Thermal QCD with external imaginary electric fields on the lattice

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Strong-interaction matter under extreme conditions

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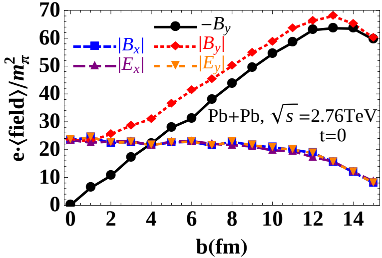
Outline

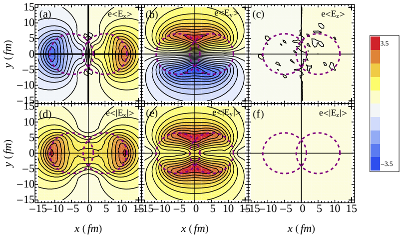
- introduction
- equilibrium with electric fields?
- issues at finite temperature
- Taylor expansion: susceptibility via vacuum polarization
- preliminary results
- summary

Introduction

Electric fields

- electromagnetic fields in the early stage of heavy-ion collisions
  Deng et al. '12

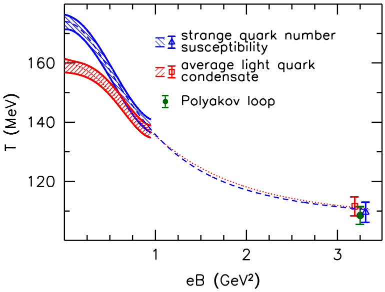




- impact of electric field enhanced for asymmetric systems (for example Cu+Au at RHIC) Voronyuk et al. '14

Magnetic/electric fields

- effect of magnetic fields on QCD thermodynamics well understood Bali et al. '11 D'Elia et al. '11 Endrődi '15



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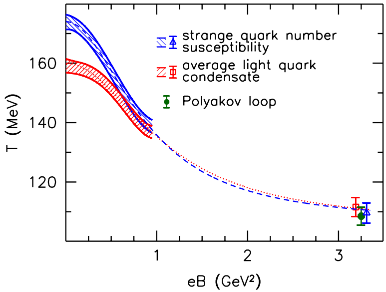


- effect of electric fields:
  NJL model calculations 🔗 Suganuma et al. '91 🔗 Babansky et al. '98
  lattice QCD with opposite charges 🔗 Yamamoto '12
  electric polarizability of hadrons 🔗 Engelhardt et al. '07
  🔗 Lujan et al. '14 🔗 Niyazi et al. '21
- here: electric susceptibility

Setup

Euclidean electric fields

- covariant derivative (electric charge q)

$$D_\mu = \partial_\mu + iqA_\mu + i\mathcal{A}_\mu^{\rm gluon}$$

- Minkowskian electric field

$$E_i = F_{i0} = \partial_i A_0 - \partial_0 A_i$$

- Wick rotation

$$\partial_0 \to i\partial_4, \qquad A_0 \to iA_4$$

- Euclidean electric field

$$E_i^{\rm Eucl} = \partial_i A_4 - \partial_4 A_i \equiv -iE_i$$

Sign problem

- consider constant $\mathbf{E} = E\,\mathbf{e_1}$ in a static gauge

$$A_4 = -iE\,x_1$$

- electromagnetic links

$$u_4 = \exp\left[iaqA_4\right] = \exp\left[aqEx_1\right]$$

- hermiticity relation

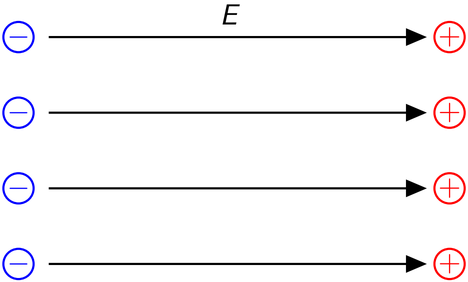
$$\not{D}^\dagger(E) = -\not{D}(-E^*)$$

- determinant

$$\det \not{D}(E) \in \mathbb{C}, \qquad \det \not{D}(E^{\rm Eucl}) \in \mathbb{R}$$

