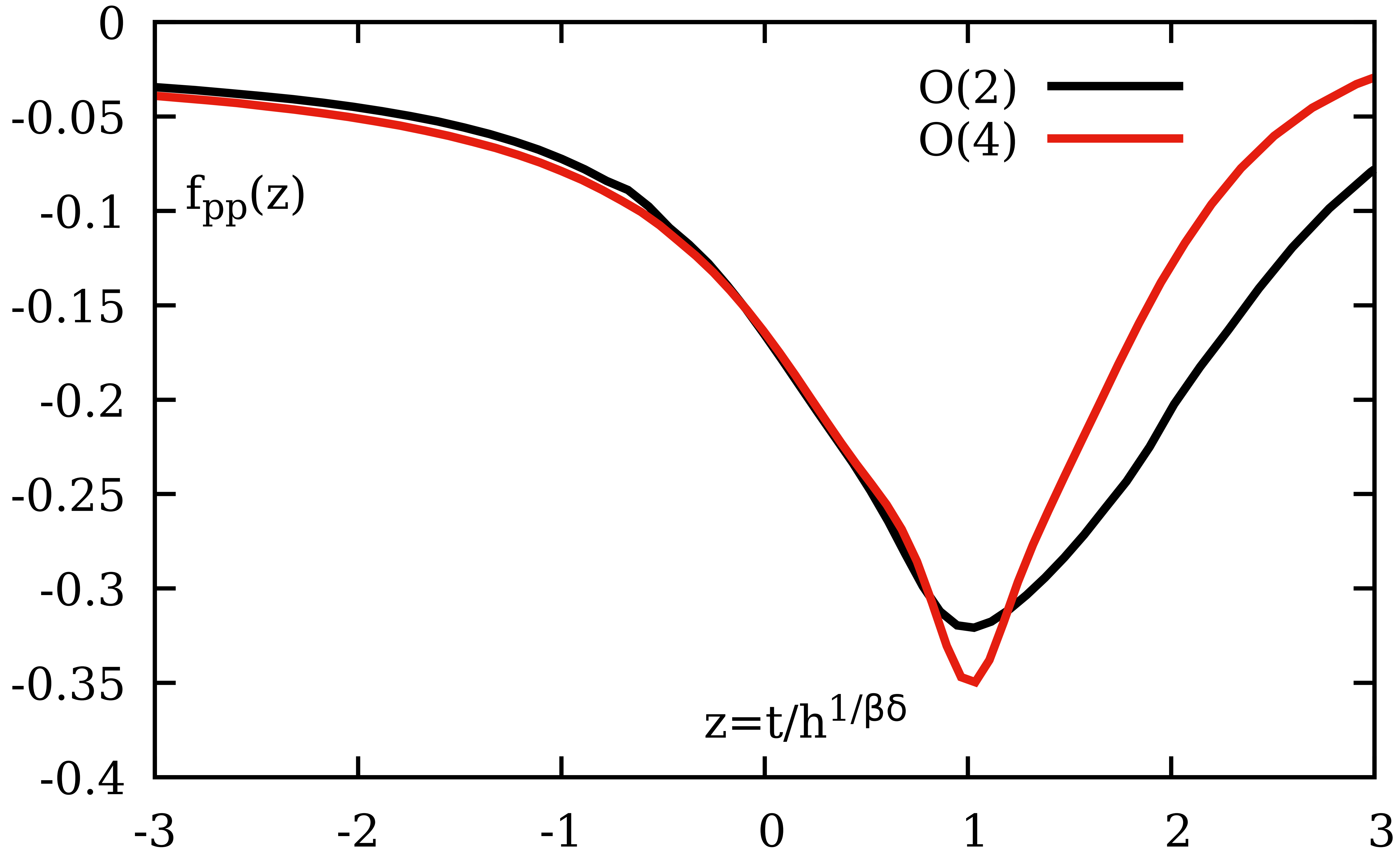
Dilute instanton gas picture is not valid as getting closer to T_c

Temperature dependence of $\partial\chi_{\rm disc}/\partial m_l$

$$\frac{\partial\chi_{\rm disc}}{\partial m_l} = \int_0^\infty {\rm d}\lambda \frac{4m_l\partial^2\rho/\partial m_l^2}{\lambda^2+m_l^2} + \int_0^\infty {\rm d}\lambda \frac{4(\lambda^2-m_l^2)\partial\rho/\partial m_l}{(\lambda^2+m_l^2)^2}$$

