In-medium static quark potential from spectral functions on realistic HISQ ensembles

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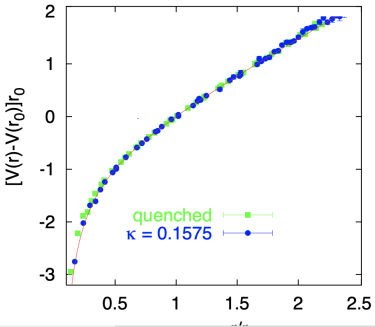
- If seperation of scales is present: Use EFTs (NRQCD and pNRQCD): describe physics in form of potential?

- At T=0 Schrodinger like potential picture has been observed (G. Bali Phys.Rept. 343 (2001) 1-136).

$$i\partial_t W_\square(t,r) = \Phi(t,r) W_\square(t,r)$$
$$V(r) = \lim_{t\to\infty} \Phi(t,r)$$



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$$W_\square(r,t) = \int d\omega e^{-i\omega t} \rho_\square(r,\omega)$$
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- Computation of spectral function : Ill-Posed Inverse problem
- In HTL regime there exists a complex potential with screened real part (M.Laine et. al JHEP 03 (2007), 054).

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- Using highly improved staggered quark (HISQ) action.
- $N_\sigma^3 \times N_\tau$ lattices. $N_\tau = 10, 12, 16$ and $N_\sigma / N_\tau = 4$.
- Calculate Wilson loop and Wilson Line correlator in Column Gauge.
- Fix box approach; temp range - 140MeV to 2GeV.

Cumulants and HTL comparison

- Define cumulants of the correlation function:

$$m_1(r,\tau,T) = -\partial_\tau \ln W(r,\tau,T),$$
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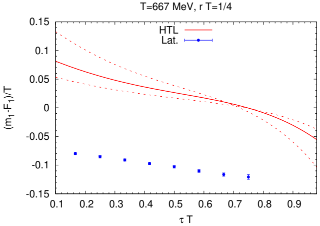
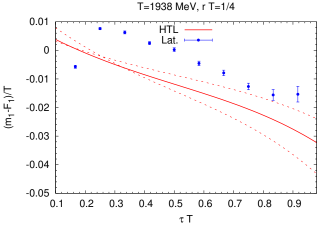
- First cumulant does not agree with HTL results, situation gets better with increasing Temp.





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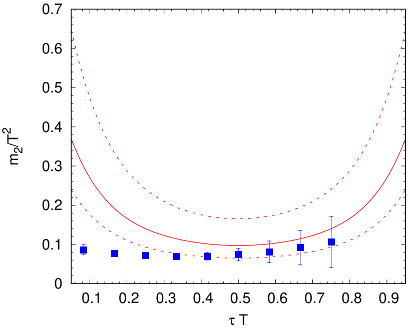

- Second cumulant on $N_\tau = 12$, $rT = 1/4$
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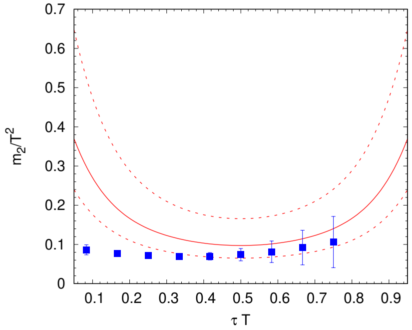

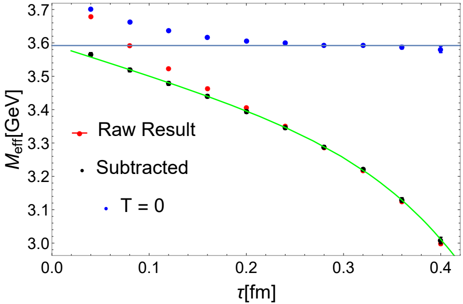

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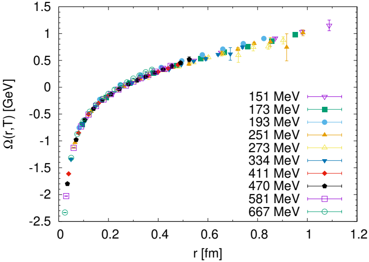
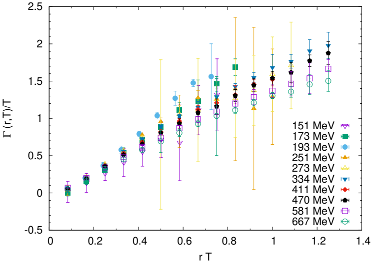
- The linear falloff at small τ part in m_1 suggests a Gaussian peak.

