Study of the EoS of dense QCD

in an external magnetic field

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Outline

Introduction

Extreme conditions:

- high T
- $\mu_B \neq 0$
- large eB.

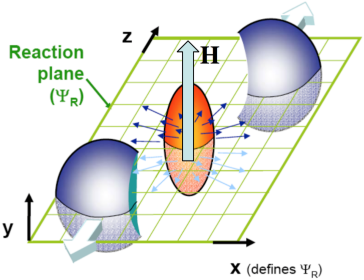
There is success in study of EoS

- at finite density

  D. Bollweg et al., Nucl. Phys. A 1005, 121835 (2021) [arXiv:2002.01837 [hep-lat]]; others

- at nonzero magnetic field

  M. D'Elia et al., Phys. Rev. D 98, no.5, 054509 (2018) [arXiv:1808.07008 [hep-lat]]; others



$eB \sim 0.1$ GeV^2

$\Downarrow$

Strongly influences QCD properties, in particular, phase diagram

There are much less investigations at both $\mu_B \neq 0$ and $eB \neq 0$

H. T. Ding et al., Eur. Phys. J. A 57, no.6, 202 (2021) [arXiv:2104.06843 [hep-lat]]

We want to obtain EoS from 2+1 LQCD accounting for both $\mu_B \neq 0$ and $eB \neq 0$

Thermodynamics on the lattice

$$p = -\frac{\Omega}{V} = \frac{T}{V}\ln\mathcal{Z}$$ ⟵ cannot be measured directly

/⋆ $\mathcal{Z} = \int \mathcal{D}[U]\mathcal{D}[\psi,\bar\psi]\,\mathrm{e}^{-S(\psi,\bar\psi,U)}$ ⋆/

Derivatives of p can be measured!

$$n_q = \frac{N_q}{V} = \frac{\partial p}{\partial \mu_q}$$ – quark number density

$$\mu_B = \mu_u + 2\mu_d \qquad n_B = (n_u + n_d + n_s)/3$$
$$\mu_Q = \mu_u - \mu_d \qquad n_Q = (2n_u - n_d - n_s)/3$$
$$\mu_S = \mu_d - \mu_s \qquad n_S = -n_s$$

J. N. Günther et al., Nucl. Phys. A **967**, 720 (2017) [arXiv:1607.02493 [hep-lat]]

$$\frac{p}{T^4} = c_0(T) + c_2(T)\left(\frac{\mu_B}{T}\right)^2 + c_4(T)\left(\frac{\mu_B}{T}\right)^4 + c_6(T)\left(\frac{\mu_B}{T}\right)^6 + \mathcal{O}(\mu_B^8)$$

$$\frac{n}{\mu_B T^2} = \frac{T}{\mu_B}\cdot\frac{d(p/T^4)}{d(\mu_B/T)} = 2c_2 + 4c_4\left(\frac{\mu_B}{T}\right)^2 + 6c_6\left(\frac{\mu_B}{T}\right)^4$$ ← coefficients can be found from fit

(μ_u, μ_d, μ_s are such that $\langle n_S\rangle = 0$, $\langle n_Q\rangle = 0.4\langle n_B\rangle$)

Calculation of c_0:

S. Borsanyi et al., Phys. Lett. B **730**, 99 (2014) [arXiv:1309.5258 [hep-lat]].

G. S. Bali et al., JHEP **08**, 177 (2014) [arXiv:1406.0269 [hep-lat]].

Our choice of chemical potentials:

$$\mu_u = \mu_d = \mu_q; \quad \mu_s = 0$$
$$\Downarrow$$
$$\mu_B = 3\mu_q; \quad \mu_Q = 0; \quad \mu_S = \mu_q$$

Lattice setup

- Tree level improved Symanzik gauge action.
- Staggered $2+1$ fermionic action.
- Stout smearing improvement.
- Imaginary chemical potential: $\mu_q = i\mu_I$.
- External magnetic field:

$$\vec{B} = B\vec{e}_z; \qquad B = \text{const} \qquad\qquad A_y^{\text{ext}} = Bx/2, \quad A_x^{\text{ext}} = -By/2, \quad A_\mu^{\text{ext}} = 0, \;\; \mu = z, t$$

