(2+1+1)-flavor QCD equation of state on coarse lattices

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Nuclear Matter equation of state on the lattice

The quark-gluon plasma (QGP), the high-temperature phase of bulk nuclear matter, has been studied in ultra-relativistic heavy-ion collision (HIC) experiments at RHIC (BNL), LHC (CERN) for many years, and will be probed after their upgrades and in future experiments such as FAIR (GSI) and NICA (JINR), too. At vanishing baryon density the transition between the hadron gas and the QGP takes place as a broad chiral crossover around a temperature of $T_{\rm pc} = 156.5(1.5)$ MeV at the physical point [1]. The thermodynamic properties of QGP are given in terms of its equation of state (EoS), which has been studied extensively on the lattice in pure gauge theory (without sea quarks) [2], or with 2+1 dynamical flavors (*i.e.* light quarks in the isopin limit, and a physical strange quark) of sea quarks [3, 4, 5]; after clearing up discrepancies between early lattice calculations due to a poorly controlled continuum limit, ***good agreement was achieved in (2+1)-flavor QCD***.

Heavy quarks in the bulk

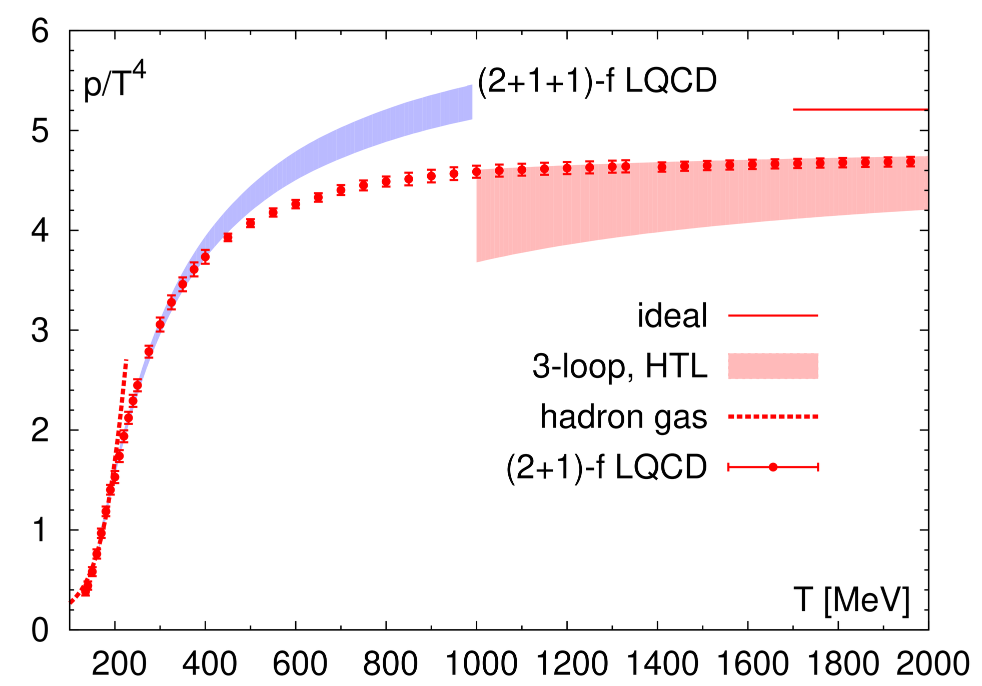

Figure 1: The EoS in (2+1)-flavor [5] or (2+1+1)-flavor lattice QCD [6] differs at $T \gtrsim 400$ MeV; it is still about 10% below ideal gas limit at $T \gtrsim 1$ GeV.

Heavy quarks are negligible in nuclei. Instead, they are produced in hard processes during early stages of the HIC. Future HIC experiments at larger $\sqrt{s}$ will lead to higher temperature and copious production of charm. Thus, it is urgent to include dynamical charm quarks on the lattice. Heavy quarks are challenging due to the large discretization errors associated with their mass, see e.g. the difficulty of the continuum limit for moments of pseudoscalar charmonium correlators [7]. At $T \gtrsim 2T_{\rm pc}$ the previously dominant gluon contribution and the light or strange quark contributions die down rapidly, whereas the ***contribution from charm quarks catches up as thermal scales, i.e. πT, approach its mass*** ($\overline{\rm MS}$: $m_c(m_c, N_f = 4) = 1.2735(35)$ GeV [8]). Charm quarks dominate the EoS ***before weak coupling becomes reliable***, see Fig. 1.

Although results in (2+1+1)-flavor QCD (*i.e.* with a charm sea) have been obtained already some time ago [6], ***no independent cross-check*** through a calculation using another discretization for the charm sea is ***available yet***. We report on an ongoing (2+1+1)-flavor QCD study [9, 10] with ***highly improved staggered quark*** (HISQ) action [11] optimized for controlling heavy-quark mass discretization effects.