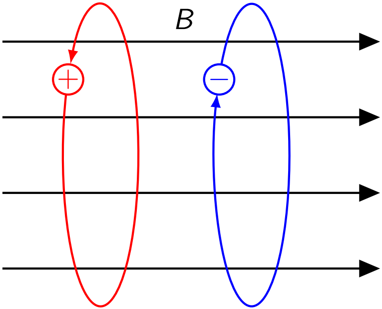
Sign problem

- electric fields cause a sign problem



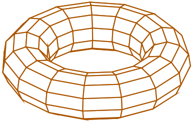
Sign problem

- electric fields cause a sign problem
- magnetic fields cause no sign problem



Flux quantization

- in a periodic volume



- flux of field necessarily quantized 🔗 't Hooft '79 🔗 D'Elia et al. '11

  magnetic field $\mathbf{B} \parallel \hat{z}$:

$$qB \cdot L_1 L_2 = 2\pi N_B, \qquad N_B \in \mathbb{Z}$$

  imaginary electric field $i\mathbf{E} \parallel \hat{x}$:

$$iqE \cdot L_1 L_4 = 2\pi N_E, \qquad N_E \in \mathbb{Z}$$

Flux quantization

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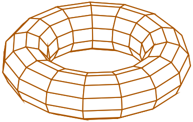

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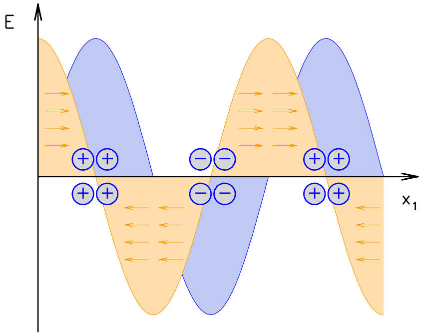
- E is discrete variable; naive Taylor expansion not applicable

Issues: equilibrium

Equilibrium

- constant E leads system out of equilibrium
- instead: $E(x) = E \cdot \cos(p_1 x_1)$ with $p_1 = 2\pi n / L_1$, $n \in \mathbb{Z}^+$

  equilibrium exists



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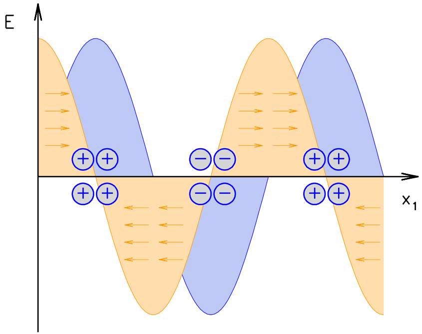

- interpretation: polarization described by electric permittivity

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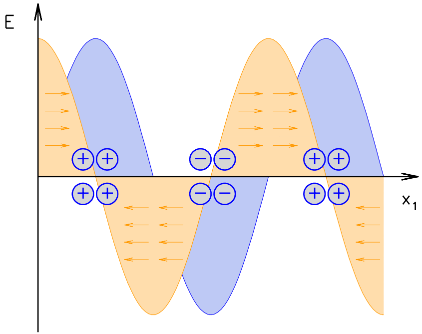


- interpretation: polarization described by electric permittivity
- take thermodynamic limit, then $p_1 \to 0$ limit

