Normalizing flows for the real-time sign problem

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in collaboration with **Scott Lawrence**

based on "*Normalizing flows and the real-time sign problem*"
arXiv:2101.05755[hep-lat]

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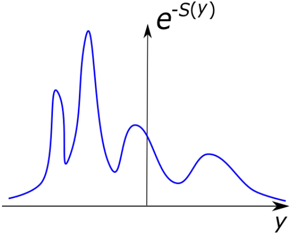
Contents:

Trivializing Map[1]

Some distribution

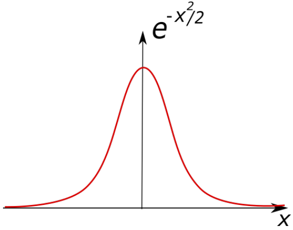


Map

$\leftrightarrow$

$y(x)$

Simple Gaussian distribution



Trivializing map:

$$dy\ e^{-S(y)} = dx\ \frac{dy(x)}{dx}\ e^{-S(y(x))} = dx\ \mathcal{N} e^{-x^2/2}$$

Expectation values:

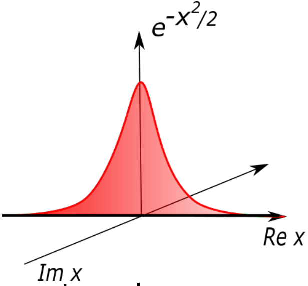
$$\langle \mathcal{O} \rangle = \frac{\int dy\ e^{-S(y)} \mathcal{O}(y)}{\int dy\ e^{-S(y)}} = \frac{\int dx\ e^{-x^2/2} \mathcal{O}(y(x))}{\int dx\ e^{-x^2/2}}$$

[1] M. Albergo et.el. Phys. Rev. D 100, 034515(2019)
K. A. Nicoli, et.el. Phys. Rev. E 101, 023304(2020)

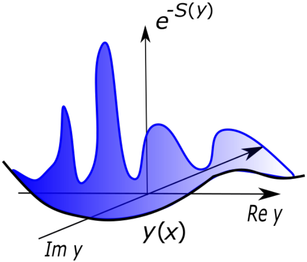
Complex Trivializing Map

Application to complex actions:

$$dx\,\mathcal{N}e^{-x^2/2} = dx\,\frac{dy(x)}{dx}\,e^{-S(y(x))} = dy\;e^{-S(y)}$$



Map
$\leftrightarrow$
$y(x)$



Expectation values:

$$\frac{\int_{\mathbb{R}} dx\;e^{-x^2/2}\mathcal{O}(y(x))}{\int_{\mathbb{R}} dx\;e^{-x^2/2}} = \frac{\int_{y(\mathbb{R})} dy\;e^{-S(y)}\mathcal{O}(y)}{\int_{y(\mathbb{R})} dy\;e^{-S(y)}} \stackrel{?}{=} \langle\mathcal{O}\rangle$$

Contour of integration changes!

Constraints on Trivializing Map[2]

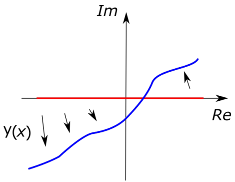
Trivializing maps give the correct $\langle \mathcal{O} \rangle$

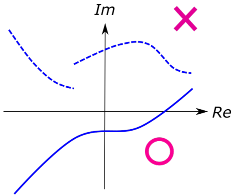
$$\langle \mathcal{O} \rangle = \frac{\int_{\mathbb{R}} dy\ e^{-S(y)} \mathcal{O}(y)}{\int_{\mathbb{R}} dy\ e^{-S(y)}} = \frac{\int_{y(\mathbb{R})} dy\ e^{-S(y)} \mathcal{O}(y)}{\int_{y(\mathbb{R})} dy\ e^{-S(y)}}$$

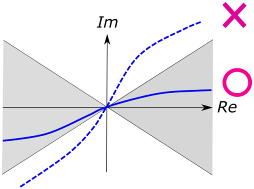
only when:

- The induced contour (—) is a continuous manifold
- The induced contour (—) is in "asymptotically safe" region
- Both $e^{-S(y)}$ and $e^{-S(y)}\mathcal{O}(y)$ are holomorphic functions in the region between (—) and (—)

