Lee-Yang edge singularities in 2+1 flavor QCD with imaginary chemical potential.

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LATTICE 21

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Bielefeld-Parma Collaboration:
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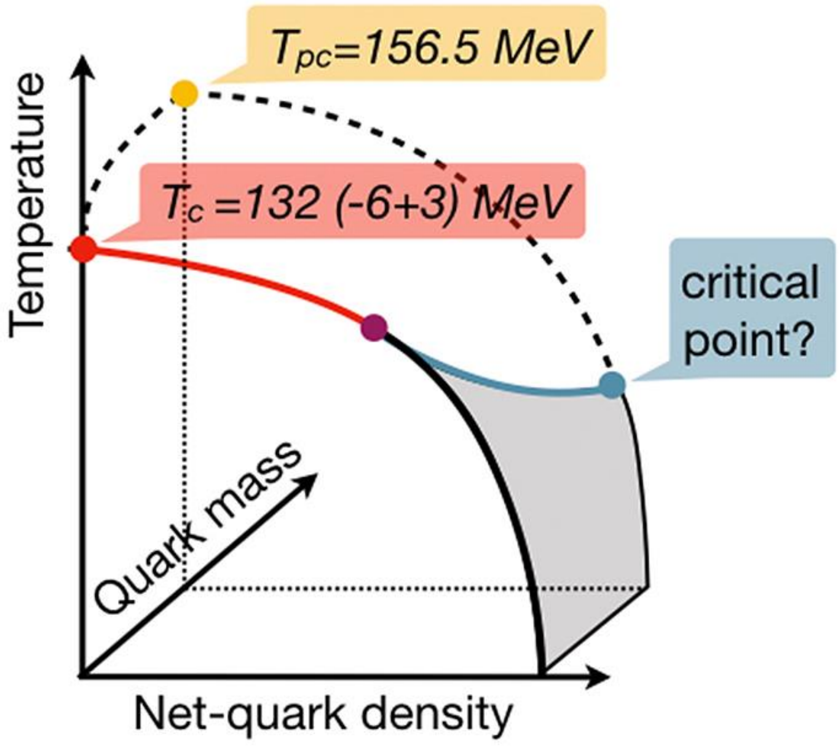
UNIVERSITÄT BIELEFELD
Fakultät für Physik

CRC-TR 211
Strong-interaction matter under extreme conditions

Introduction



In the search of the critical point.

Sign problem:
Introduce a quark chemical potential into the action;
Determinant of the Dirac's Matrix become complex;
No probabilistic interpretation for MC simulations.

How we manage it:
Simulations with imaginary baryon chemical potential in the time direction.

What interests us:
Universal critical behavior of phase transitions in the complex plane.

Lattice settings and observables

2+1 flavors, HISQ, Rational Hybrid Monte Carlo (RHMC);
Bielefeld GPU code;
$N_\sigma^3 \times N_\tau$: 24³x4, 36³x6.

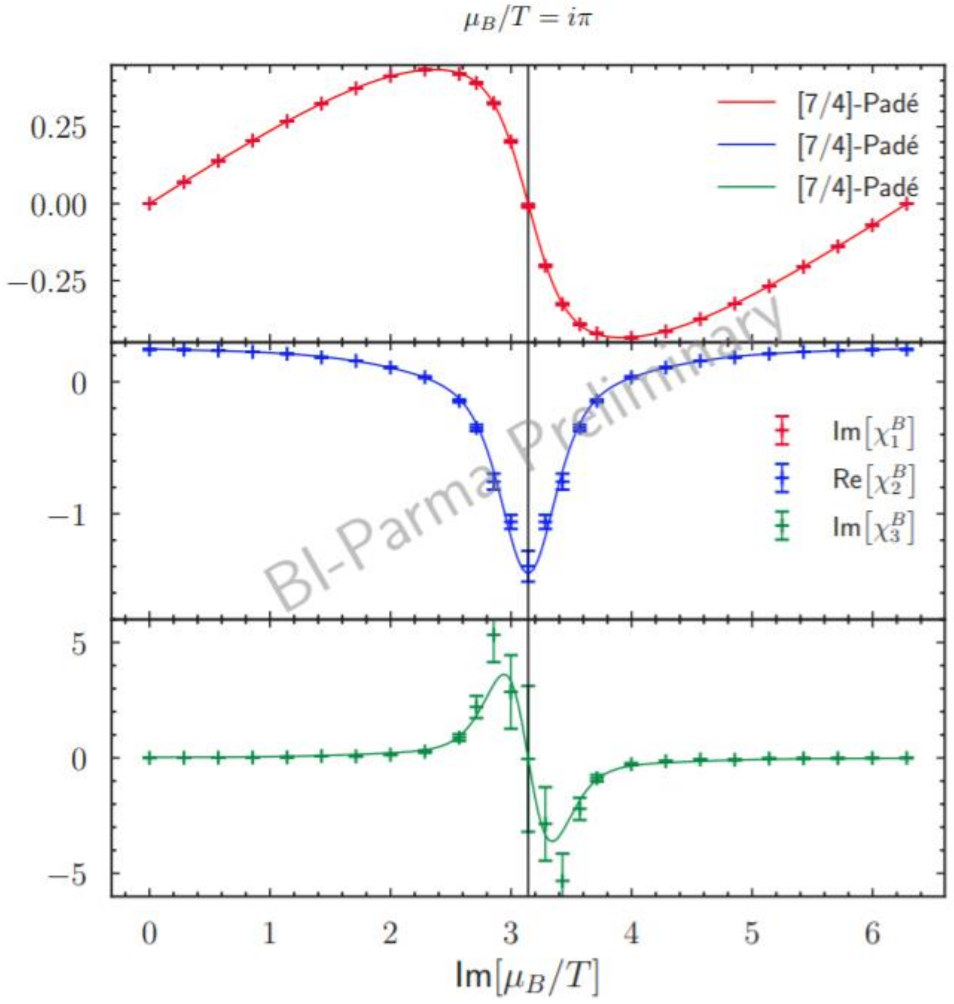
$$Z(T,V,\mu) = \int DU \cdot det(M_l(i\mu))^{1/2} \cdot det(M_s(i\mu))^{1/4} \cdot e^{-S_G}$$

