The complex potential from 2+1 flavor QCD from HTL inspired approach

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outline

- One important probe to study QGP is the quarkonium (bound states of heavy quark-antiquark).
- The thermal static $Q\bar{Q}$ potential is an important quantity to study quarkonia inside QGP.
- Spectral function for $\bar{Q}\gamma_\mu Q$ can be obtained from the thermal static $Q\bar{Q}$ potential in leading order of $\frac{1}{m_q}$.

  M. Laine, O. Philipsen, P. Romatschke & M. Tassler, JHEP 0703 (2007) 054.

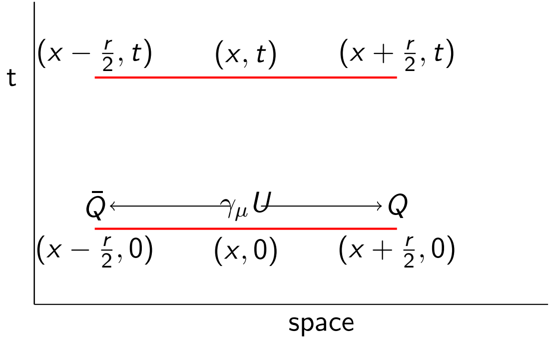
$$\frac{d\Gamma_{\mu^+\mu^-}}{d^4Q} \propto \rho(Q)$$

- Thermal potential is also an essential ingredient for the open quantum system studies of quarkonia in QGP.

  Y. Akamatsu and A. Rothkopf, Phys. Rev. D 85, 105011

- Consider state created by the point split version of vector $\bar{Q}\gamma_\mu Q$ operator,

$$|\psi(\vec{x},\vec{r};t),n\rangle = \bar{Q}(\vec{x}-\frac{\vec{r}}{2};t)\gamma_\mu U(\vec{x}-\frac{\vec{r}}{2},\vec{x}+\frac{\vec{r}}{2};t)Q(\vec{x}+\frac{\vec{r}}{2};t)\,|n\rangle$$



- The $C_>(\vec{r},t) = \frac{1}{Z_{QCD}}\int d^3\vec{x}\sum_n \exp(-\beta E_n)\,\langle\psi(\vec{x},\vec{r};t),n|\psi(\vec{x},\vec{r};0),n\rangle$,

- In the free theory (g=0), in leading order $\frac{1}{m_q}$

$$-\frac{\nabla^2}{m_q} C_>(\vec{r},t) = i\,\frac{\partial C_>(\vec{r},t)}{\partial t}$$

M. Laine, O. Philipsen, P. Romatschke & M. Tassler, JHEP 0703 (2007) 054.

- Potential is defined by the modified equation,

$$-\frac{\nabla^2}{m_q} C_>(\vec{r},t) + V(r,T;t)\, C_>(\vec{r},t) = i\,\frac{\partial C_>(\vec{r},t)}{\partial t}$$

- A potential is exists when

$$\lim_{t\to\infty} V(r,T;t) = V(r,T)$$

- $C_>(\vec{r},t)$ is not $Q\bar{Q}$ wave function, $|C_>(\vec{r},t)|^2$ is not probability.

- In the static limit $m_q \to \infty$, $C_>(\vec{r}, t) \propto W_T(\vec{r}, t)$,
- The static potential is then given by if it exists

$$V(r,T) = i \lim_{t\to\infty} \frac{\partial \log(W_T(r,t))}{\partial t}$$

- The potential is complex in leading order perturbation theory for $r \sim \frac{1}{m_d}$.
  M. Laine, O. Philipsen, P. Romatschke & M. Tassler, JHEP 0703 (2007) 054.

$$V_{re}(r) = -\frac{\alpha}{r} e^{-m_d r} - m_d \alpha$$

$$V_{im}(r) = -\frac{4\alpha_s}{3} T \int_0^\infty dz \, \frac{2z}{(z^2+1)^2} \left[1 - \frac{\sin zx}{zx}\right]$$

- On the lattice, we can calculate W_T in Euclidean time τ.

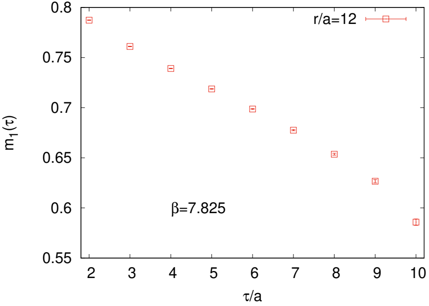
$$W_T(r,\, \tau) = \int_{-\infty}^{\infty} d\omega\, \rho(r, \omega)\, \exp(-\tau\, \omega)$$

$$W_T(r,\, t) = \int_{-\infty}^{\infty} d\omega\, \rho(r, \omega)\, \exp(-i\, t\, \omega)$$

A. Rothkopf, T. Hatsuda, & S. Sasaki, Phys. Rev. Lett. 108, 162001

- The inversion is an unstable problem without any prior.
- Various possible spectral function exists which describe lattice data.
- Here we assume potential exist non-perturbatively.
  DB & S. Datta, Phys. Rev. D 101, 034507
- Here we will use Wilson line correlator fixed in Coulomb gauge instead of Wilson loop. Y. Burnier & A. Rothkopf, Phys. Rev. D 87, 114019
- HISQ action with $m_l = m_s/20$ and $N_\tau = 12$ has been used.

- At zero temperature the dominant peak is $\rho(r,\omega) \sim \delta(\omega - V(r))$.
- V(r) can be obtained from the plateau of the effective mass $m_1(r,\tau) = -\frac{\partial \log W_T(r,\tau)}{\partial \tau}$.
- Above the crossover temperature no such plateau exist.



