QCD topology and axion's properties from Wilson twisted mass lattice simulations

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Motivation: Strong CP problem

$$\mathcal{L} = \mathcal{L}_{QCD} + \theta \frac{g^2}{32\pi^2} F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a$$

$q(x) = \frac{g^2}{32\pi^2} F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a$ – topological charge density

How to test θ?

Neutron electric dipole moment:

$d_n = 2.4 \times 10^{-16}\theta$ e cm – QCD sum rules

$d_n = 3.3 - 3.6 \times 10^{-16}\theta$ e cm – ChPT

Experimental value:

$d_n = 0.0 \pm 1.1(\text{stat}) \pm 0.2(\text{sys}) \times 10^{-26}$ e cm

$$\theta < 0.5 \times 10^{-10}$$

- Pospelov & Ritz, NPB **573**, 177 (2000)
- Pich & de Rafael, NPB **367**, 313 (1991)
- Abel et al. (nEDM), PRL **124**, 081803 (2020)

(Possible) Solution: the axion

- Peccei & Quinn, PRL **38**, 1440; PRD **16** 1791 (1997)
- Weinberg, PRL **40**, 223; Wilczek PRL **40**, 279 (1978)]

$$\mathcal{L} = \mathcal{L}_{QCD} + \theta \frac{g^2}{32\pi^2} F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a + \partial^2_\mu a^2 + \frac{a}{f_A}\frac{g^2}{32\pi^2} F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a$$

$a(x) = f_A\theta(x)$ – axion field, $f_A \gtrsim 4 \times 10^8$ GeV.

Axion mass:

$$m^2_A(T) f^2_A = \left.\frac{\partial^2 F(\theta, T)}{\partial\theta^2}\right|_{\theta=0} \equiv \chi_{top}(T).$$

At low and high temperatures $F(\theta, T)$ is available from LO ChPT and DIGA.

At $T = 0$:

$$m_A(T = 0) = \frac{\sqrt{\chi_{top}}}{f_A} = 56.9(5)\,\frac{10^{11}\text{GeV}}{f_A}\ \mu\text{eV}.$$

Our setup

- finite-T simulation with fixed scale approach
- Wilson twisted mass fermions *at maximal twist*
- physical strange and charm masses
- 4 values of pion mass, including physical one

Ensemble	m_π [MeV]	a [fm]
M140	139(1)	0.0801(4)
D210	213(9)	0.0646(7)
A260	261(11)	0.0936(13)
B260	256(12)	0.0823(10)
A370	364(15)	0.0936(13)
B370	372(17)	0.0823(10)
D370	369(15)	0.0646(7)

- for lattice spacing and other parameters we rely on ETMC $T = 0$ results [PRD **90**, 074501 (2014); **103**, 034509 (2021)]

Observables

Chiral condensate:

$$\langle\bar\psi\psi\rangle = \langle\bar u u\rangle + \langle\bar d d\rangle = \frac{T}{V}\frac{\partial Z}{\partial m_l} = \frac{1}{N_t N_s^3}\langle \text{Tr}\, M^{-1}\rangle.$$

Chiral susceptibility: $\chi_L = \frac{\partial}{\partial m_l}\langle\bar\psi\psi\rangle$.

We define

$$\langle\bar\psi\psi\rangle_3 = \langle\bar\psi\psi\rangle - m_l\,\chi_L,$$

which is free from linear additive renormalization as well as from linear correction to scaling.