Issues: nonzero temperature

Gauge transformations

- one flavor for simplicity
- consider some nonzero temperature
- at $E = 0$ the partition function depends on μ

$$\log \mathcal{Z}(E = 0, \mu)$$

- including $E > 0$ implies gauging the symmetry
  gauge freedom:

  $$A_0 = E\, x_1 \quad \leftrightarrow \quad A_0 = E\, x_1 + \mu$$

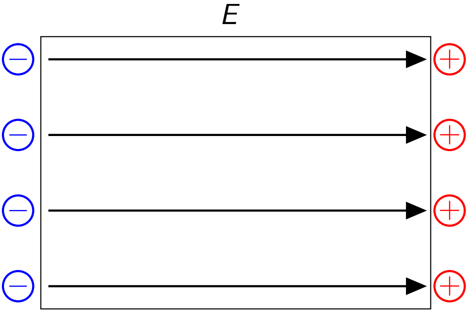
  therefore

  $$\log \mathcal{Z}(E > 0, \not{\mu})$$

- similarly, no $i\mu$ dependence at $iE > 0$

How to compare $E = 0$ and $E > 0$?

- at $E = 0$ thermodynamic variable: μ
- at $E > 0$ integration over μ is automatic
  $\rightsquigarrow$ forces zero density $N = 0$



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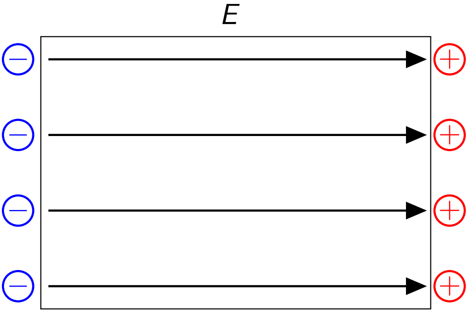
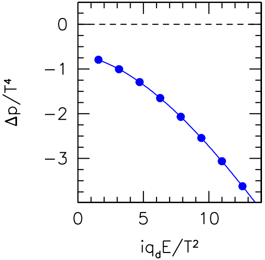


- we need to project to $N = 0$ already at $E = 0$

How to compare $E = 0$ and $E > 0$?

- explicitly calculating $\log\det[\not{D} + m]$ in the free case



$iE > 0$

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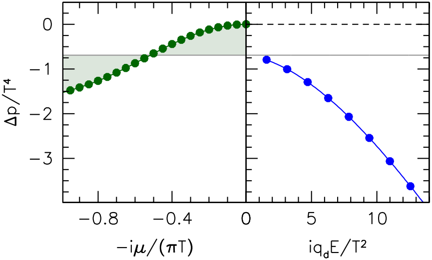

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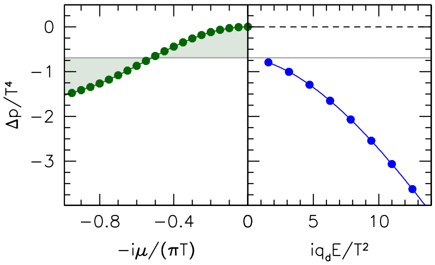

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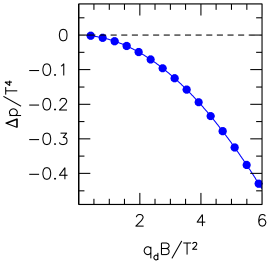

$B > 0$

How to compare $E = 0$ and $E > 0$?

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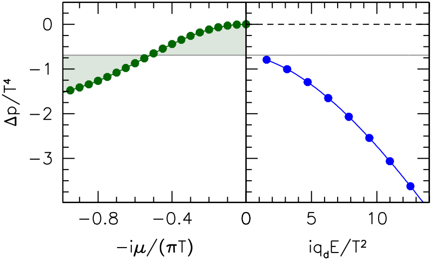

$iE > 0$

- mismatch term in $\partial^2 \log \mathcal{Z}/\partial E^2$ is

$$\propto \frac{T^4}{E_{\rm min}^2} = \frac{T^4}{(2\pi T/L)^2} \propto \boxed{L^2 T^2}$$

Taylor expansion approach

Taylor expansion

- approach constant iE via oscillatory fields
  see same approach for B Bali, Endrődi, Piemonte '20
- magnetic field case

$$\chi = \left.\frac{\partial^2 \log \mathcal{Z}}{\partial B^2}\right|_{B=0} = \int_0^{L/2} \mathrm{d}x_1\, x_1^2 G_{22}(x_1), \quad G_{\mu\nu}(x_1) = \int \mathrm{d}x_2 \mathrm{d}x_3 \mathrm{d}x_4\, \langle j_\mu(x) j_\nu(0)\rangle$$