$\rightarrow$ **Cauchy's integral theorem!**



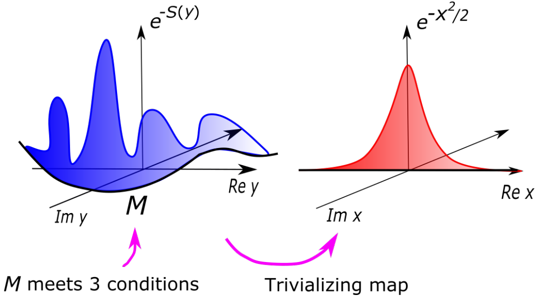




[2] A. Alexandru et.el., Phys. Rev. D. 98, 034506(2018)

Trivializing Map and The Generalized Thimble Method

$\mathcal{M}$ is of exactly the sort used in the generalized thimble method



"Average sign" on the manifold $\mathcal{M}$ is

$$\langle\sigma\rangle = \frac{\int_{\mathcal{M}} dy\ e^{-S(y)}}{\int_{\mathcal{M}} dy\ e^{-\operatorname{Re} S(y)}} = \frac{\int_{\mathbb{R}} dx\ e^{-x^2/2}}{\int_{\mathbb{R}} dx\ e^{-x^2/2}} = 1$$

So the manifold $\mathcal{M}$ has no sign problems.

Trivializing maps exist $\leftrightarrow$ Perfect manifolds exist

Existence of Trivializing Maps

For an **action S which is finite except at infinity**, with no sign problems

Fact: Trivialiaing maps exist.

Conjecture:
Trivializing maps are analytic functions of the parameters of the action.

Example: Scalar field theory $S(y; M, \Lambda) = y_i\, M_{ij}\, y_j + \lambda\, \Lambda_i\, y_i^4$

Perturbative flow:

$$y_i(x; M, \Lambda) = x_i - \lambda \left(\sum_j \frac{1}{2} M_{ij}^{-1} \Lambda_j x_j^3 + \frac{3}{4} M_{ij}^{-1} M_{jj}^{-1} \Lambda_j x_j \right)$$

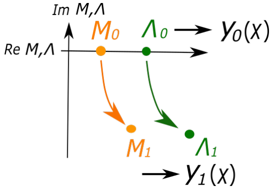
Conjecture implies:

Perfect manifolds exists for $M, \Lambda \in \mathcal{C}$

Caveat:

When manifolds intersect with singularity of S,

Trivializing maps may not exist



For actions without singularities at finite field values, perfect manifolds exist with $M, \Lambda \in \mathcal{C}$

An Attempt to Find a Map via Machine Learning

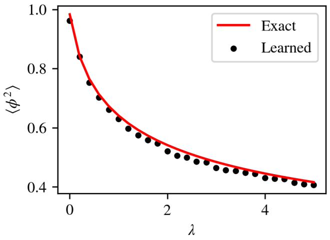
To estimate $\langle \mathcal{O} \rangle_S$ for the action S, let us define $S' = S + \alpha\,\mathcal{O}$.
A perturbing map $\vec{y}(x)$ from $S'(x + \alpha\,\vec{y}(x))$ to $S(x)$ satisfies

$$\nabla \cdot \vec{y}(x) - \vec{y}(x) \cdot \nabla S(x) - \mathcal{O}(x) + \langle \mathcal{O} \rangle_S = 0$$

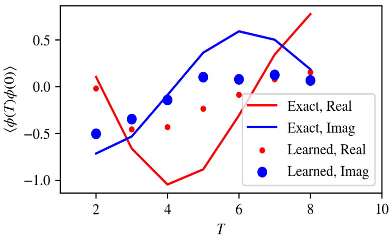
Solve the ODE for $\vec{y}(x)$ and $\langle \mathcal{O} \rangle_S$ via machine learning:

- Represent $\vec{y}(x)$ with neural networks (parameter w)
- Train neural networks as well as $\langle \mathcal{O} \rangle_S$ with respect to the cost function:

$$C(w, \langle \mathcal{O} \rangle_S) = \sum_x |\nabla \cdot \vec{y}_w(x) - \vec{y}_w(x) \cdot \nabla S(x) - \mathcal{O}(x) + \langle \mathcal{O} \rangle_S|^2$$



$0+1\text{d},\ m = 0.5,\ N_\beta = 10,\ N_T = 0$



$0+1\text{d},\ m = 0.5, \lambda = 0.5, N_\beta = 2$

Thank you!