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The upper right corner of the Columbia plot with staggered fermions

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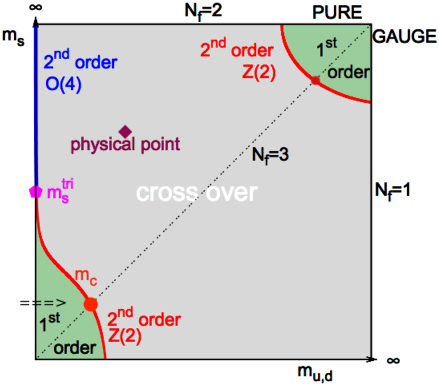
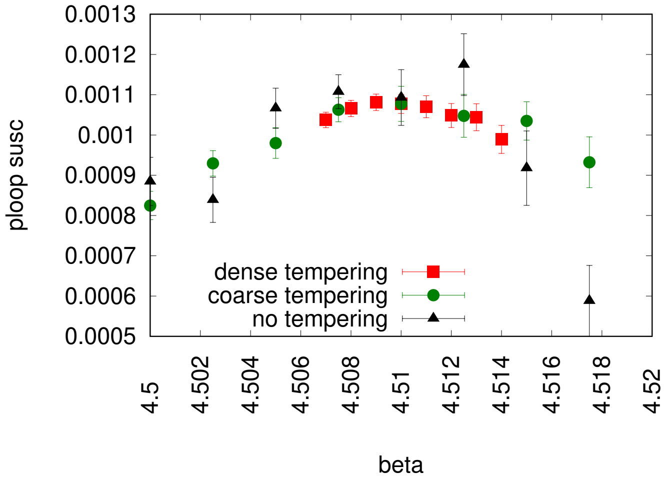
Lattice Field Theory 2021



Forcrand et. al. 1702.00330

Quenched QCD

- Latent heat known in conti. lim. Shirogane et. al. [1605.02997] ⇒ 1st order
- Decreasing quark masses ⇒ transition gets weaker
- Investigations for $N_f = 2$ Wilson fermions Cuteri et. al. [2009.14033]
- Goal: Determination of the critical mass m_c for $N_f = 3$



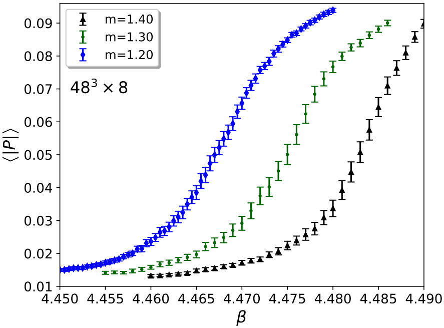
Tempering algorithms

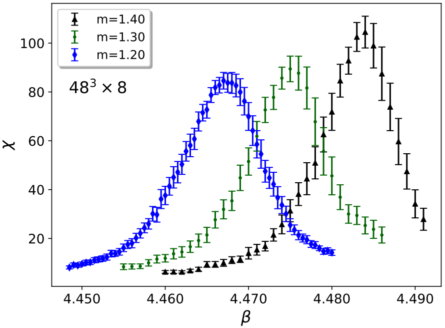
- Several simulations at different couplings β [Marinari 9205018] [Joó 9810032]
- Swap configurations between sub-ensemble pairs $\Rightarrow$ reduced auto-corr.

Observables: Polyakov loop and its susceptibility

$$P = \frac{1}{N_s^3}\sum_{\vec{x}} P_{\vec{x}} = \frac{1}{N_s^3}\sum_{\vec{x}} \mathrm{tr}\left[\prod_{\tau} U_4(\vec{x},\tau)\right] \qquad \chi = N_s^3\left(\langle|P|^2\rangle - \langle|P|\rangle^2\right)$$

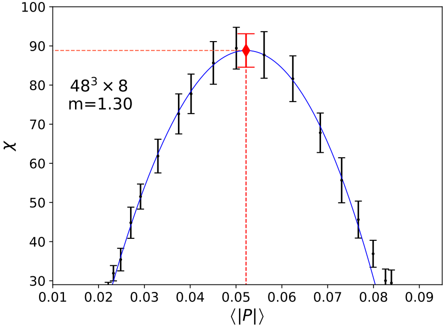
How to determine precisely the peak of χ ?