- imaginary electric field case

$$\xi = \left.\frac{\partial^2 \log \mathcal{Z}}{\partial (iE)^2}\right|_{E=0} = \int_0^{L/2} \mathrm{d}x_1\, x_1^2 G_{44}(x_1) - \frac{2}{L}\int_0^{L/2} \mathrm{d}x_1 x_1^2 \int \mathrm{d}y_1\, G_{44}(y_1)$$

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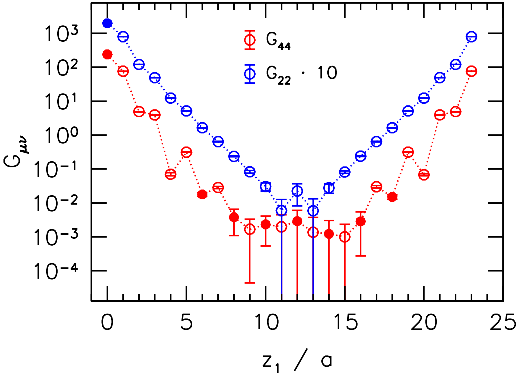
- mismatch term $\propto \boxed{L^2 T^2}$ containing

$$c = \left.\frac{\partial^2 \log \mathcal{Z}}{\partial (\mu/T)^2}\right|_{\mu=0}$$

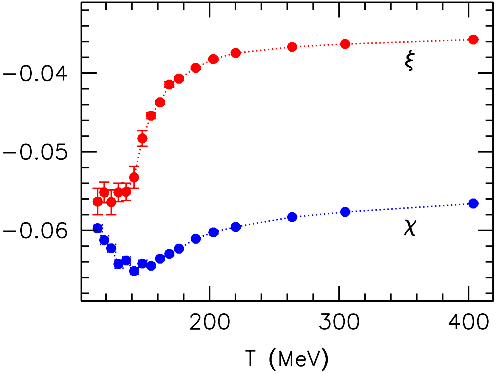
Results

Correlators



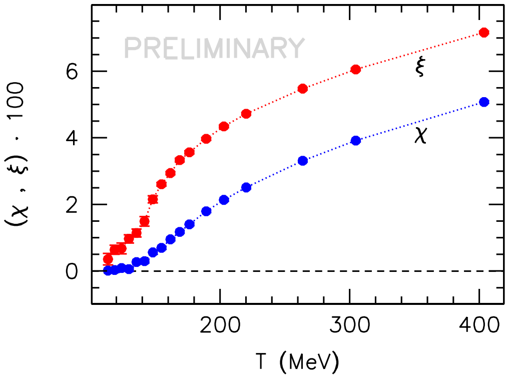
- on $24^3 \times 6$ lattice, at temperature $T = 176$ MeV
- connected as well as disconnected contributions included
- filled (empty) points: positive (negative)

Bare susceptibilities



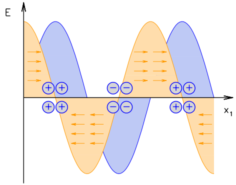
Renormalized susceptibilities

- divergences cancel in $\chi_r = \chi(T) - \chi(T=0)$ and $\xi_r = \xi(T) - \chi(T=0)$
- $\chi(T=0, a)$ determined in Bali, Endrődi, Piemonte '20



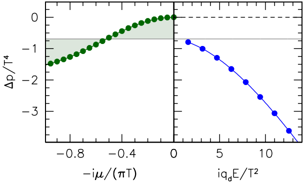
Summary

- equilibrium exists for oscillatory electric fields



- mismatch when comparing $E > 0$ and $E = 0$

  $\rightsquigarrow$ impact for hadron polarizabilities away from $T = 0$



- renormalization by $T = 0$ subtraction