Measuring topology: Fermionic definition + symmetry arguments

$$Q = m_l \int d^4x\, \bar\psi(x)\gamma_5\psi(x)$$

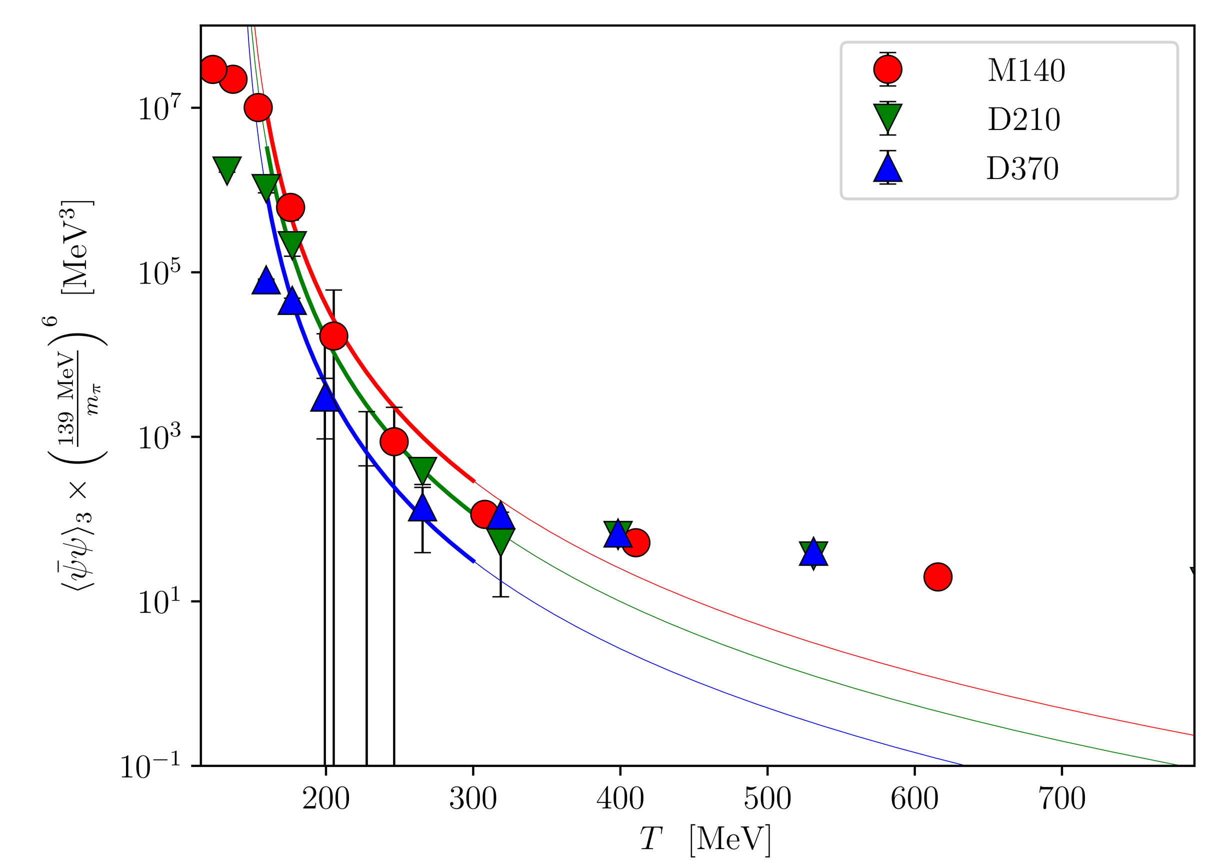
$$\chi_{top} = \frac{\langle Q^2\rangle}{V_4} = m_l^2\,\chi_{5,disc}$$

After chiral transition: $\chi_{5,disc} = \chi_{disc}$

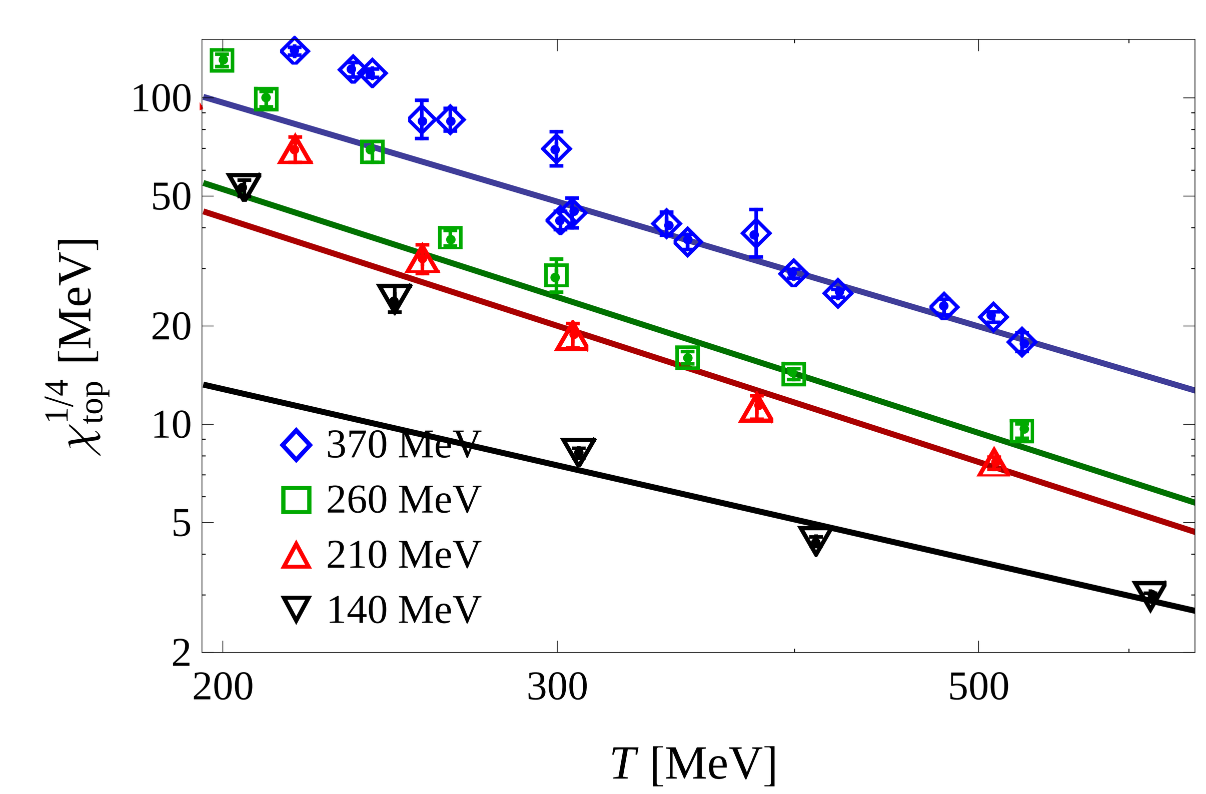
$$\chi_{top}(T \gtrsim T_c) = m_l^2\,\chi_{disc} = m_l^2 \frac{V}{T}\left(\langle(\bar\psi\psi)^2\rangle_l - \langle\bar\psi\psi\rangle^2_l\right)$$