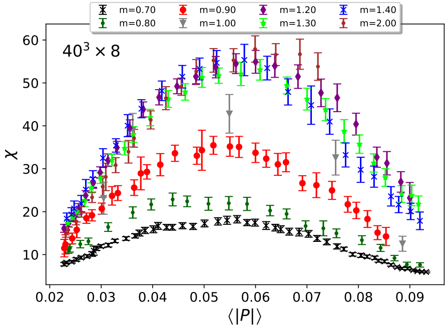




Determination of χ_{max}

- Cubic spline of $\langle|P|\rangle\,(\beta)$ and $\chi(\beta) \Longrightarrow \chi(\langle|P|\rangle)$
- Low-order polynomial fit $\Longrightarrow$ Precise determination of χ_{max}
- Peak position weakly mass dependent
- Peak height grows weakly for highest masses

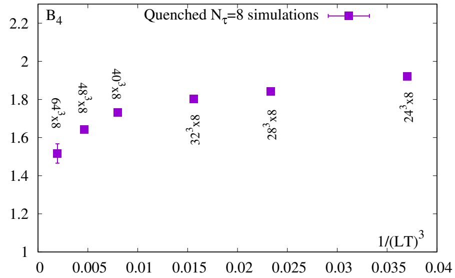


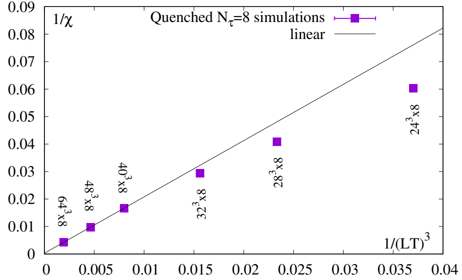


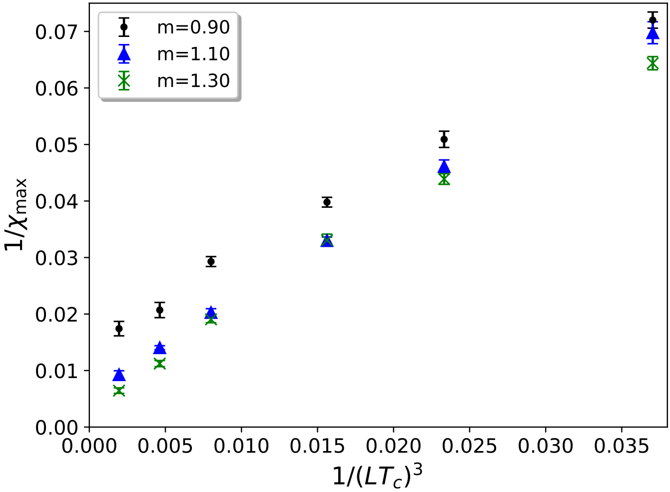
Recall: Quenched case

Polyakov loop and its moments

$$B_n = \frac{\langle (|P| - \langle |P| \rangle)^n \rangle}{\langle (|P| - \langle |P| \rangle^2 \rangle^{\frac{n}{2}}} \qquad \chi = N_s^3 \left(\langle |P|^2 \rangle - \langle |P| \rangle^2 \right)$$