$$\chi_n^B = \frac{1}{n!VT^3} \frac{\partial^n lnZ}{\partial(\mu_B/T)}$$

From Taylor series at different purely imaginary baryon chemical potential we have built multi-point Padé approximants:

$$R_n^m(x) = \frac{P_m(x)}{1+Q_n(x)} = \frac{\sum_{i=0}^{m} a_i\, x^i}{1+\sum_{j=1}^{n} b_j\, x^j}$$

[Simran Singh, Lattice 2021, Tuesday at 6.45 EST]



Magnetic Equation of State (EoS) and Lee-Yang Edge singularity

[Michael E. Fisher, Phys. Rev. Lett. 40, 1610 (1978)]

We can write the order parameter as:

$$M(t,h) = h^{\frac{1}{\delta}} f_G(z) + M_{reg}(t,h)$$ **Magnetic EoS**

t = field responsible for the deviation from criticality without explicit symmetry breaking;
h = symmetry breaking field;
$f_G(z)$ = scaling function;
z = scaling variable $t/h^{1/\beta\delta}$;
$\beta,\ \delta$ = universal critical exponent.

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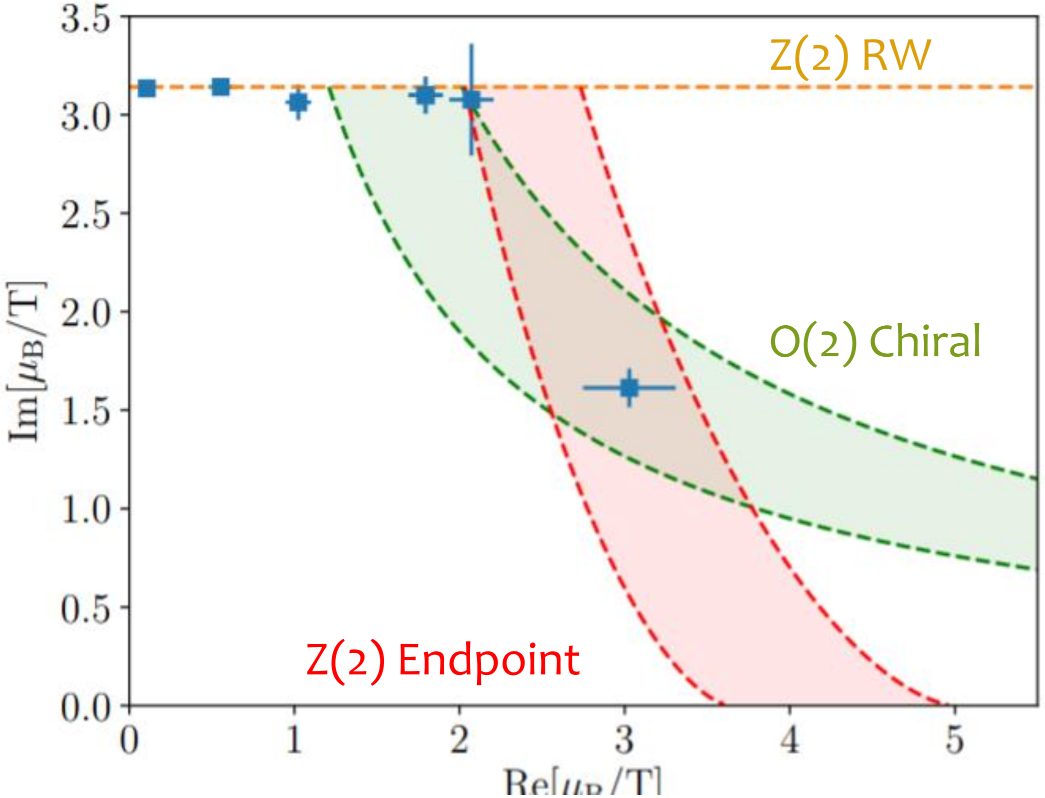
β, δ = universal critical exponent.