- Splitting of the rooted determinant:

$$\mathcal{Z} = \int \mathcal{D}U \, \mathrm{e}^{-S_G} \left[\det D(B, m_u, q_u)\right]^{\frac{1}{4}} \left[\det D(B, m_d, q_d)\right]^{\frac{1}{4}} \left[\det D(B, m_s, q_s)\right]^{\frac{1}{4}}$$

$$D(n|f) = \frac{1}{2a} \sum_\mu \eta_\mu(n) \left[u_\mu(B, q, n)\, \mathrm{e}^{ia\mu_I \times \delta_{\mu 4}}\, U_\mu(n) \delta_{f, n+\hat{\mu}} - u_\mu^\star(B, q, f)\, \mathrm{e}^{-ia\mu_I \times \delta_{\mu 4}}\, U_\mu^\dagger(f) \delta_{f, n-\hat{\mu}} \right] + m\, \delta_{f,n}$$

$$u_x(B, q, n_x, n_y, n_z, n_t) = \mathrm{e}^{-ia^2 qB\, n_y/2}, \quad n_x \neq N_x - 1, \qquad u_y(B, q, n_x, n_y, n_z, n_t) = \mathrm{e}^{ia^2 qB\, n_x/2}, \quad n_y \neq N_y - 1,$$

$$u_x(B, q, N_x - 1, n_y, n_z, n_t) = \mathrm{e}^{-ia^2 qB(N_x+1) n_y/2}, \qquad u_y(B, q, n_x, N_y - 1, n_z, n_t) = \mathrm{e}^{ia^2 qB(N_y+1) n_x/2}.$$

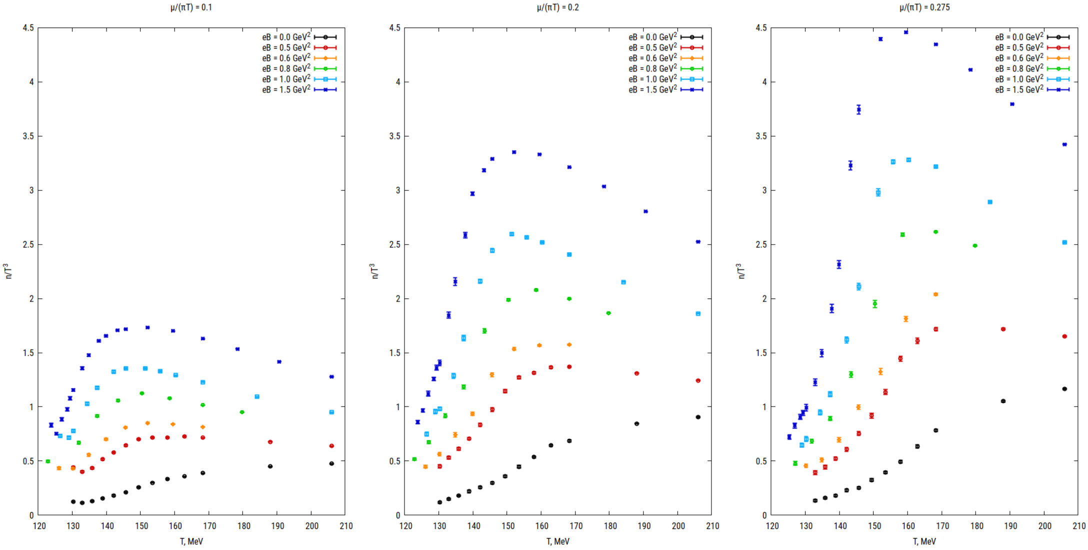
Periodic boundary conditions $\Rightarrow$ $eB = \dfrac{6\pi\, k}{N_x\, N_y\, a^2}, \qquad k \in \mathbb{Z}$

Simulation parameters:

