Radius of convergence at finite chemical potential with rooted staggered fermions

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Outline

Introduction

- Taylor expansion of the pressure is a popular method to study the QCD phase diagram
- The Taylor series has a finite radius of convergence ($R_{\rm conv}$) in $\hat{\mu}_B = \mu_B/T$
- $R_{\rm conv}$ is the distance of the closest singularity of the pressure to the origin
- $R_{\rm conv}$ is not only a limitation but also provides a lower bound to the location of the CEP

Introduction

One can determine R_{conv} in different ways:

1. using the Taylor coefficients
   - BUT the widely used 'ratio' method does not always converge
   - $\rightarrow$ improved estimators (Giordano, Pasztor, 2019)
   - high orders are needed for convergence
2. find the leading singularity of $\log Z$ directly
   - numerically challenging

In both cases staggered rooting causes problems (Golterman, Shamir, Svetitsky, 2006)

Radius of convergence in QCD

For 'typical' finite V statistical physics systems Z is a polynomial (up to an $e^{-kV\hat{\mu}}$ factor):

$$Z(\hat{\mu}) = \sum_{n=-kV}^{kV} Z_n \, e^{n\hat{\mu}} = e^{-kV\hat{\mu}} \sum_{m=0}^{2kV} Z_{m-kV} \, e^{m\hat{\mu}}$$

The zeroes of this polynomial generate the singularities of $\log Z$ and provide R_{conv}

In QCD

$$Z = \int \mathcal{D}U \, \mathrm{det}M(\hat{\mu}) e^{-S_g}$$

with ultra-local actions $\mathrm{det}M$ (and Z) has the desired properties, e.g. for staggered:

$$\mathrm{det}M = e^{-3V\hat{\mu}} \, \mathrm{det}(P - e^{\hat{\mu}}) = e^{-3V\hat{\mu}} \prod_i (\xi_i - e^{\hat{\mu}})$$

Geometric matching

However:

$$\sqrt{\mathrm{det} M} = \sqrt{e^{-3V\hat{\mu}} \prod (\xi_i - e^{\hat{\mu}})}$$

is not a polynomial, is not even unambiguous and has unwanted singularities!

$\rightarrow$ The leading singularity determining R_{conv} may be non-physical.

Proposed solution: geometric matching

close to the continuum ξ_i come in quartets

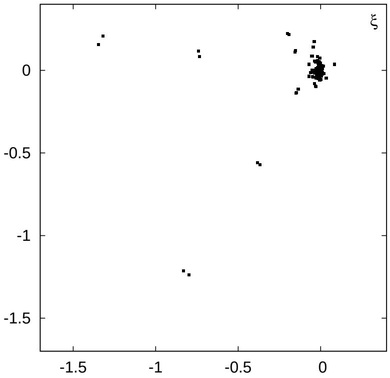
$\rightarrow$ keep only one/two from each quartet

In general find closest ξ_i, ξ_j pairs and replace them by $\bar{\xi} = \sqrt{\xi_i \xi_j}$

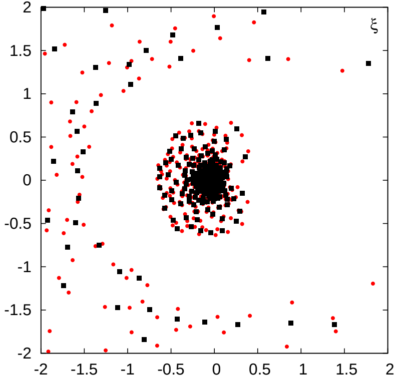
Define rooted determinant as a polynomial:

$$\sqrt{\mathrm{det} M_P} \equiv e^{-3V\hat{\mu}/2} \prod (\bar{\xi}_i - e^{\hat{\mu}})$$

Illustration of geometric matching



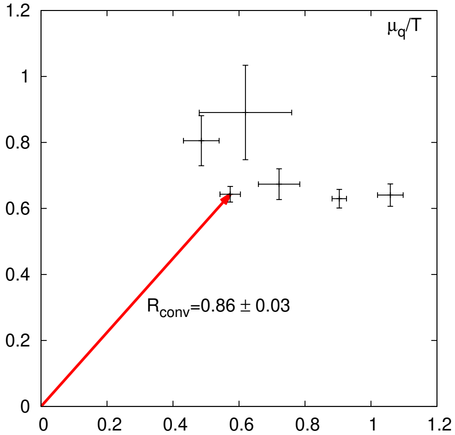
pairs of eigenvalues after 50 smearing steps



finding pairs

Results

- determine $R_{\rm conv}$ on $N_t = 4$ lattices around T_c
- Symanzik gauge + 2-stout fermion action with physical quark masses
- use geometric matching to define rooted LY polynomial
- numerically find its roots and determine $R_{\rm conv}$
- finite size dependence from $N_s = 6, 8, 10, 12$



Results

Volume and temperature (β) dependence of R_{conv}