Approaching the critical point, $M_{reg}(t,h)$ does not have singularities in this region.

$h^{\frac{1}{\delta}} f_G(z)$ governs the non analytic behavior.
The singularity of the scaling function in $z = z_c$ is called **Lee Yang edge sigularity** (LYE).

Now we have to **choose the scaling fields** *t*, *h* in agreement with the transition.

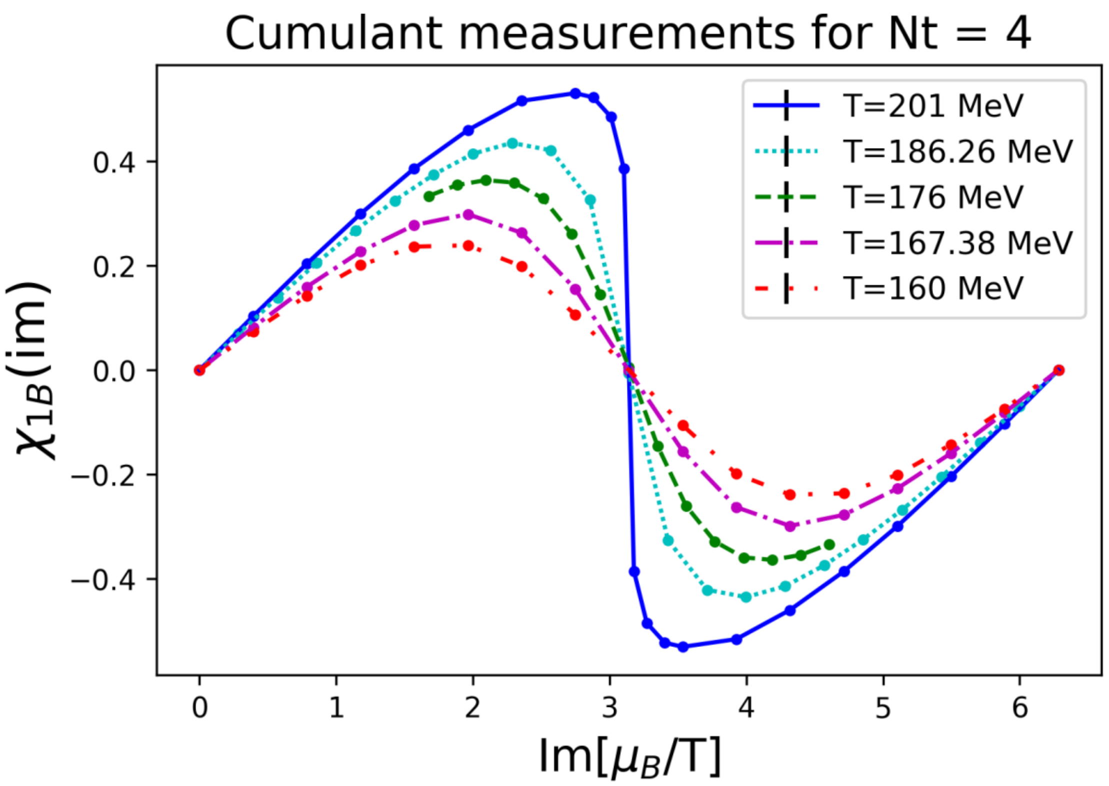


Roberge-Weiss transition scaling

Lattice size: 24^3 x 4

T_{RW}=201 MeV

[Jishnu Goswami et al, PoS CORFU2018 (2019) 162]



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Symmetry of the transition: Z(2)