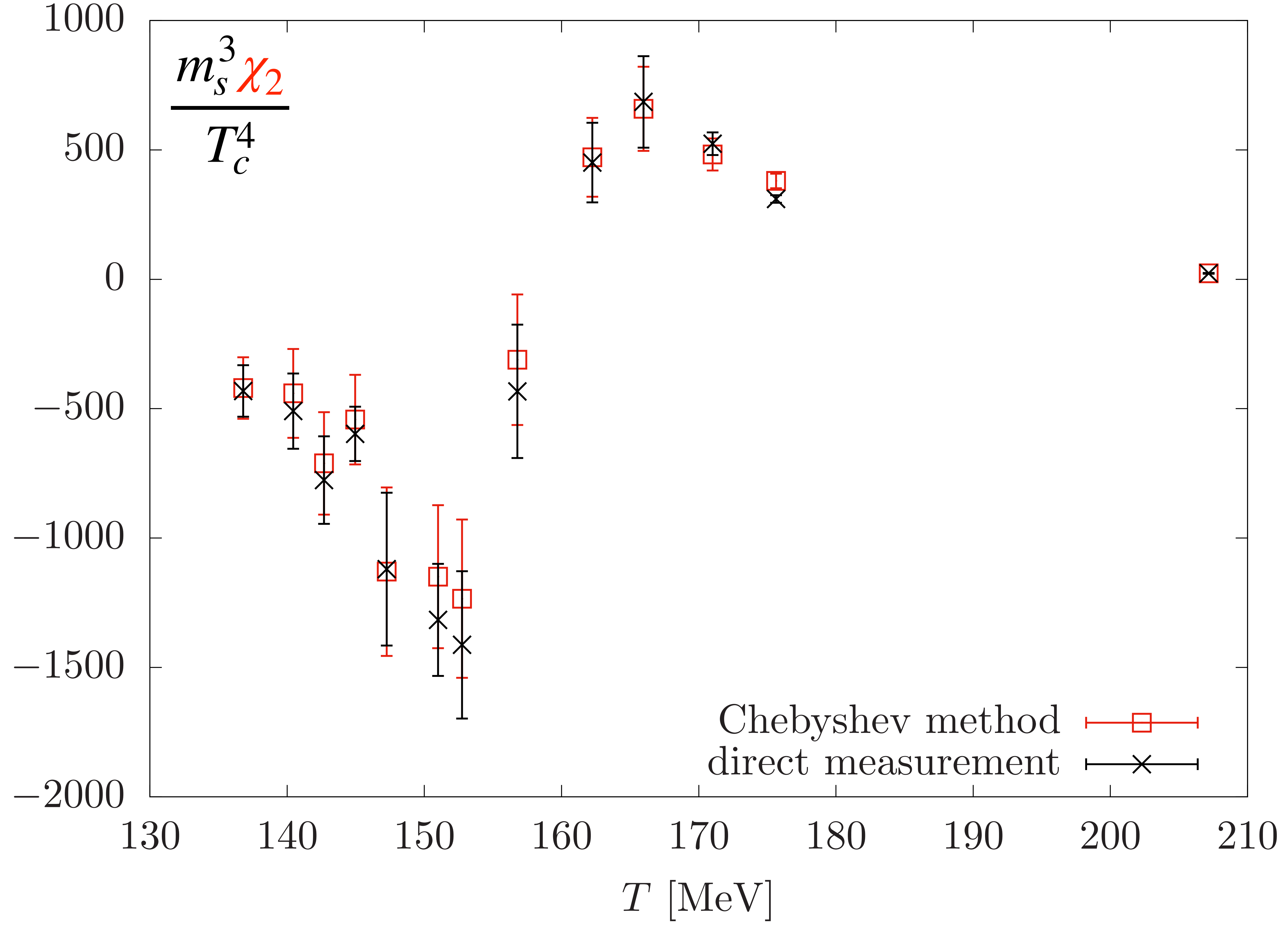
$$\chi'_M = \frac{1}{h_0^2}\frac{1}{\delta}h^{1/\delta-2}\overbrace{\left[(1/\delta-1)f_G(z) + \left(\frac{z}{\beta}+\frac{z}{\beta^2\delta}-2\cdot\frac{z}{\beta\delta}\right)f'_G(z) + \frac{z^2}{\beta^2\delta}f''_G(z)\right]}^{f_{pp}(z)}$$

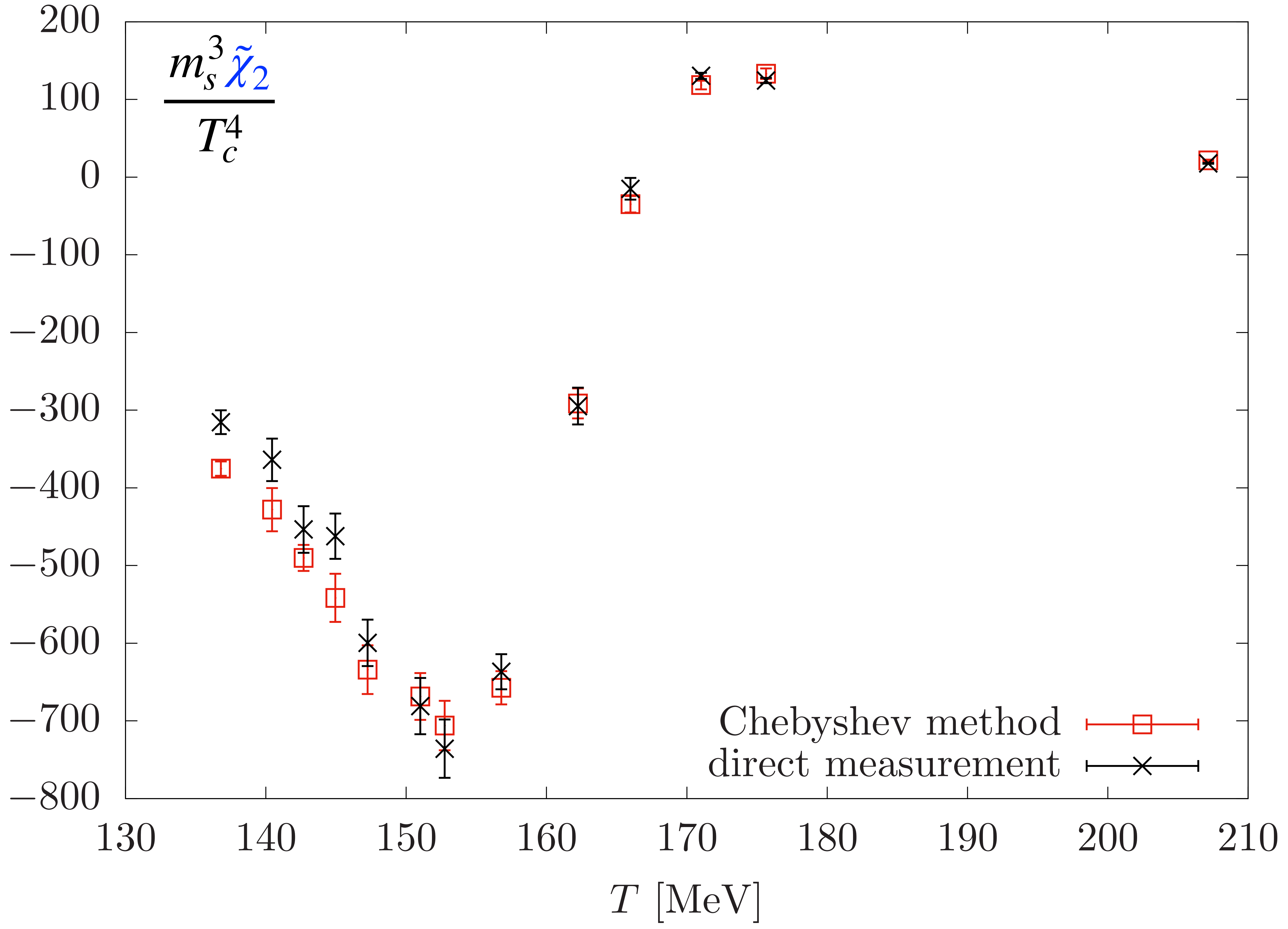




Temperature dependence of $\partial\chi_{\rm disc}/\partial m_l$

$$\frac{\partial\chi_{\rm disc}}{\partial m_l} = \underbrace{\int_0^\infty d\lambda \frac{4m_l \partial^2\rho/\partial m_l^2}{\lambda^2 + m_l^2}}_{\equiv \chi_2} + \underbrace{\int_0^\infty d\lambda \frac{4(\lambda^2 - m_l^2)\partial\rho/\partial m_l}{(\lambda^2 + m_l^2)^2}}_{\equiv \tilde{\chi}_2}$$