Lattice setup, action parameters and scale setting

Any lattice calculation of the EoS is computationally demanding. In the traditional approach that we follow, i.e. the ***integral method***, both $T > 0$ and $T = 0$ ensembles with high statistics are needed at each bare gauge coupling to cancel UV divergences. We use coarse $T > 0$ lattices with aspect ratio $N_\sigma/N_\tau = 4$ and ***temporal extent*** $N_\tau = 6$ ***or*** 8; the temperature is set as $T = 1/(aN_\tau)$. The data set is anchored to a set of existing, high statistics MILC ensembles [12] at $T = 0$ along the line of constant physics (LCP) with a ***light quark mass*** $m_l = m_s/5$, i.e. $m_\pi \approx 300$ MeV in the continuum limit. We combine ***HISQ*** [11] with a tadpole one-loop improved gauge action. HISQ suppresses taste exchanges and diminishes mass splittings in the pion sector; this improves the approach to the continuum limit at low temperatures. HISQ is $O(a^2)$-improved at tree-level due the Naik (three-link) term, which improves scaling at high temperatures [5], and contains a mass-dependent correction ϵ_N for the charm quark [11], which reproduces the ***correct charm dispersion relation*** at tree-level up to $O((am_c)^4)$.

We compute the static energy at $T = 0$ to set the lattice spacing a using the scale $r_1 \simeq 0.31$ fm [13]. Strange and charm quark masses are tuned to physical values by using masses of π, K, and the spin average of η_c and J/ψ. The tadpole factor defined from the trace of the plaquette $u_0 = \langle {\rm Tr}\, U_p/3\rangle^{1/4}$ is determined during thermalization of the $T = 0$ ensembles.
We cover a window of $T \in [149, 967]$ MeV with $N_\tau = 6$ and $T \in [136, 725]$ MeV with $N_\tau = 8$.

Trace anomaly and vacuum subtraction

In the traditional approach the EoS can be obtained from the trace of the energy-momentum tensor (EMT), $\Theta^{\mu\mu} = \varepsilon - 3p$, where ε or p are energy density or pressure. $\Theta^{\mu\mu}$ is related to the partition function as

$$\frac{\Theta^{\mu\mu}}{T^4} = -\frac{T}{V}\frac{d\ln Z}{d\ln a}, \quad Z = \int DU D\bar\psi D\psi\, e^{-S_g - S_f}. \tag{1}$$

The temperature-independent divergences of any individual contribution X to $\Theta^{\mu\mu}$ due to mixing with the identity operator can be removed by subtracting the vacuum result for this operator X, i.e.

$$\Delta(X) = \langle X\rangle_\tau - \langle X\rangle_0\,. \tag{2}$$

The ***vacuum-subtracted trace anomaly*** is given in terms of the basic ingredients of the action,

$$\frac{\Theta^{\mu\mu}}{T^4} = -R_\beta(\beta)\left[\Delta(S_g) + R_u(\beta)\Delta\left(\frac{dS_g}{du_0}\right)\right] + R_\beta(\beta)R_{m_s}(\beta)\left[2m_l\Delta(\bar\psi_l\psi_l) + m_s\Delta(\bar\psi_s\psi_s)\right]$$
$$+ R_\beta(\beta)R_{m_c}(\beta)\left[m_c\Delta(\bar\psi_c\psi_c) + R_{\varepsilon_N}(\beta)\Delta\left(\bar\psi_c\left[\frac{dM_c}{d\varepsilon_N}\right]\psi_c\right)\right], \tag{3}$$

after the lattice spacing derivatives have been rephrased in terms of β functions and action parameter derivatives. Changes of the lattice spacing and the action parameters along the LCP are controlled by lattice β-functions:

$$R_\beta(\beta) = T\frac{{\rm d}\beta}{{\rm d}T} = -a\frac{{\rm d}\beta}{{\rm d}a} = (r_1/a)(\beta)\left(\frac{{\rm d}(r_1/a)(\beta)}{{\rm d}\beta}\right)^{-1}, \tag{4}$$

$$R_{m_q}(\beta) = \frac{1}{am_q(\beta)}\frac{{\rm d}am_q(\beta)}{{\rm d}\beta} \quad \text{for } q = s, c\,, \quad R_u(\beta) = \beta\frac{{\rm d}u_0(\beta)}{{\rm d}\beta}, \quad R_\epsilon(\beta) = \frac{{\rm d}\epsilon_N(\beta)}{{\rm d}\beta}. \tag{5}$$