We set:

$$h = \frac{1}{h_0}\frac{Im(\frac{\mu_B}{T}) - \pi}{\pi} \qquad t = \frac{1}{t_0}\frac{T - T_{RW}}{T_{RW}}$$

Temperature scaling:

$$Re[\mu_B/T] = \pi\left(\frac{z_0}{|z_c|}\right)^{\beta\delta}\left|\frac{T_{RW} - T}{T_{RW}}\right|^{\beta\delta}$$

$$Im[\mu_B/T] = \pi$$

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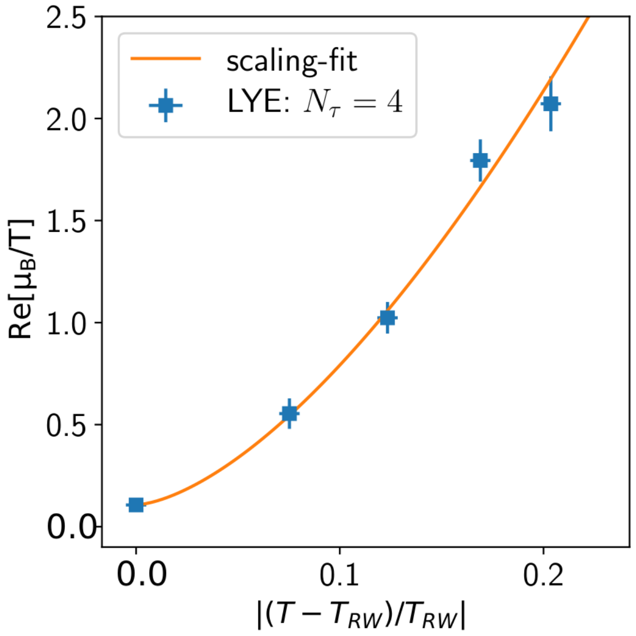

Fit $\quad f(x) = a \cdot t^{\beta\delta} + b$

Known 3D Ising critical exp:

$$\beta = 0.3264$$
$$\delta = 4.789$$
$$z_c = 2.452 \pm 0.025$$

[Andrew Connelly et al, arXiv:2006.12541v2]

Preliminary result:

$$\chi^2_{test} \approx 0.8$$
$$z_0 = 9.28 \pm 0.26$$

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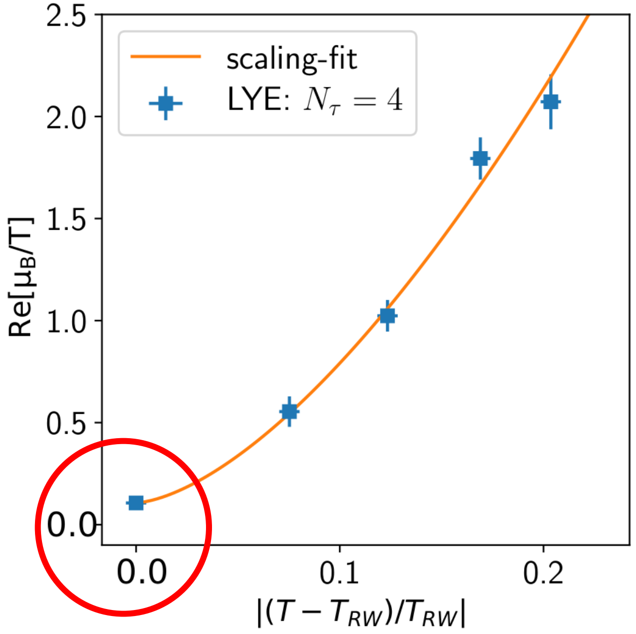

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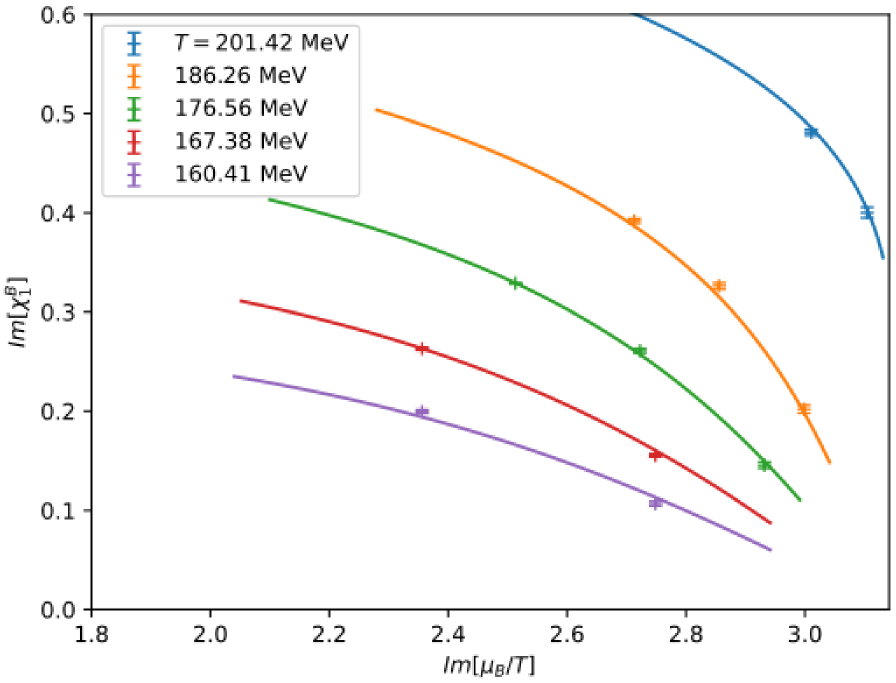
Fit parameters **slightly** correlated;
Non zero singularity in *t*=0.