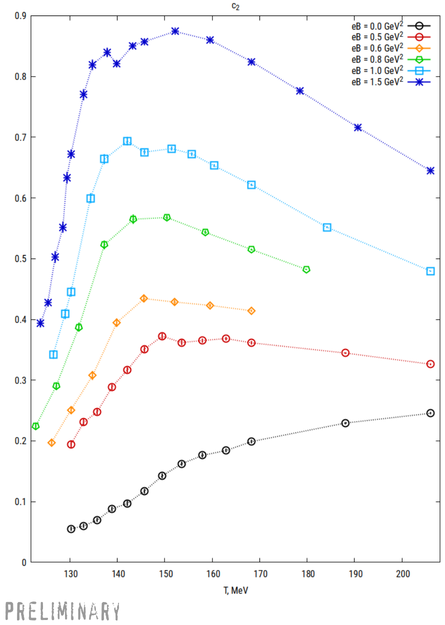
Lattice	6×24^3	8×32^3	10×40^3
eB, GeV^2	0.0, 0.5, 0.6, 0.8, 1.0, 1.5	0.6	0.6
T, MeV	123 – 206	126 – 198	126 – 198

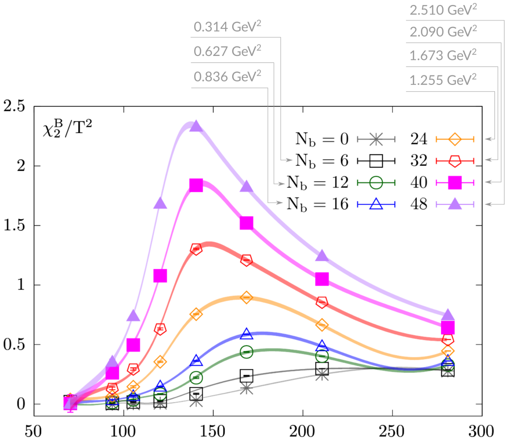
physical quark masses.

Results: densities at 6×24^3 lattice



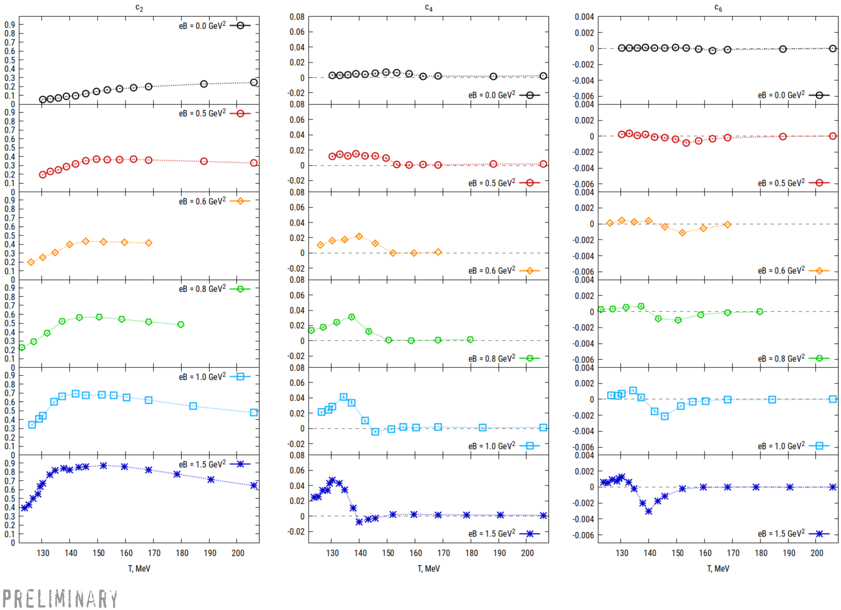
Results: expansion coefficient c_2 at 6×24^3 lattice