Fine lattices are required to avoid artifacts.

Results

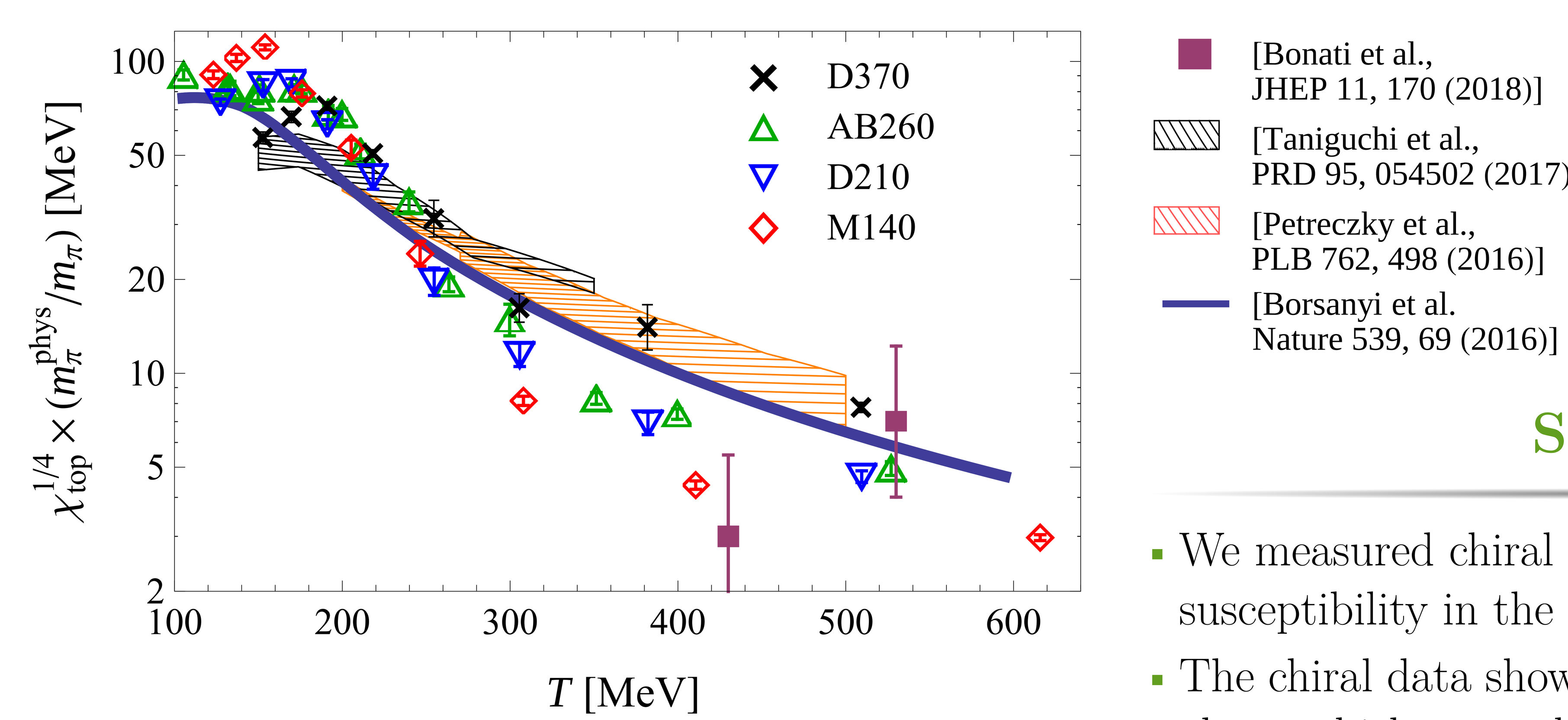


Fit is to 3D $O(4)$ behaviour: $\langle\bar\psi\psi\rangle_3 \propto (T - T_0)^{-\gamma-2\beta\delta}$.