Magnetic EoS as alternative method to obtain z_0

Lattice size: 24^3 x 4



$$f_G(z) = \frac{M(t,h) - M_{reg}(t,h)}{h^{\frac{1}{\delta}}}$$

with $M_{reg}(t,h) = a_2 \cdot t \cdot h$

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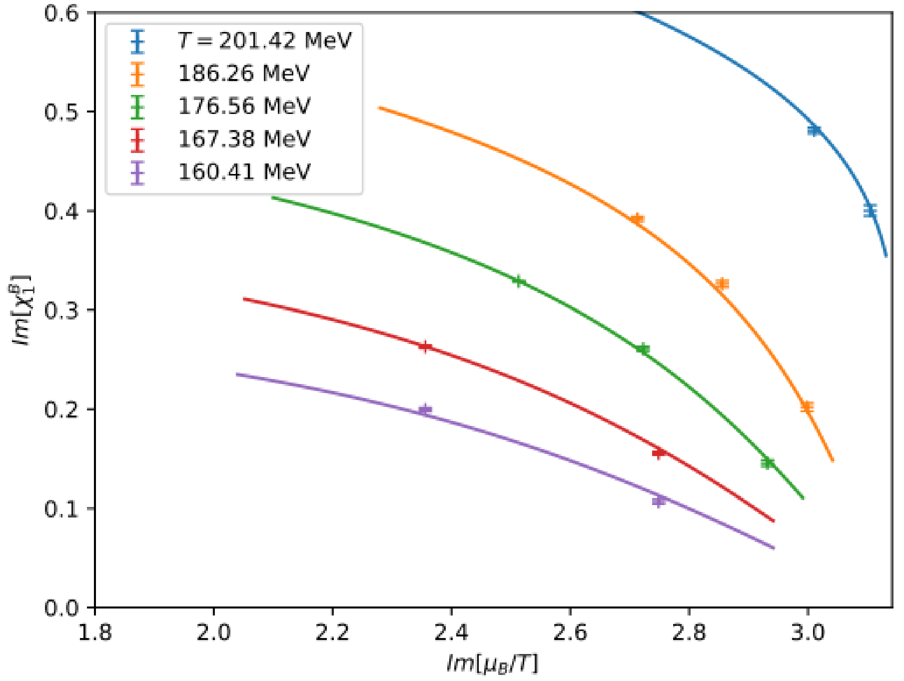