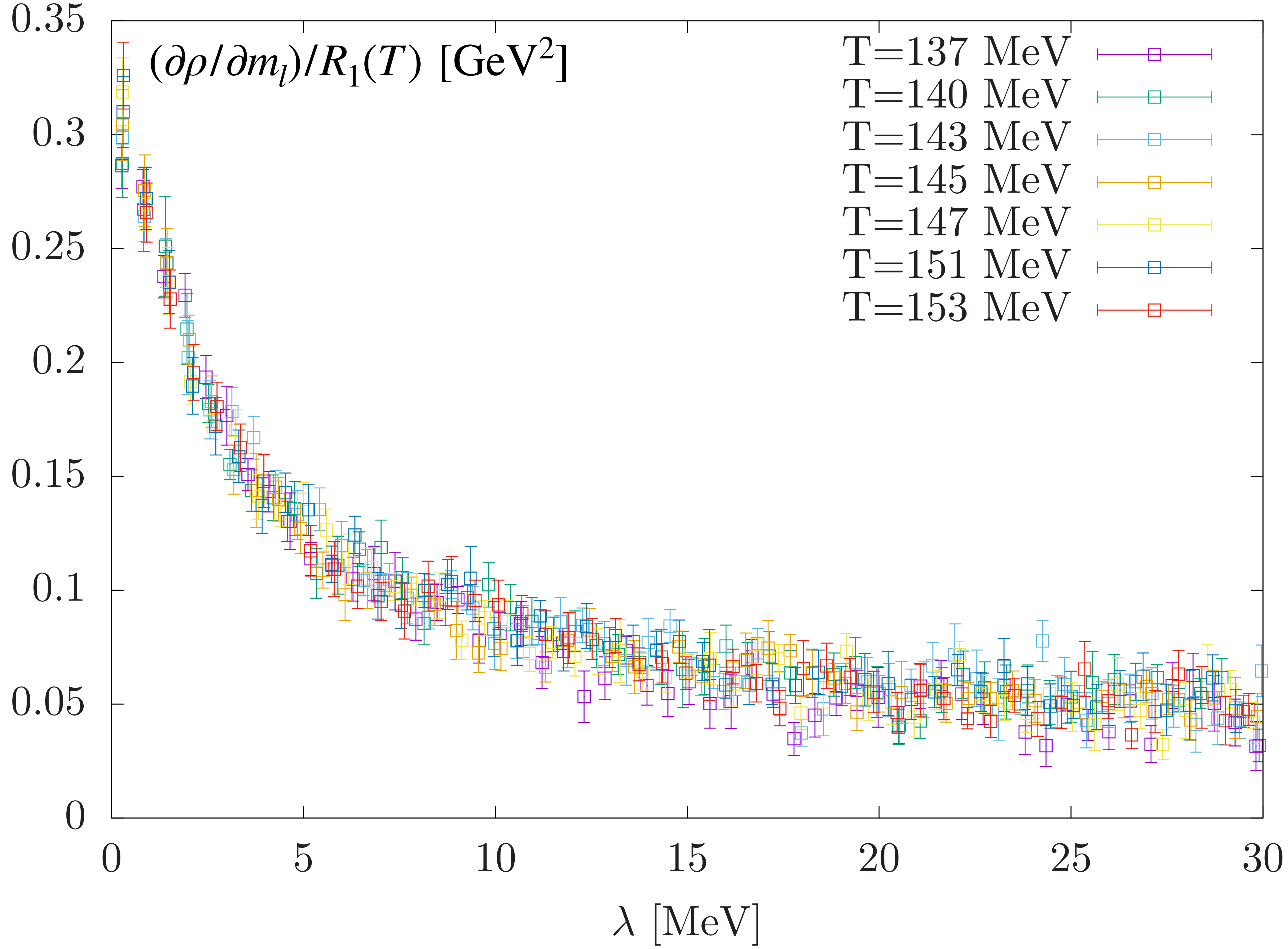


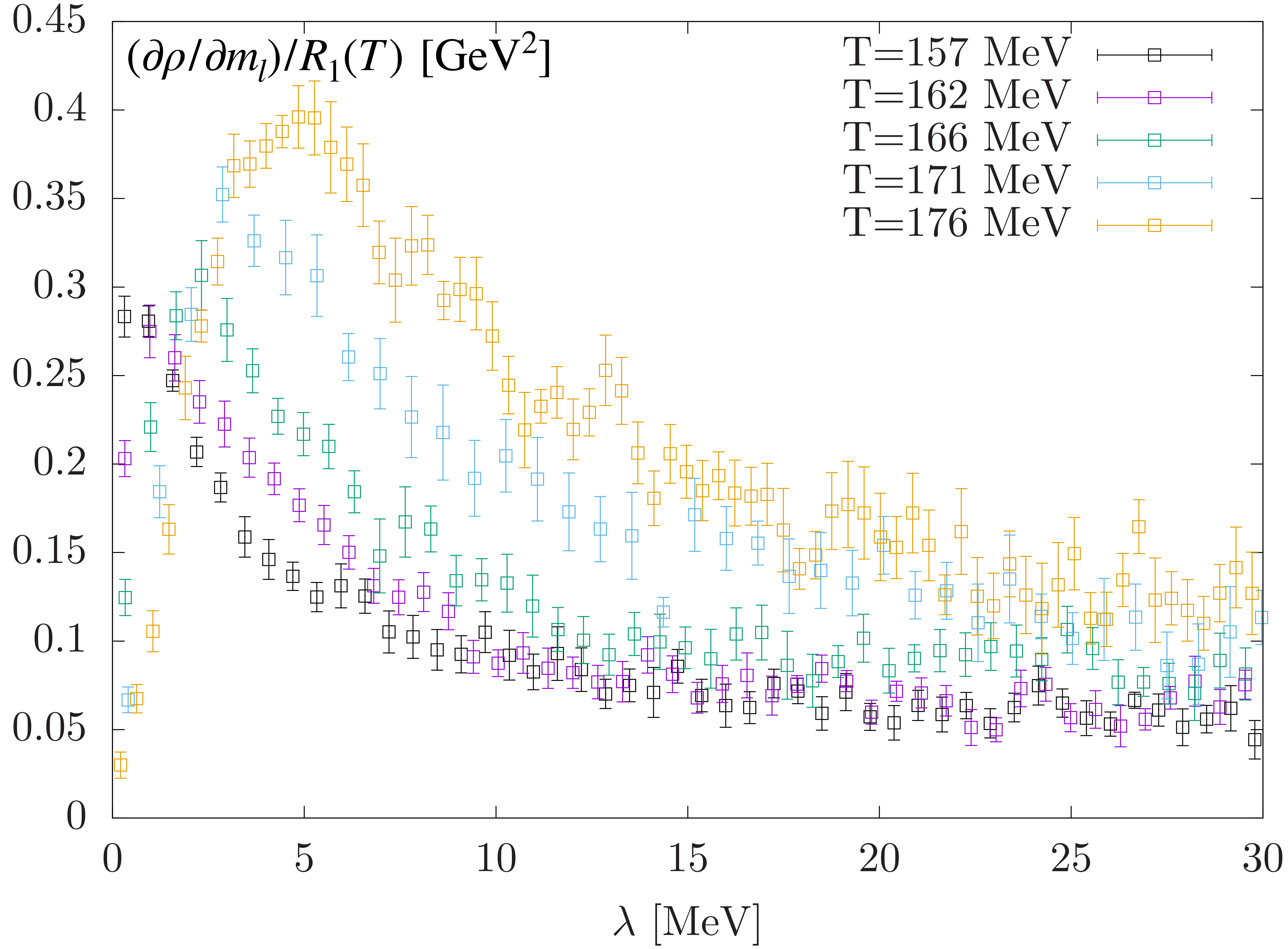
Temperature dependence of $\partial\rho/\partial m_l$

assume

$$\chi_{\rm disc}(T) = \int_0^\infty d\lambda \frac{4m_l \cdot \partial\rho/\partial m_l}{\lambda^2 + m_l^2} = \int_0^\infty d\lambda \frac{4m_l \cdot f_1(T)\cdot g_1(\lambda, m_l)}{\lambda^2+m_l^2} \Longrightarrow R_1(T) \equiv \frac{\chi_{\rm disc}(T)}{\chi_{\rm disc}(T_0)} = \frac{f_1(T)}{f_1(T_0)}$$

$$\Longrightarrow \frac{\partial\rho/\partial m_l}{R_1(T)} = f_1(T_0) \times g_1(\lambda, m_l) \text{ (no } T \text{ dependence here)}$$