Using Spectral Function Model fits

- Lattice data sensitive only to peak position Ω and effective width Γ.

$$W(r,\tau,T) = A_P \exp(-\Omega(r,T)\tau + \Gamma_G(r,T)^2\tau^2/2) + A_{cut}(r,T)\exp(-\omega_{cut}(r,T)\tau),$$





- No T dependence in Ω, Γ shows a linear dependence.

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- Transform the Euclidean correlator into Matsubara frequency space.
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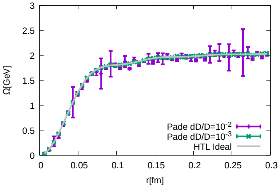
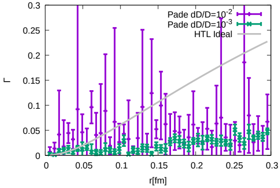
- This is actually interpolation of data and not fitting. Does not require minimization.
- Obtain pole structure from rational function: Directly related to Ω and Γ.
- Select the pole that is closest to the real frequency axis. This will lead to the dominant peak structure.

Results from Pade Extraction

- Noisy HTL mock data results.





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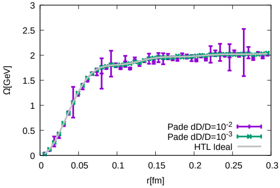
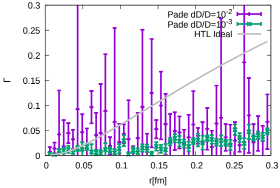
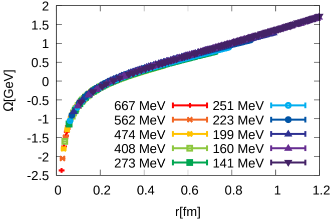




- Lattice data results.



- The results we get for Ω are similar to T=0 and Γ extraction is underestimated.

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Theorem (Bayes Theorem)

$$P[\rho|D, I] \propto P[D|\rho, I]P[\rho|I] = \exp[-L + \alpha S_{\mathrm{BR}}],$$

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- L is the usual quadratic distance used in χ^2 fitting.
- The prior probability $P(\rho|I) = \exp(\alpha S_{BR})$ acts as a regulator

$$S_{\mathrm{BR}} = \int d\omega \Big(1 - \frac{\rho(\omega)}{m(\omega)} + \log\Big[\frac{\rho(\omega)}{m(\omega)}\Big]\Big).$$

- $m(\omega)$ is the default model ; choose to use the most uninformative default model $m = const$.

Spectral Function Extraction using Bayesian Method (contd)

- Tune the hyperparameter α such that the likelihood takes the value $L = N_\tau/2$ (Morozov Criteria).

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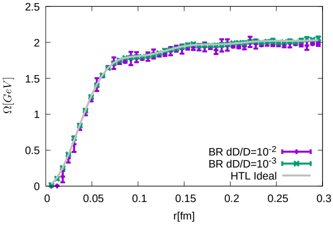
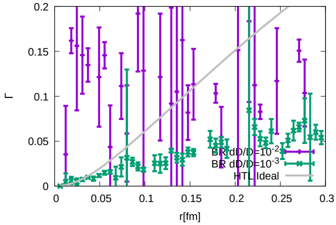
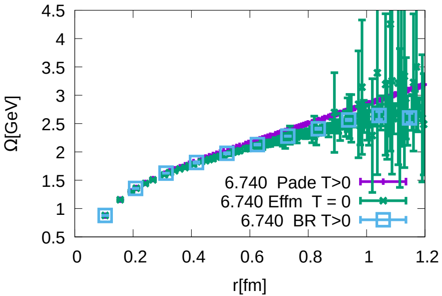
- We look for the most probably spectrum by locating the extremum of the posterior.
- Comparison with HTL benchmark .

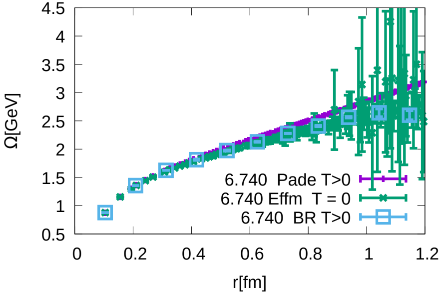




- HTL results reproduced: can we extract something from Lattice?



- Comparison of BR and Pade Ω for $\beta = 6.740$.



- Comparison of BR and Pade Ω for $\beta = 6.740$.
- At high T, the non-monotonous behavior in effective mass plots at small distances is a manifestation of positivity violation in spectral function.

Conclusion

- First Study of Wilson loop/line spectral properties on high precision HISQ ensembles.
- Inverse problem is a challenge for reliable spectral reconstruction.
- Gaussian fit and pade Ω obtained differs from previous studies.. More investigation is needed.
- Γ shows a linear increase with T.
- See Dibyendu's talk for a different approach inspired by HTL (up next).