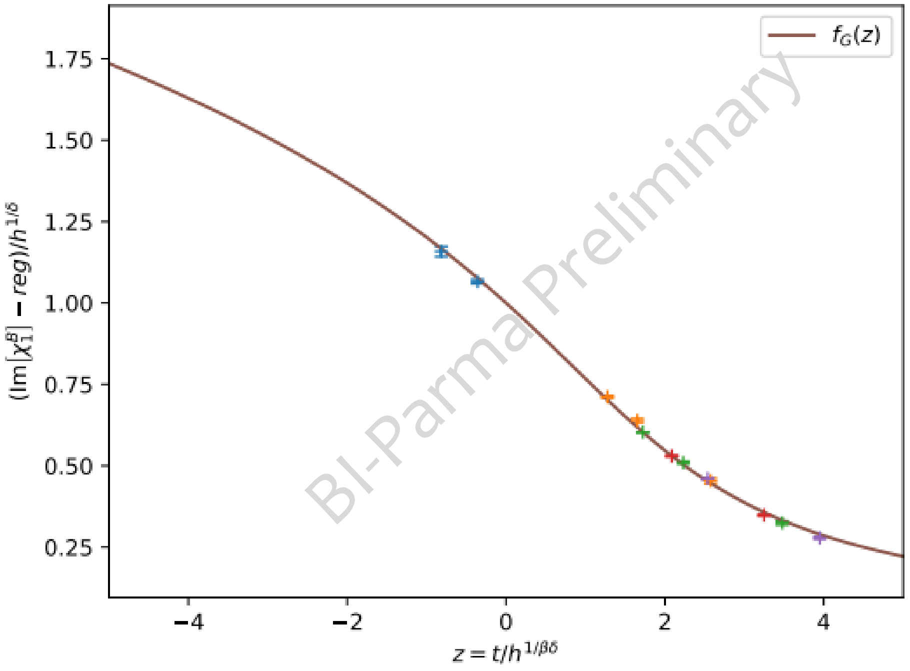

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Preliminary

$z_0 = 5.316 \pm 0.017$

$\chi^2_{test} \approx 6$

LYE singularity method:

$(z_0 = 9.28 \pm 0.26)$

Higly correlated fit parameters!

Chiral transition scaling

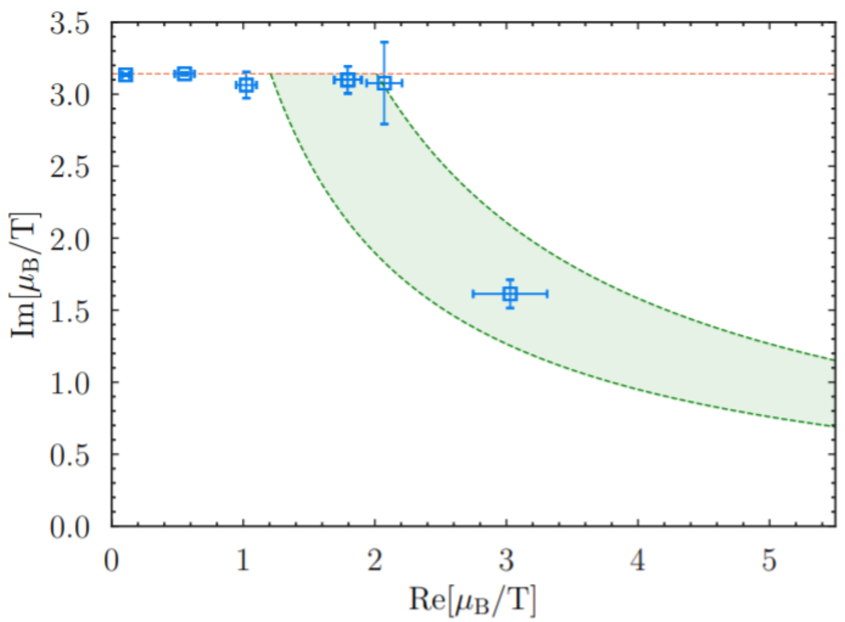
Lattice 36^3 x 6, T_C= 147 MeV

Symmetry of the transition: O(2)

$$t = \frac{1}{t_0}\left[\frac{T-T_c}{T_c} - \kappa_2^B\left(\frac{\mu_B}{T}\right)^2\right] \qquad h = \frac{1}{h_0}\left(\frac{m_l}{m_s}\right)$$

We find the scaling: $\frac{\mu_B}{T} = \left[\frac{1}{\kappa_2^B}\left(\frac{T-T_c}{T_c} - \frac{z_c}{z_0}\left(\frac{m_l}{m_s}\right)^{1/\beta\delta}\right)\right]^{1/2}$

[S. Mukherjee & V. Skokov, arxiv 1909.04639v2]



