Spectral Reconstruction in NRQCD via the Backus-Gilbert Method

Ben Page (Speaker)
Swansea University

FASTSUM

Gert Aarts, Chris Allton, Benjamin Jäger, Seyong Kim, Maria Paola Lombardo, Sam Offler, Sinead M. Ryan, Jon-Ivar Skullerud, Thomas Spriggs

Lattice 2021 - July 30, 2021

Spectral Reconstruction in NRQCD

In non-relativistic QCD, the spectral representation of the correlation function may be written

$$G(\tau) = \int_{-2M}^{\infty} \rho(\omega) e^{-\omega\tau} \mathrm{d}\omega \tag{1}$$

This can be treated via the Backus-Gilbert method with kernel $K(\omega, \tau) = e^{-\omega\tau}$.

- This is an inverse Laplace problem
- Is ill-conditioned in nature

Backus-Gilbert: Given some data $G(\tau) = \int \rho(\omega) K(\omega, \tau) d\omega$, construct an estimate of the localised average of $\rho(\omega)$ (denoted $\hat{\rho}$).

Overview - The Backus-Gilbert Method

- Generate *averaging functions* $A(\omega, \omega_0) = \sum_\tau c_\tau K(\omega, \tau)$ such that

$$\hat{\rho}(\omega_0) = \int_{\omega_{\text{min}}}^{\omega_{\text{max}}} A(\omega, \omega_0)\rho(\omega)\ \mathrm{d}\omega \tag{2}$$

- Backus-Gilbert estimate of the localised average:

$$\hat{\rho}(\omega_0) = \sum_\tau c_\tau G(\tau) \tag{3}$$

$\implies$ Backus-Gilbert method performed for every ω_0 sampled

Calculating the averaging coefficients c_τ

The averaging coefficients c_τ are found by minimising the *width* of $A(\omega, \omega_0)$, subject to some definition:

- Backus and Gilbert's spread function[1]:

$$W(\omega_0) = 12 \int (\omega - \omega_0)^2 A(\omega, \omega_0)^2 d\omega \tag{4}$$

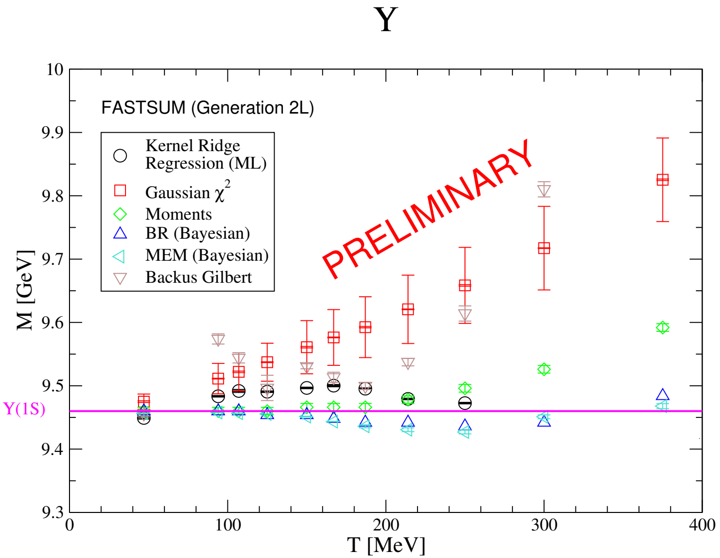
  subject to the condition $\int A(\omega, \omega_0) d\omega = 1$.