- This could be due to thermal broddening of the peak.

- Motivated from HTL we write near $\tau \sim \frac{\beta}{2}$,

$$\log(W_T(r,\tau)) = -V_{re}(r,\tau)\tau + \int_{-\infty}^{\infty} d\omega\sigma(r,\omega)[\exp(\omega\tau) + \exp(\omega(\beta-\tau))]$$

M. Laine, O. Philipsen, P. Romatschke & M. Tassler, JHEP 0703 (2007) 054.

- A potential will exist when

$$\lim_{t\to\infty} i\frac{\partial \log W_T(r,it)}{\partial t} = finite$$

We will need, $\sigma(\omega) \sim \frac{1}{\omega^2}$ as $\omega \to 0$

- Parametrizing using HTL structure of $\sigma(\omega)$ we can write,

$$\sigma(\omega) = n_b(\omega)\left(\frac{V_{im}(r,T)}{\omega} + c_1\omega + c_3\omega^3 + ...\right)$$

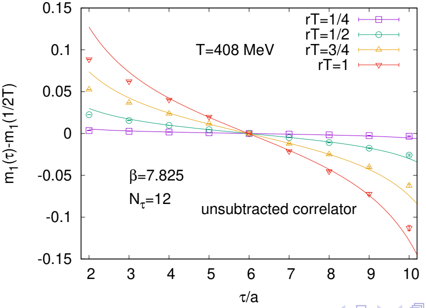
- Using this parametrization we expect near $\tau \sim \frac{\beta}{2}$,

$$m_1 = -\frac{\partial \log W_T(r,\tau)}{\partial \tau} = V_{re}(r,T) + V_{im}(r,T)\cot\left(\frac{\pi\tau}{\beta}\right) + \ldots$$

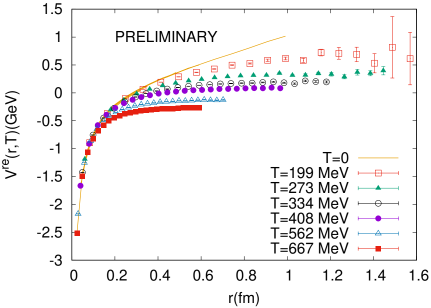
DB & S. Datta, Phys. Rev. D 101, 034507

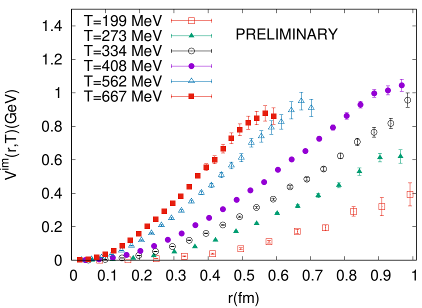
- This form will allow us to have a potential,

$$V(r,T) = \lim_{t\to\infty} m_1(r,it) = V_{re}(r,T) - iV_{im}(r,T)$$



- Real part shows medium modification.
- The imaginary part increases both with temperature and distance.



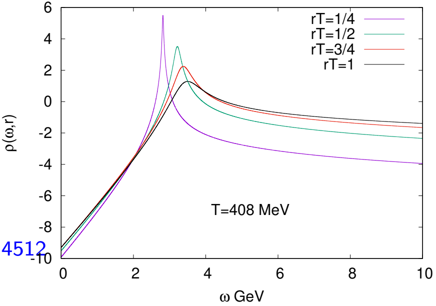


- Similar analysis has also been performed in quenched approximation on hybrid state. DB & S. Datta, Phys. Rev. D 103, 014512

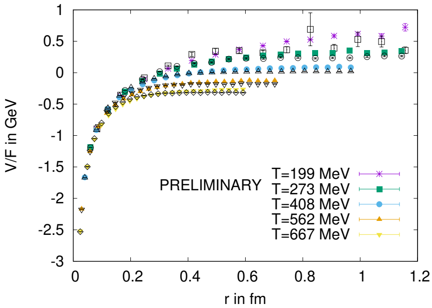
- $$W_T(r,\tau) \sim \exp\left(-V_{re}(r,T)\,\tau + \frac{\beta V_{im}(r,T)}{\pi}\log\left[\sin\left(\frac{\pi\tau}{\beta}\right)\right]\right)$$
- $$\rho(r,\omega) = \int_{-\infty}^{\infty} dt\, \exp(i\omega t)\, W_T(r,it)$$
- $$\rho(r,\omega) \sim \frac{V_{im}}{(\omega - V_{re})^2 + V_{im}^2},$$

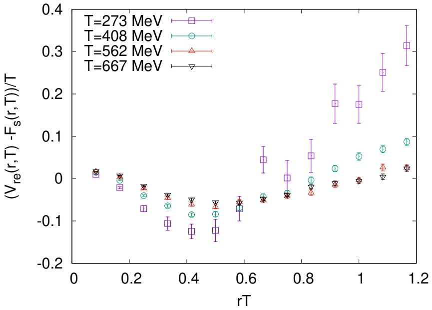
  for $\omega \sim V_{re}$

  DB & S. Datta, Phys. Rev. D 103, 014512



- Singlet Free Energy $F_s(r, T) = -T \log(W_{coulomb}(r, \beta))$.
- In leading order HTL, $V_{re}(r, T) = F_s(r, T)$.





- Similarfeature has been observed in,

A. Bazavov, N. Brambilla, P. Petreczky, A. Vairo, and J. H. Weber, Phys. Rev. D 98, 054511

- The data near $\tau \sim \beta/2$ can be fitted with the HTL parametrization.
- The real part shows medium modification and close to the singlet free energy.
- The imaginary part increases with distance and temperature.
- The spectral function near the peak is Lorentzian.