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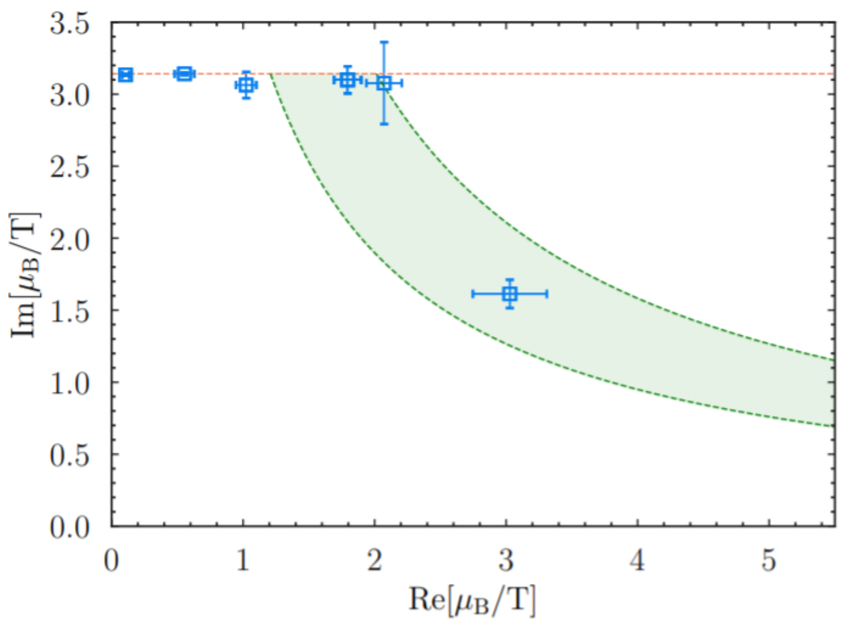
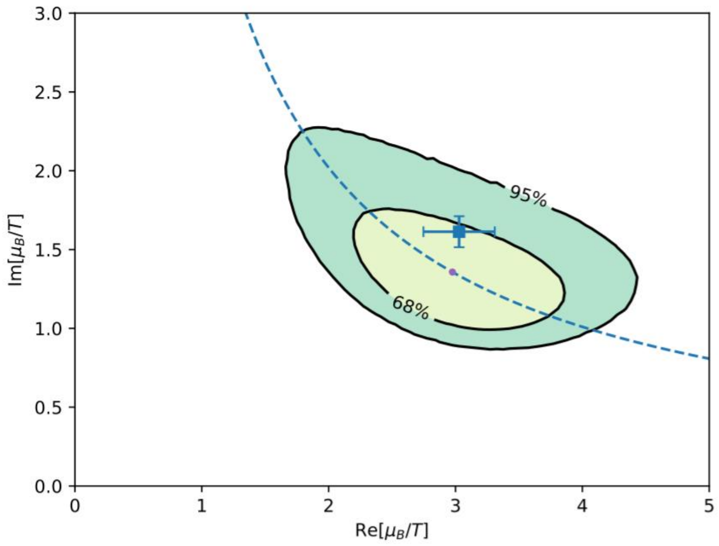


How do we know that is a chiral LYE?
Parameter free prediction from HotQCD collaboration

[HotQCD private comunications]



They are in good agreement

Critical endpoint scaling: an outlook

Hypotetical Scenario

Symmetry of the transition: Z(2)
Unknown map to universal theory

Linear map: $t = \alpha_t(T - T_{cep}) + \beta_t(\mu_B - \mu_B^{cep})$ $\quad h = \alpha_h(T - T_{cep}) + \beta_h(\mu_B - \mu_B^{cep})$ [Gokce Basar, arXiv:2105.08080]

Scaling law:

$$\mu_{LY}(T) \approx c_1(T - T_{cep}) + ic_2|z_c|^{-\beta\delta}(T - T_{cep})$$

Many parameters are unknown!

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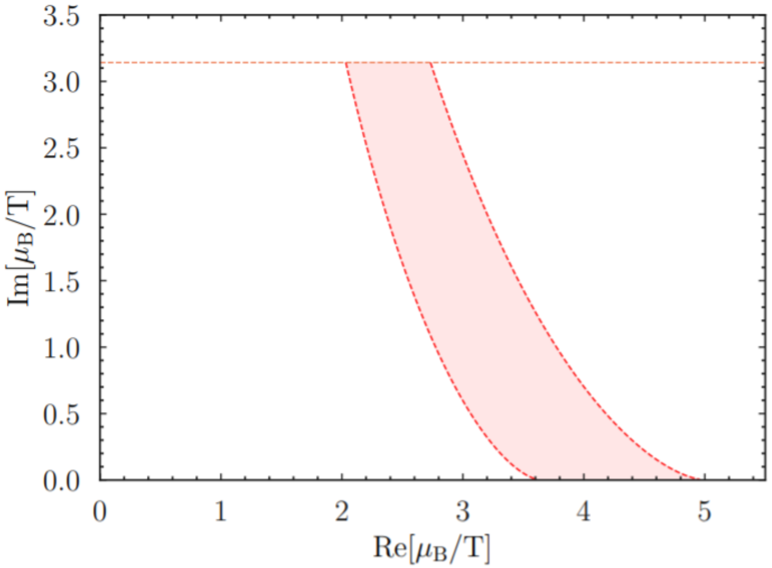

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Many parameters are unknown!

