Bottomonium spectral widths at non-zero temperature using maximum likelihood

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We study the spectrum of the bottomonium system at non-zero temperature using the NRQCD approximation. A maximum likelihood method is used with a Gaussian ansatz for the ground state spectral contribution rather than the traditional delta function. This gives access to the state's width. We apply this approach to the FASTSUM's anisotropic ensembles and compare results for the ground state masses and widths for S- and P-wave states with those from other methods.

Introduction

Study of in-medium properties of mesons at colliders has been an extensive area of research. For a more complete background see [1]. In-medium properties of mesonic bound states are encoded in their spectral function, *ρ(ω,T)*, which is related to the Euclidean correlator *C(τ)*, which is calculable on the lattice for a given kernel, *K(τ,ω,T)*, via

$$C(\tau,T) = \int_0^\infty d\omega \rho(\omega,T) K(\tau,\omega,T)\ . \quad (1)$$

However, the number of data that comprises *C(τ)* is O(10) yet for *ρ(ω,T)* we would like more like O(1000). And thus, this problem is ill-posed [2].

Zero Temperature Ansatz

One simple method for circumventing the inversion problem is to assume a specific parametrisation for the spectral function and use the maximum likelihood estimation to fit the physical parameters of the ansatz to the data; the data in this case is the two-point Euclidean correlation function, *C(τ)*.

To motivate the choice of parametrisation we follow a similar approach to [3], where we consider the energy spectrum of a meson at zero temperature on a finite lattice. In this case we could expect the spectrum to consist only of peaks resembling the Dirac δ-function, where the lowest energy peak corresponds to the ground state, and the intermediate energy peaks to some excited states and finally the highest energy peaks we would expect to form the continuum. Thus, a spectral function could look like

$$\rho(\omega) = \sum_i A_i \delta(\omega - M_i) + \theta(\omega - s_0)\rho_{cont}(\omega). \quad (2)$$

Where *i* labels both ground and excited states, and s_0 is some energy threshold above which the continuum effects are salient. To further simplify this, we notice that the higher energy contribution from the continuum spectral function will be exponentially suppressed in the Laplace transform, and only the first few Euclidean time steps will be sensitive to this behaviour. Therefore, at zero temperature we can just consider the first term.

To extend this logic to thermal correlation functions we can impose that the parametrisation be sensitive to a finite width on the ground state. To do this we replaced the ground state delta function with a Gaussian, and to keep the ansatz robust and contain as few fit parameters as possible we only include one exited state in the ansatz, which remains a Dirac δ-function. What remains is to take the Laplace transform of our parametrisation and fit it to the data, leaving the full ansatz that is fit to the correlator

$$C(\tau) = \mathcal{A}\exp\left(-\tau\left(M_{ground} - \left(\frac{\sigma^2\tau}{2}\right)\right)\right) + \mathcal{A}'\exp\left(-\tau M_{exicted}\right). \quad (3)$$

Results

The correlators used are the FASTSUM Generation-2L NRQCD bottomonium correlators, with the two states of interest being the Υ_0 (S wave) and the χ_{b1} (P wave). These were generated from ensembles on an anisotropic lattice with an M_π approximately 240MeV [4].

Effective Mass

To isolate the ground state in such a fit one must have a region where it is sensible to postulate that the ground state is the dominant contribution to the correlator. The effective mass, defined through

$$M_{eff} = \frac{\log(C(\tau))}{\log(C(\tau+1))}, \quad (4)$$

acts as the indicator here. A plateau in the effective mass over a certain region in time is indicative of only one state contributing significantly, apparent through a combination of equations (1) and (2). Figure 2 shows that even at temperatures above the critical temperature the effective mass does flatten off, if not completely. So, one should be able to extract ground state properties.

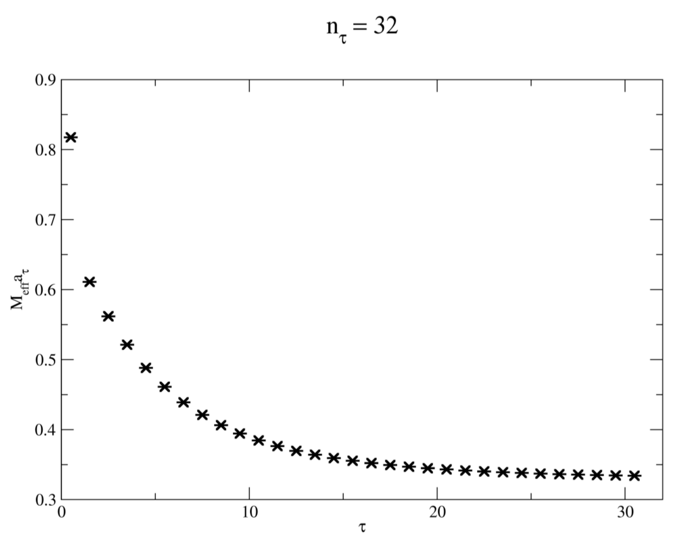

Figure 1: The effective mass as a function of time for the S-wave state Υ_0 at a temperature of 187 GeV.