- The Dirichlet (least sq.) criterion[2]:

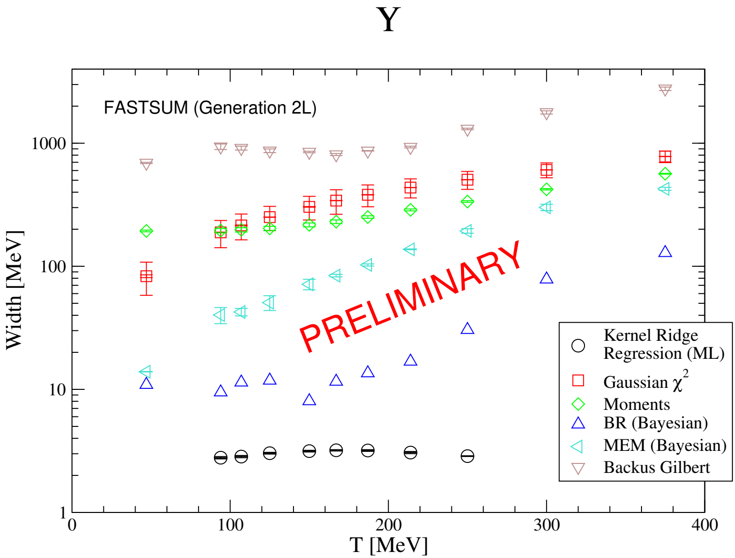
$$J(\omega_0) = \int [A(\omega, \omega_0) - \delta(\omega - \omega_0)]^2 d\omega \tag{5}$$

[1] Backus, G., & Gilbert, F. (1968). The Resolving Power of Gross Earth Data.
[2] Oldenburg, D. W. (1984). An Introduction to Linear Inverse Theory.

Performance comparison - Υ $b\bar{b}$ - vs. FASTSUM

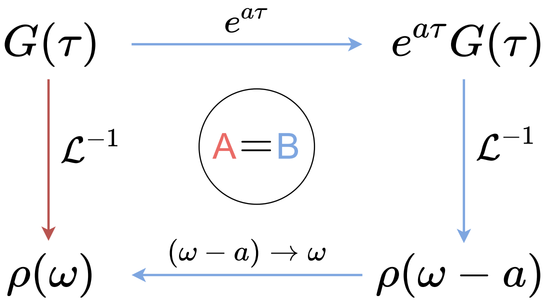


Performance comparison - Υ $b\bar{b}$ - vs. FASTSUM



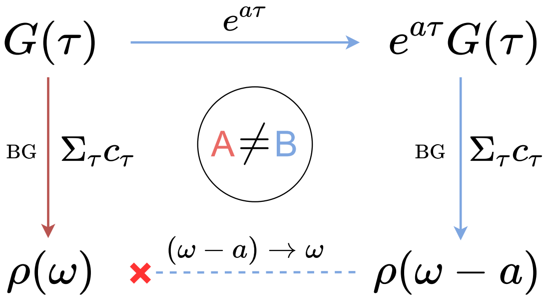
Increasing resolution via Laplace shifting

Using standard Laplace transform rules, we can consider two ‘routes’ between $G(\tau)$ and $\rho(\omega)$:



Increasing resolution via Laplace shifting

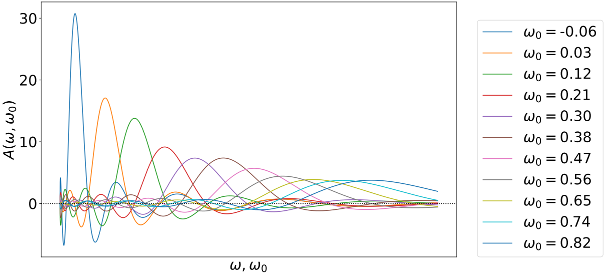
Naively, we can replace the inverse transform with the BG method:



However, as $A(\omega, \omega_0) \neq \delta(\omega - \omega_0)$ we find $\rho_A(\omega) \neq \rho_B(\omega)$.