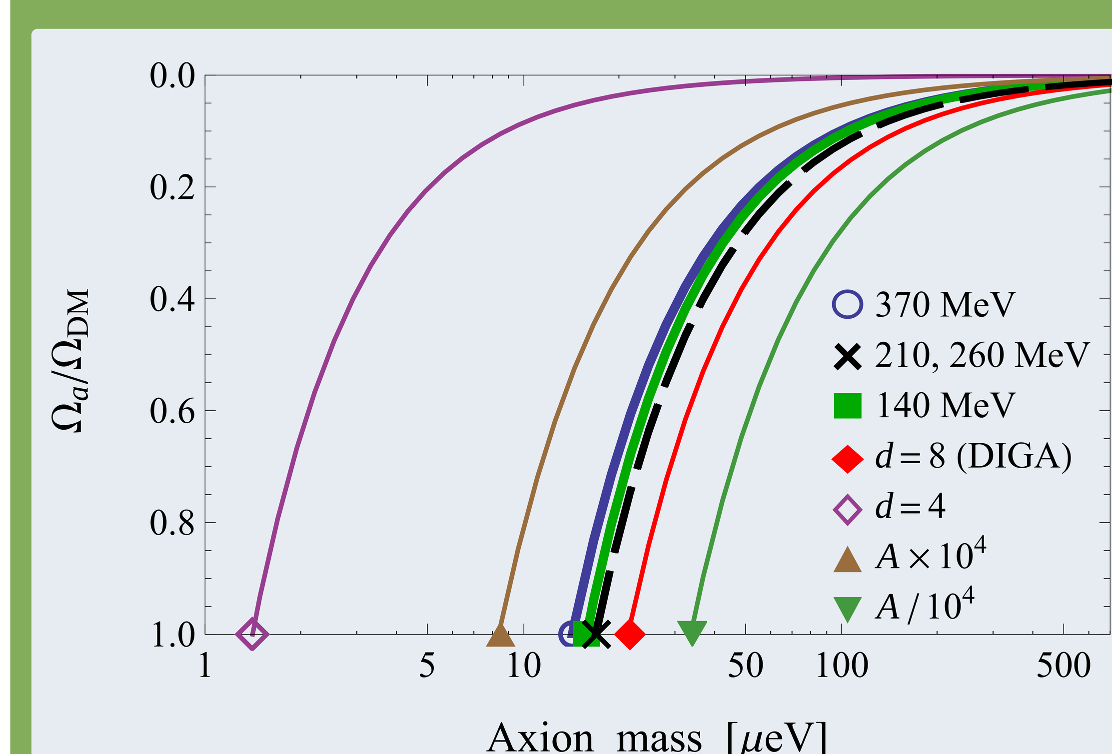
Fit is to DIGA behaviour: $\chi_{top}(T) \simeq A\,T^{-d}$.

The breaking of $O(4)$ scaling for $\langle\bar\psi\psi\rangle_3$ and the onset of DIGA-like decay for χ_{top} both occur at $T \simeq 300$ MeV.



$\chi_{top} \propto m_\pi^4$ is suggested by DIGA and, more generally, by the analyticity of $\bar\psi\psi$.

Axion mass



The prediction for axion mass can be read off from the plot using axion density Ω_a as an input. For the lower bound $\Omega_a = \Omega_{DM}$, attributing all Dark Matter to the axions, we have $m_a \simeq 20\ \mu$eV.

Summary

- We measured chiral observables and topological susceptibility in the region $120 \lesssim T \lesssim 600$ MeV.
- The chiral data shows clear threshold at $T \simeq 300$ MeV, above which a trend consistent with $O(4)$ scaling gives way to a simple leading order Griffith analytic scaling.
- Around the same point $T \simeq 300$ MeV the topological susceptibility starts to follow DIGA power-law decay.
- The high-T topological results from different studies are in the same ball park, but lacking complete quantitative agreement.
- The prediction for axion mass is rather insensitive to these differences. The same holds for different pion masses once the proper scaling is applied.

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