Comparison With Other Methods

Figure 2 contains the results of the fitted ground state mass of the S-wave Υ_0, in lattice units, as a function of N_τ, a proxy for inverse temperature. Overlaid are the other, preliminary, results from other methods performed by colleagues in the FASTSUM collaboration. The agreement between all the other methods is clear, with the method from this poster also agreeing qualitatively. All show an increase in mass with temperature and even that the onset of this is above the critical temperature – corresponding to $N_\tau < 37$. The degree to which the mass seems to depend on the critical temperature varies with method.

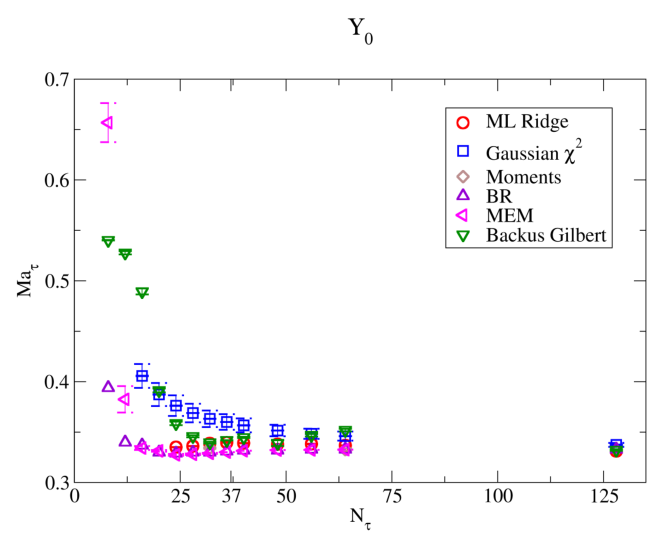

Figure 2: A preliminary comparison of the ground state mass for the Υ_0 particle against N_τ for various spectral reconstruction methods performed by the FASTSUM collaboration. Statistical errorbars are given by solid lines and systematic ones by dashed lines.

Systematic Errors

We note that this method rigidly assumes that the excited state can be modelled with a single exponential. This will become a poor approximation which can be tested by varying the time window.

A fit window not necessarily needed, but for many reasons it can be beneficial. Two such reasons are that the initial time step is more of an imposed initial condition than anything physical, so its inclusion is not necessarily beneficial; and the earlier times contain the largest excited state contribution, and these are not the states we consider as signal, we care mostly for the ground state.

Clearly, by reference to Figure 3, the fitted value of mass is sensitive to the time window chosen, acting as a source of systematic error.

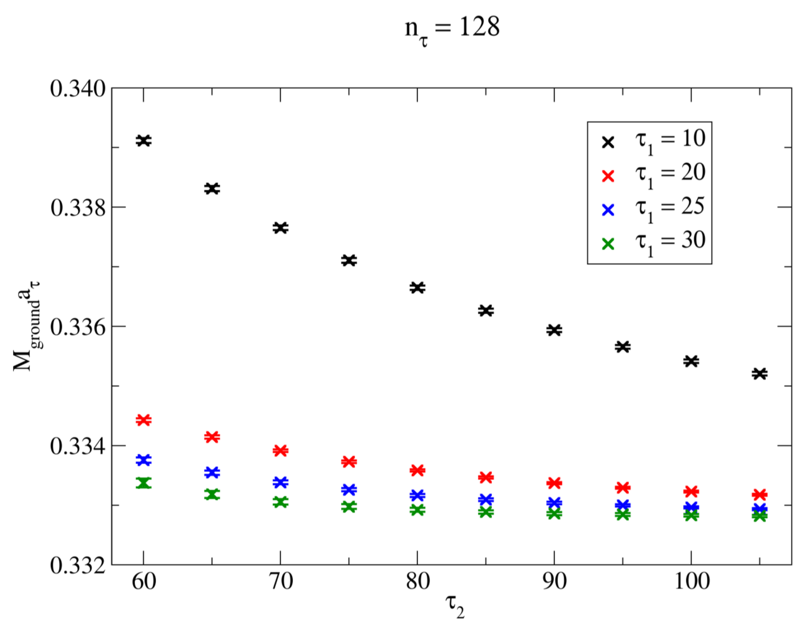

Figure 3: The fitted value of the ground state mass for the Υ_0 particle at a temperature of 47 GeV, where the fit is performed over a range of time windows, starting at τ_1 and ending at τ_2. The errors shown are purely statistical, and from the trend of the mass changing significantly with the systematic change of time window one can see how the systematic errors greatly outweigh the statistical ones.

Future Works

Currently there is work underway to replace the single exponential that acts as the excited state with an NRQCD sum rules inspired ansatz.

There is also mass and width values for P-wave states for each method shown in Figure 2.

[1] Mocsy, A., Petreczky, P. and Strickland, M., 2013. Quarkonia in the quark gluon plasma. *International Journal of Modern Physics A*, *28*(11), p.1340012.
[2] Hadamard, J., 1902. Sur les problèmes aux dérivées partielles et leur signification physique. *Princeton university bulletin*, pp.49-52
[3] Larsen, R., Meinel, S., Mukherjee, S. and Petreczky, P., 2019. Thermal broadening of bottomonia: Lattice nonrelativistic QCD with extended operators. *Physical Review D*, *100*(7), p.074506.
[4] Aarts, G. *et al*. 2019. Spectral quantities in thermal QCD: a progress report from the FASTSUM collaboration. *arXiv preprint arXiv:1912.09827*.