Lattice β functions

We have determined the β-functions by fitting the data to the following Ansätze. For the lattice spacing:

$$\frac{r_1}{a}(\beta) = \frac{c_r^{(0)}f(\beta) + c_r^{(2)}(10/\beta)f^3(\beta)}{1 + d_r^{(2)}(10/\beta)f^2(\beta)}, \tag{6}$$

and for the strange or charm quark masses ($q = s, c$):

$$am_q(\beta) = \frac{c_q^{(0)}f(\beta) + c_q^{(2)}(10/\beta)f^3(\beta)}{1 + d_q^{(2)}(10/\beta)f^2(\beta)}\left(\frac{20b_0}{\beta}\right)^{\frac{4}{9}}. \tag{7}$$

We use the $N_f = 3$ two-loop β-function,

$$f(\beta) = \left(\frac{10b_0}{\beta}\right)^{-b_1/(2b_0^2)}\exp\left(-\beta/20b_0\right), \tag{8}$$

and checked that the $N_f = 4$ β-function would produce statistically consistent results in Eqs. (6) and (7).
To obtain the β derivatives in Eq. (5), we fit u_0 with $u_0(\beta) = c_1 + c_2e^{-d_1\beta}$ and ϵ_N with a polynomial in β.

Numerical results

The gauge configurations are generated with the RHMC algorithm. At zero temperature we save lattices every 5 or 6 and at finite temperature every 10 molecular dynamics time units (TU). The statistics for the $N_\tau = 6$ and 8 ensembles is reaching for most of them 100 thousand TUs. After calculating the trace anomaly we interpolate it with splines and then evaluate the pressure via the integral method. In Figs. 2 and 3 we compare the pressure in (2+1+1)-flavor QCD along the line of constant physics $m_\pi \approx 300$ MeV with the pressure in (2+1)-flavor QCD along the line of constant physics $m_\pi \approx 160$ MeV [5]. Note that due to the difference in the pion mass the (2+1+1)-flavor pressure is below the (2+1)-flavor pressure at low temperatures where the contribution of the charm quark is still negligible.

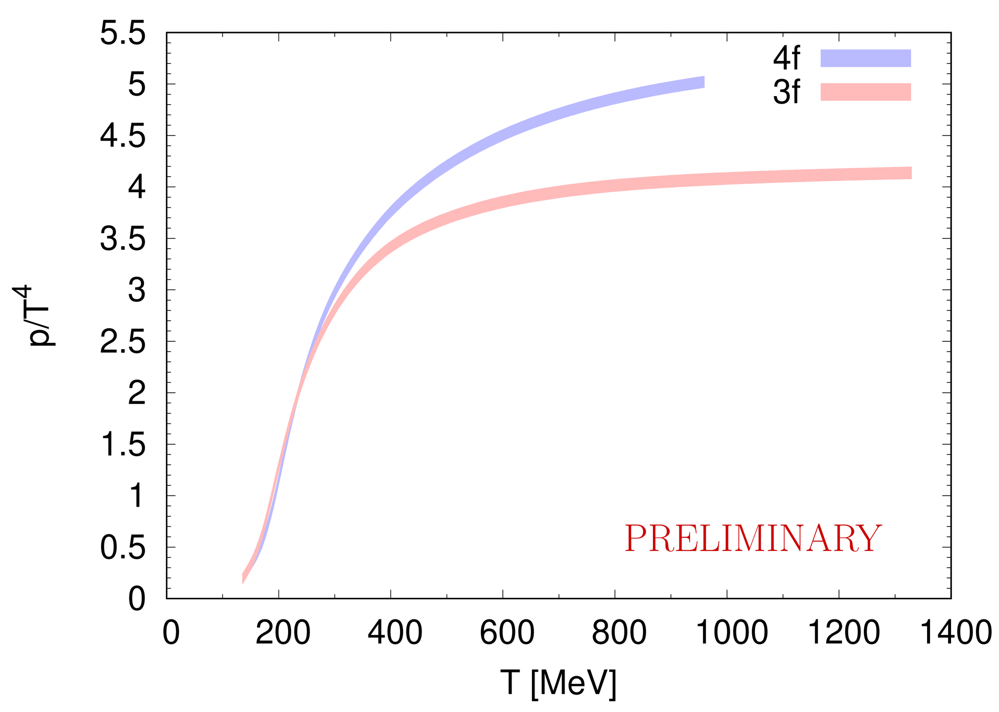

Figure 2: Pressure as function of temperature on $N_\tau = 6$ lattices for (2+1)- and (2+1+1)-flavor QCD. The errors are purely statistical.