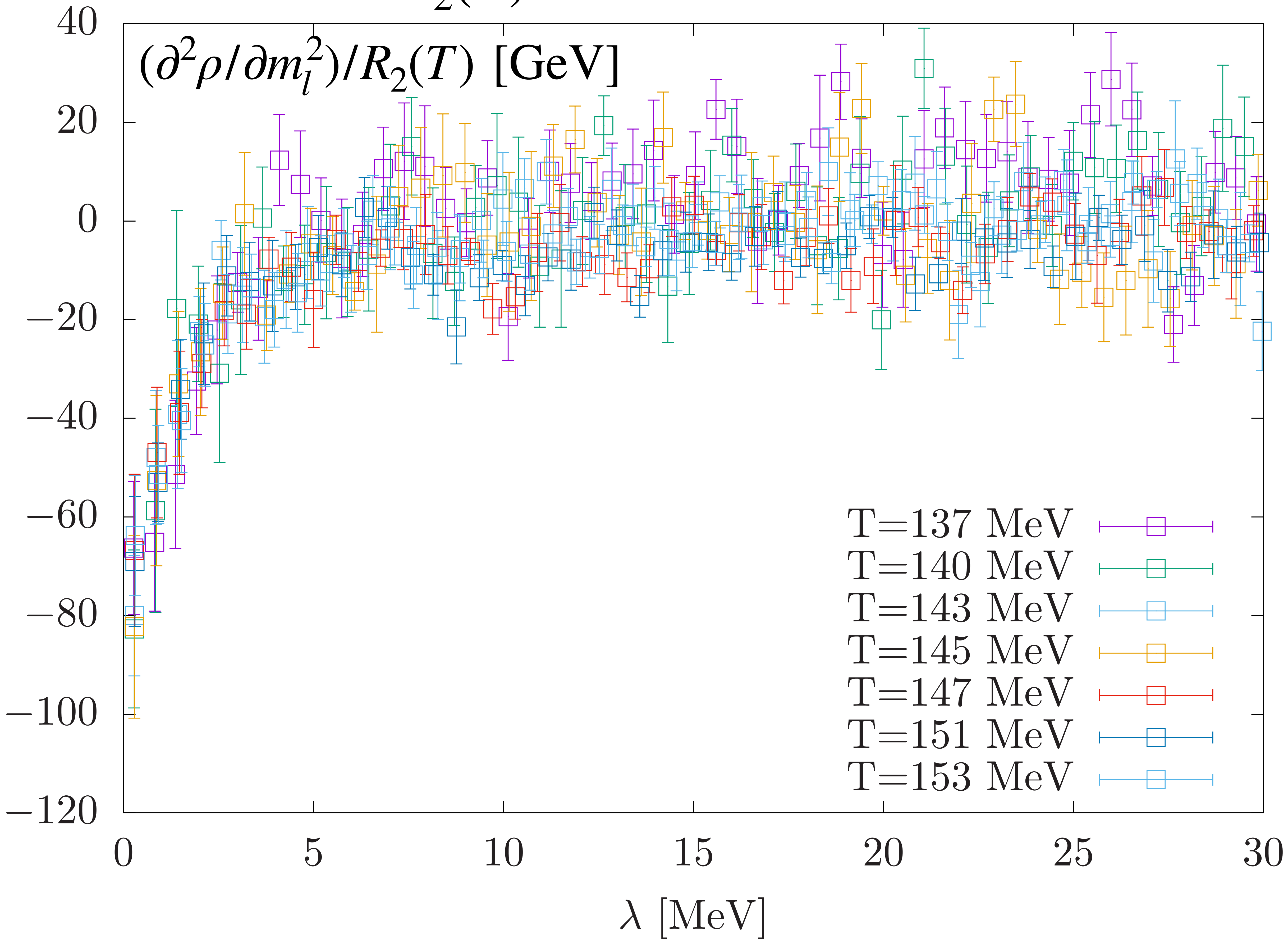


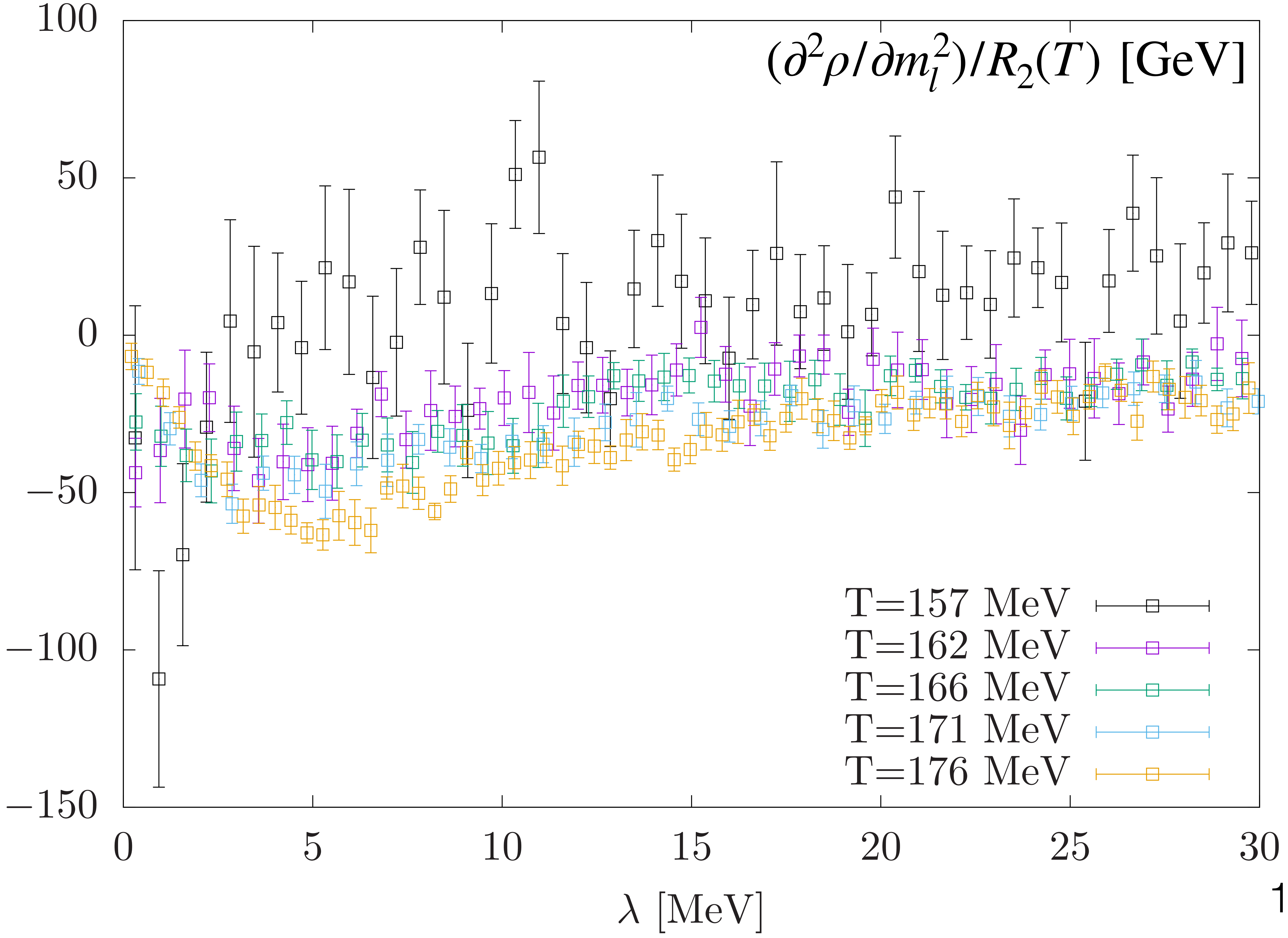
Temperature dependence of $\partial^2\rho/\partial m_l^2$