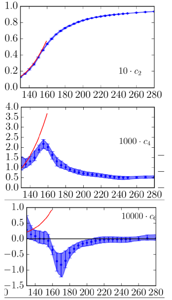




H. T. Ding et al., Eur. Phys. J. A 57, no.6, 202 (2021) [arXiv:2104.06843 [hep-lat]]

Results: expansion coefficients at 6×24^3 lattice

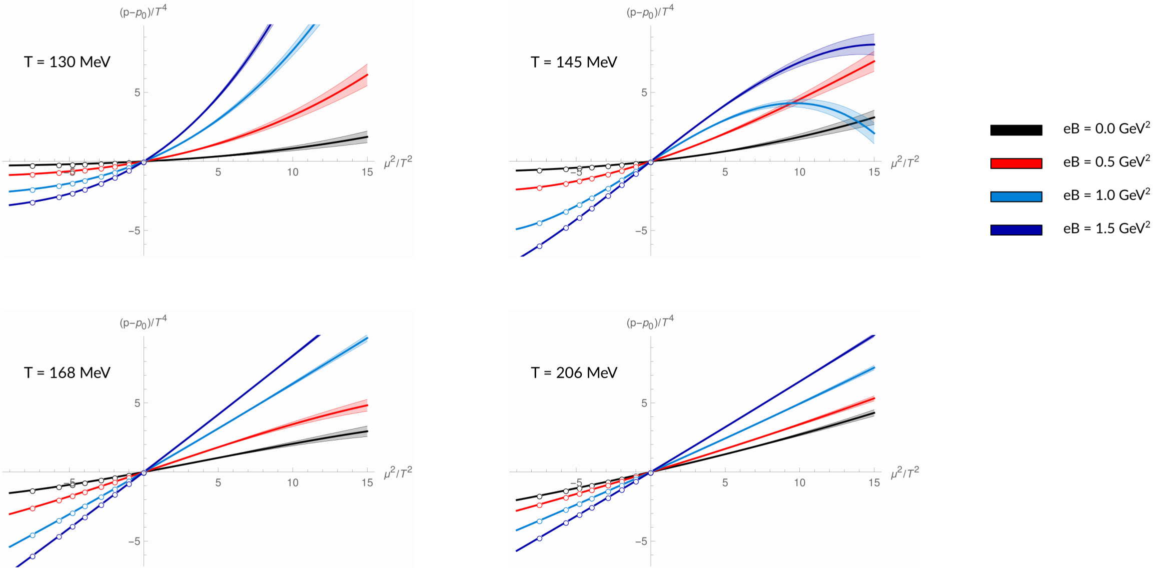




$eB = 0$ GeV^2

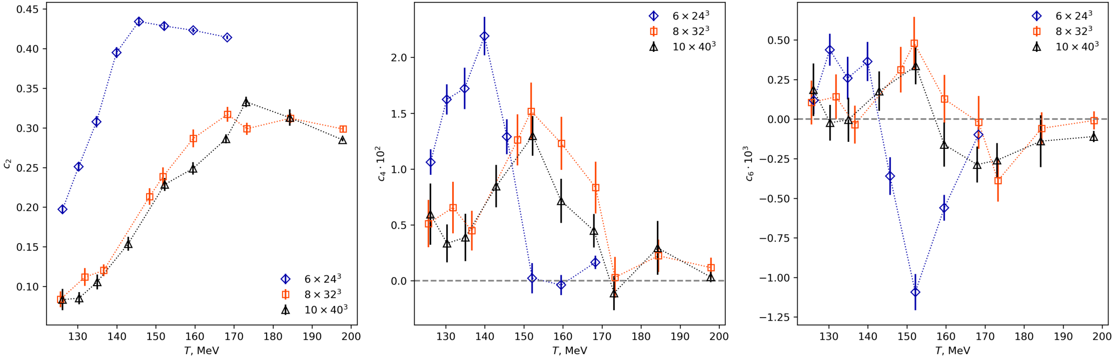
cont. extrapolated

Results: pressure



Results: comparison of coefficients for different lattices

$eB = 0.6$ GeV^2



PRELIMINARY

Conclusions

- Simulations at non-zero chemical potential and with external magnetic field have been carried out on 3 lattices.
- First results on expansion coefficients c_2, c_4, c_6 in external magnetic field have been obtained.
- Strong dependence of the EoS on magnetic field.