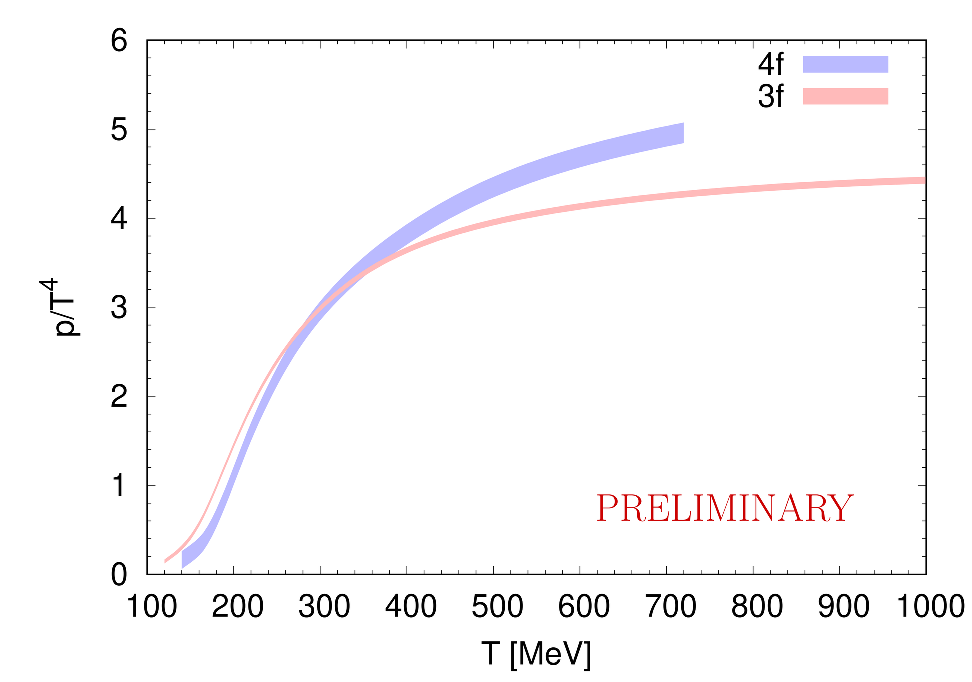

Figure 3: Pressure as function of temperature on $N_\tau = 8$ lattices for (2+1)- and (2+1+1)-flavor QCD. The errors are purely statistical.

References

[1] A. Bazavov et al. Chiral crossover in QCD at zero and non-zero chemical potentials. *Phys. Lett. B*, 795:15–21, 2019.

[2] Leonardo Giusti and Michele Pepe. Equation of state of the SU(3) Yang–Mills theory: A precise determination from a moving frame. *Phys. Lett. B*, 769:385–390, 2017.

[3] Szabocls Borsanyi, Zoltan Fodor, Christian Hoelbling, Sandor D. Katz, Stefan Krieg, and Kalman K. Szabo. Full result for the QCD equation of state with 2+1 flavors. *Phys. Lett. B*, 730:99–104, 2014.

[4] A. Bazavov et al. Equation of state in (2+1)-flavor QCD. *Phys. Rev. D*, 90:094503, 2014.

[5] A. Bazavov, P. Petreczky, and J. H. Weber. Equation of State in 2+1 Flavor QCD at High Temperatures. *Phys. Rev. D*, 97(1):014510, 2018.

[6] Sz. Borsanyi et al. Calculation of the axion mass based on high-temperature lattice quantum chromodynamics. *Nature*, 539(7627):69–71, 2016.

[7] Peter Petreczky and Johannes Heinrich Weber. Strong coupling constant from moments of quarkonium correlators revisited. 12 2020.

[8] Javad Komijani, Peter Petreczky, and Johannes Heinrich Weber. Strong coupling constant and quark masses from lattice QCD. *Prog. Part. Nucl. Phys.*, 113:103788, 2020.

[9] A. Bazavov et al. Towards a QCD Equation of State with 2 + 1 + 1 Flavors using the HISQ Action. *PoS*, LATTICE2012:071, 2012.

[10] A. Bazavov et al. Update on the 2+1+1 Flavor QCD Equation of State with HISQ. *PoS*, LATTICE2013:154, 2014.

[11] E. Follana, Q. Mason, C. Davies, K. Hornbostel, G. P. Lepage, J. Shigemitsu, H. Trottier, and K. Wong. Highly improved staggered quarks on the lattice, with applications to charm physics. *Phys. Rev. D*, 75:054502, 2007.

[12] A. Bazavov et al. *B*- and *D*-meson leptonic decay constants from four-flavor lattice QCD. *Phys. Rev. D*, 98(7):074512, 2018.

[13] R. Sommer. A New way to set the energy scale in lattice gauge theories and its applications to the static force and alpha-s in SU(2) Yang-Mills theory. *Nucl. Phys. B*, 411:839–854, 1994.

Acknowledgements

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