assume

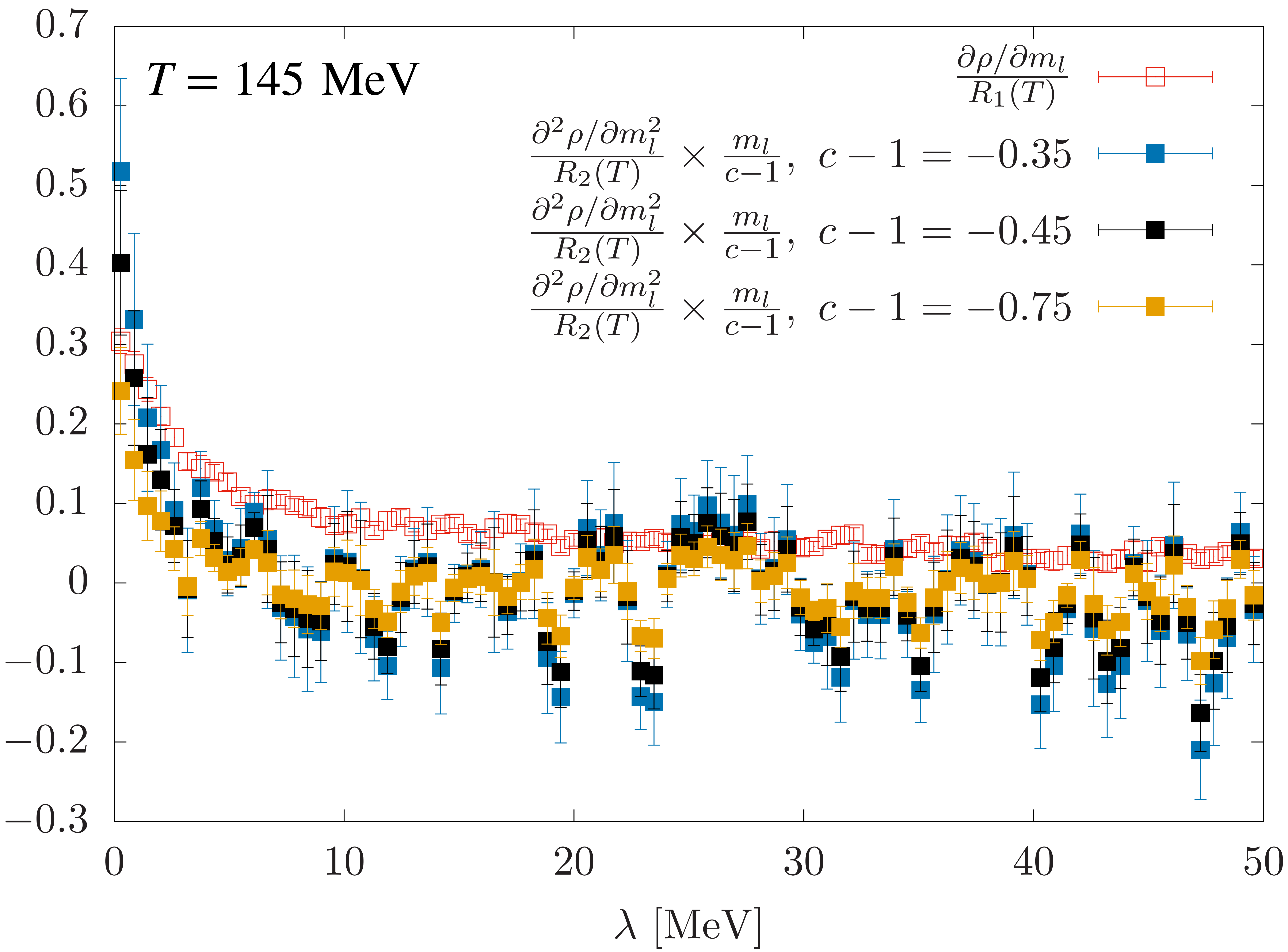
$$\chi_2(T) = \int_0^\infty \mathrm{d}\lambda \frac{4m_l \cdot \partial^2\rho/\partial m_l^2}{\lambda^2 + m_l^2} = \int_0^\infty \mathrm{d}\lambda \frac{4m_l \cdot f_2(T)\cdot g_2(\lambda, m_l)}{\lambda^2 + m_l^2} \Longrightarrow R_2(T) \equiv \frac{\chi_2(T)}{\chi_2(T_0)} = \frac{f_2(T)}{f_2(T_0)}$$

$$\Longrightarrow \frac{\partial^2\rho/\partial m_l^2}{R_2(T)} = f_2(T_0) \times g_2(\lambda, m_l) \text{ (no } T \text{ dependence here)}$$





Mass dependence of ρ : estimate c assuming $\rho \propto m_l^c$



Given $\rho \propto f_0(T) \times m_l^c$,

$$\frac{\rho}{R_0(T)} \propto m_l^c$$

$$\frac{\partial\rho/\partial m_l}{R_1(T)} \propto c \cdot m_l^{c-1}$$

$$\frac{\partial^2\rho/\partial m_l^2}{R_2(T)} \propto c \cdot (c-1) m_l^{c-2}$$

c can be estimated by comparing

$$\frac{\partial\rho/\partial m_l}{R_1(T)} \quad \text{v.s.} \quad \frac{\partial^2\rho/\partial m_l^2}{R_2(T)} \times \frac{m_l}{c-1}$$

Summary

Based on HISQ configurations $40^3 \times 8$ lattices with $M_\pi = 110\ \mathrm{MeV}$,

- ☑ Sign change and non-monotonous behavior are found in $\partial\chi_{\mathrm{disc}}/\partial m_l$
- ☑ Chiral observables can be reproduced by ρ, $\partial\rho/\partial m_l$ and $\partial^2\rho/\partial m_l^2$
- ☑ As T approaches to T_c, the m_l^2 behavior in ρ does not exist any more
- ☑ At $T \lesssim 153$ MeV, it seems that the T dependence can be factored out in the $\partial\rho/\partial m_l$ and $\partial^2\rho/\partial m_l^2$
- ☑ c in $\rho \propto m_l^c$ can be estimated by comparing $(\partial\rho/\partial m_l)/R_1(T)$ and $(\partial^2\rho/\partial m_l^2)/R_2(T) \times m_l/(c-1)$