Possible scenario:

$$T_{cep} = T_{pc}(1 - \kappa_2^B(\mu_{cep}/T_{pc})^2)$$
$$T_{pc} = 156.5\, MeV$$
$c_1 = 0.024$ (slope of the transition line at the critical point)
$c_2|z_c|^{-\beta\delta} = 0.5$ (depends on the relative angle between the *h* and *t* axes)

G. Basar used this scaling to identify the **critical point in the Gross-Neveu model.**
In principle it should work for QCD as well, if one is able to get enough **LYE in the red region**.

Summary & conclusions

Imaginary chemical potential and multi-points Padè approximants;
LYE and scaling functions introduced;
Correct scaling of our LYE in the Roberge-Weiss region;
One LYE in the chiral region in good agreement with the prediction;
Critical endpoint region was introduced.

Future prospectives

Get more points on the chiral region;
Possibly start to study the critical end point region.

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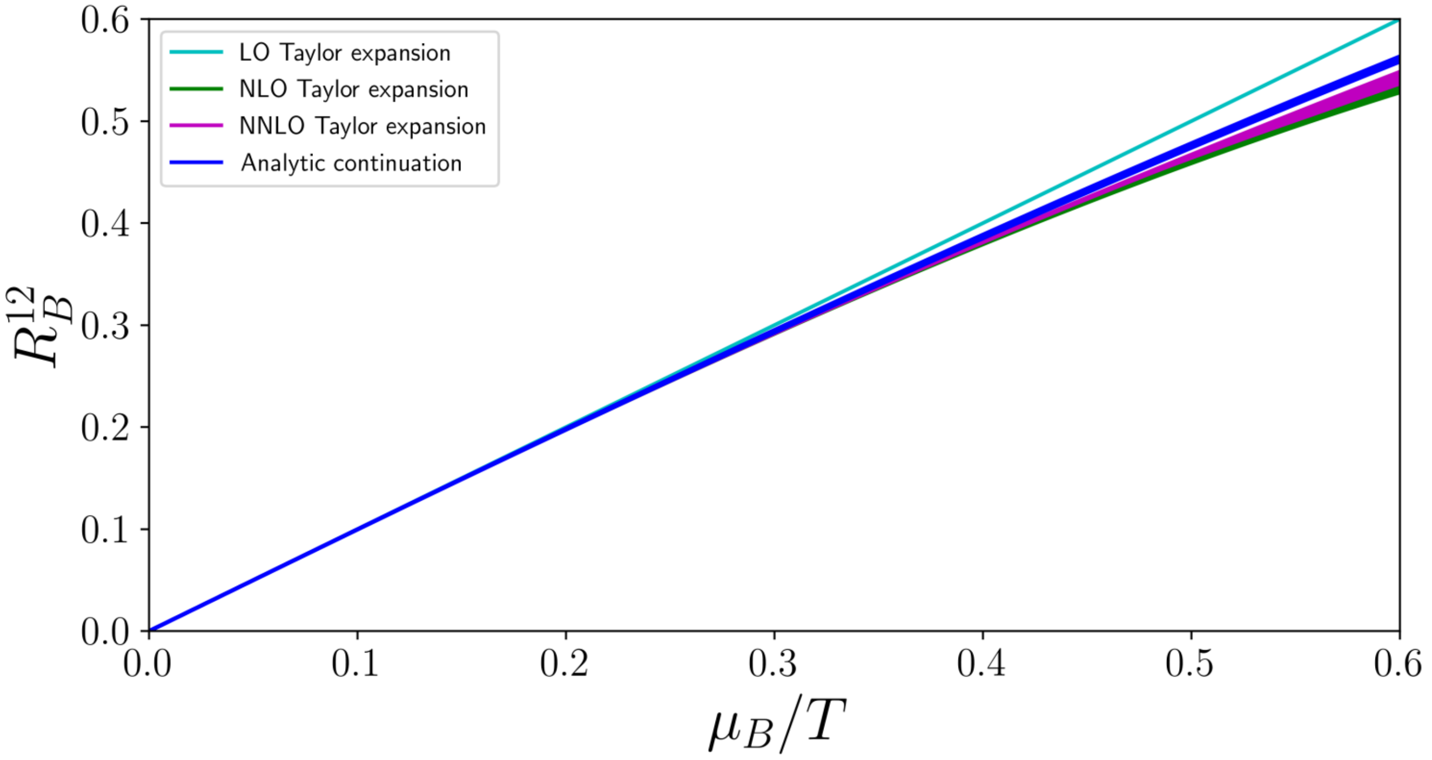
Thanks for the attention

Backup



LO Taylor expansion
NLO Taylor expansion
NNLO Taylor expansion
Analytic continuation
R_B^{12}
μ_B/T
0.0
0.1
0.2
0.3
0.4
0.5
0.6