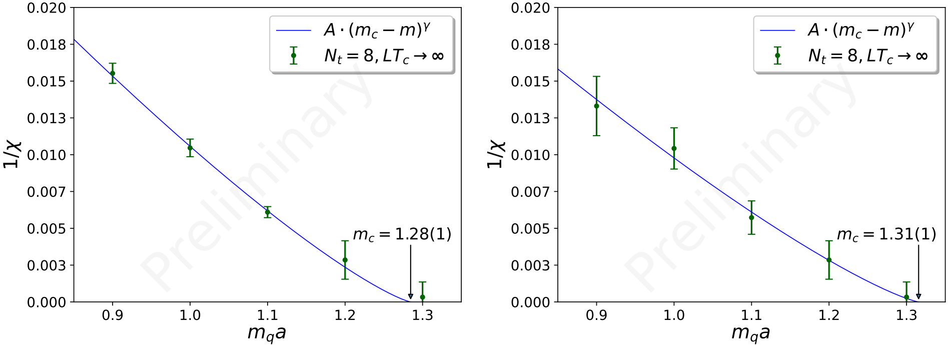




Infinite volume limit for $N_\tau = 8$

- Lattice spatial volumes: 24^3, 28^3, 32^3, 40^3, 48^3, 64^3
- Systematics: Linear and linear with effective exponent

Determination of the critical mass m_c



Infinite volume limit via exemplary two fitfunctions

<u>lhs</u>. m=0.9 , 1.00 , 1.10 linearly fitted. m=1.2, 1.3 linear fit with effective exponent

<u>rhs.</u> linear fit with effective exponent $A + B \cdot m^\alpha$

Dimensionless quantity: $m_{\mathrm{PS}}/T_c = 19.1(1)$ and $\omega_0 T_c = 0.2507(2)$

Summary

Results of dynamical quark simulations

- β tempering algorithms to reduce auto-correlation
- $\chi\left(\langle|P|\rangle\right)$ allows precise peak determination
- Linear behavior of χ_{max}^{-1} as function of $(LT_c)^{-3}$ for small masses
- Determination of the critical mass for $N_t = 8$ around $m_c \cdot a = 1.3$

Outlook

- Systematic error analyzis
- Continuum limit of the critical mass

Thank you for your attention!

Tempering algorithm

Background

- Tempering simulation is collection sub-simulations
- Overll phase space as direct product of sub-ensemble phase spaces
- Equilibrium distribution of configs. in individual sub-ensembles overlap
- Independent Markov-processes in each sub-ensemble
- Swap configs. between sub-ensemble pairs

$\Longrightarrow$ Auto-corr. is reduced

Tempering algorithm

Transition between sub-ensembles

- a is config. in sub-ensemble i, b config. in sub-ensemble j

  $(a, b) \to (b, a) \Leftrightarrow$ swap accepted

  $(a, b) \to (a, b) \Leftrightarrow$ swap rejected

Detailed balance condition

$$P_s(i, j)e^{-\mathcal{H}_i(a)}e^{-\mathcal{H}_j(b)} = P_s(j, i)e^{-\mathcal{H}_j(a)}e^{-\mathcal{H}_i(b)}$$

$$P_s(i, j) = \min\left(1, e^{\Delta\mathcal{H}}\right)$$

$$\Delta\mathcal{H} = \{\mathcal{H}_j(a) + \mathcal{H}_i(b)\} - \{\mathcal{H}_i(a) + \mathcal{H}_j(b)\}$$

Cubic spline Interpolation

$$s_i(x) = a_i + b_i x + c_i x^2 + d_i x^3 \Longrightarrow 4(N-1) \text{ conditions}$$

Smoothness conditions

$$s_i(x_{i+1}) = s_{i+1}(x_{i+1})$$
$$s_i'(x_{i+1}) = s_{i+1}'(x_{i+1})$$
$$s_i''(x_{i+1}) = s_{i+1}''(x_{i+1})$$

$\Longrightarrow 3(N-2)$ constraints

Interpolation condition

$$S(x_j) = y_j$$

$\Longrightarrow N$ conditions

Endpoints

$s_0''(x_0) = 0 = s_{N-2}''(x_{N-1}) \Longrightarrow 2$ conditions

Dimensionless calculation