Outlook

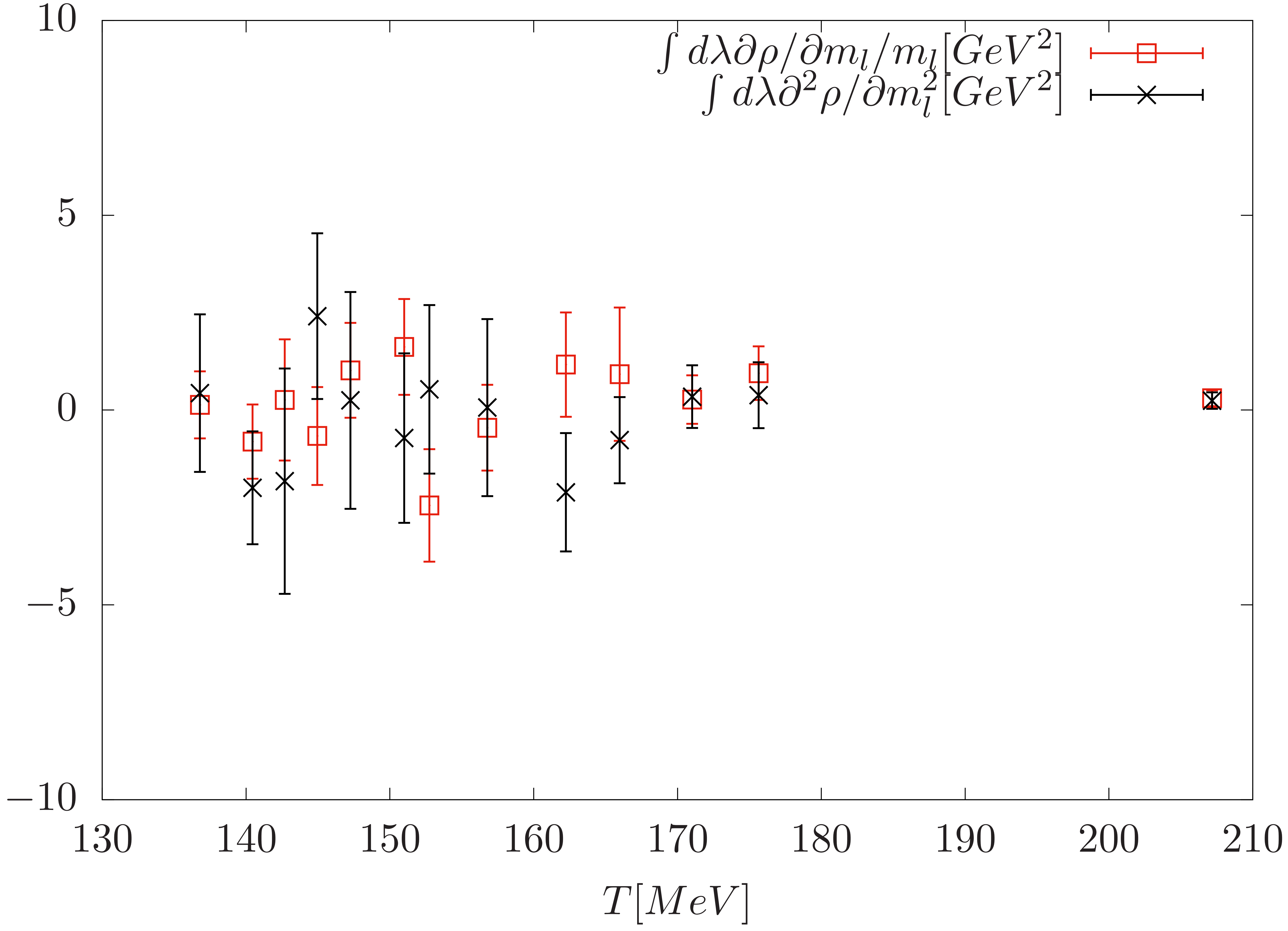
- ☐ Check the mass dependence of ρ, $\partial\rho/\partial m_l$, $\partial^2\rho/\partial m_l^2$ with different values of pion masses
- ☐ Check the scaling behavior in the Dirac eigenvalue correlations in detail

Backup

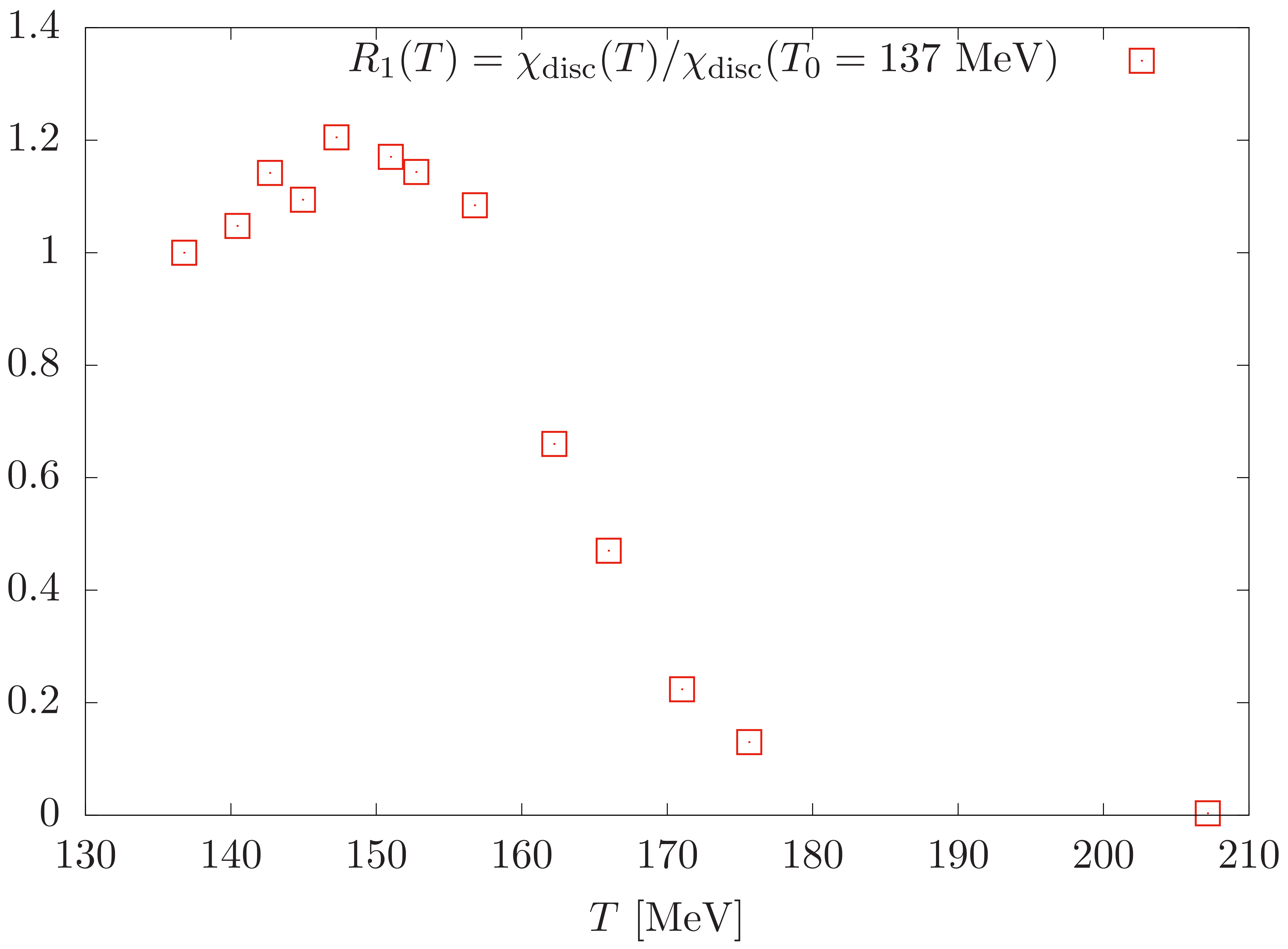
Sanity Check of $\partial\rho/\partial m_l$ and $\partial^2\rho/\partial m_l^2$

$$\int_0^\infty d\lambda_i \; C_n\left(\lambda_1, \lambda_2, \cdots, \lambda_i, \cdots, \lambda_n; m_l\right) = 0 \;, i = 1, \cdots, n$$

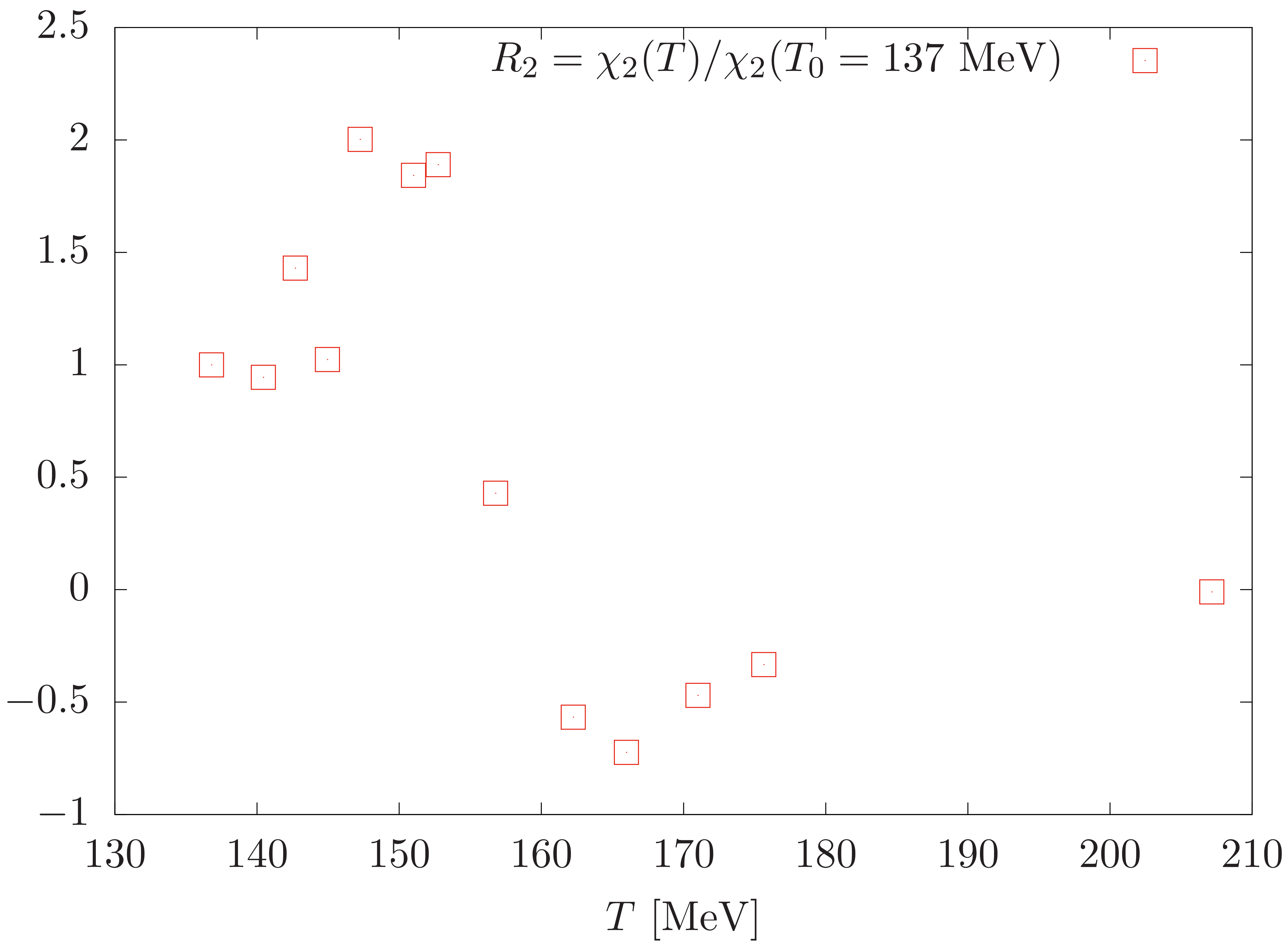
$$\Longrightarrow \int_0^\infty d\lambda \; \frac{\partial\rho}{\partial m_l} = \frac{T}{V}\int_0^\infty d\lambda_1 \; \int_0^\infty \mathrm{d}\lambda_2 \; \frac{4m_l C_2(\lambda_1, \lambda_2; m_l)}{\lambda_2^2 + m_l^2} = \frac{T}{V}\int_0^\infty d\lambda_2 \; \frac{4m_l}{\lambda_2^2 + m_l^2} \; \int_0^\infty \mathrm{d}\lambda_1 \; C_2(\lambda_1, \lambda_2; m_l) = 0$$



$R_n(T)$ v.s. T, $n = 1,2$



$R_1(T) = \chi_{\rm disc}(T)/\chi_{\rm disc}(T_0 = 137\ {\rm MeV})$
1.4
1.2
1
0.8
0.6
0.4
0.2
0
130
140
150
160
170
180
190
200
210
T [MeV]



$R_2 = \chi_2(T)/\chi_2(T_0 = 137\ {\rm MeV})$
2.5
2
1.5
1
0.5
0
−0.5
−1
130
140
150
160
170
180
190
200
210
T [MeV]