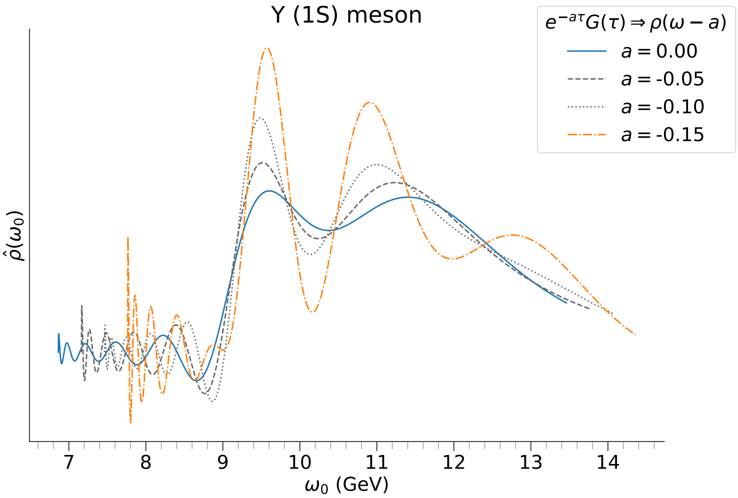
Increasing resolution via Laplace shifting

Q: Can we take advantage of this?
A: Yes (but with some caveats!)

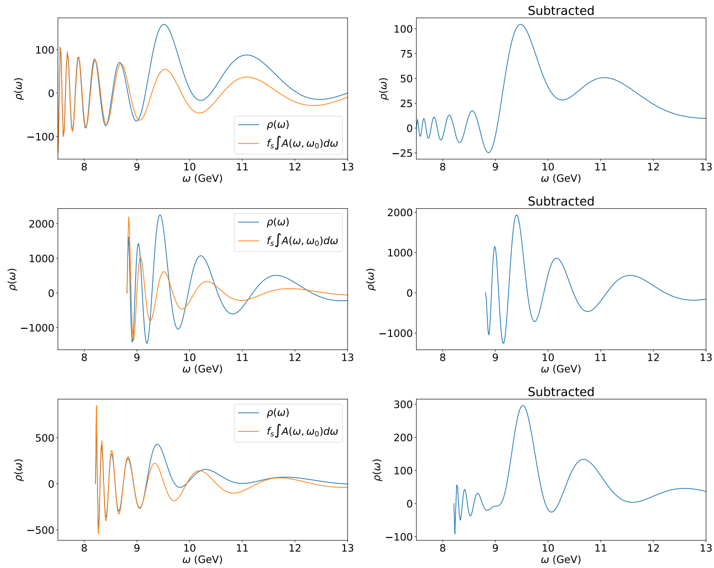


- In general, the resolving power of $A(\omega, \omega_0)$ increases as $\omega_0 \to \omega_{\text{min}}$.
- Laplace shifting moves $\rho(\omega)$ w.r.t the underlying $A(\omega, \omega_0)$.
- Must assume that $\rho(\omega) \sim 0$ for $\omega < -a$

Laplace Shifting - Example



Laplace Shifting - Noise Subtraction



Laplace Shifting - Questions

Some open questions:

- How does the shifting rule affect whitened/regularised width criteria?
- What is the optimal value of shift parameter a?
- What causes the ‘noise’ in the shifted signal? Why can’t we reliably subtract this?
- How do Laplace-shifted results compare with original BG estimates of masses and widths?

Conclusion and comments

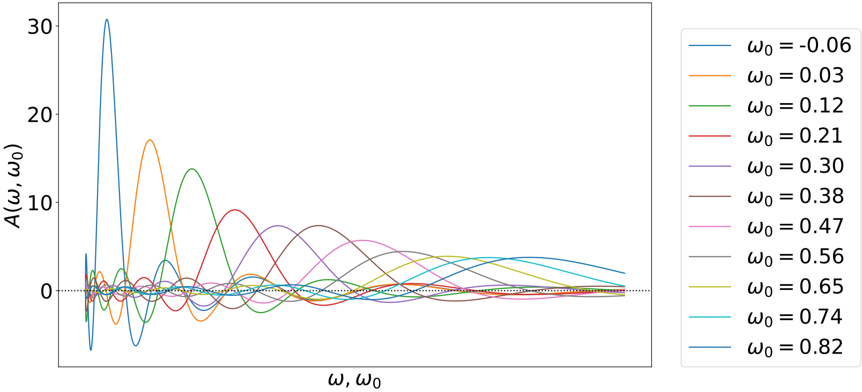
- Backus-Gilbert not competitive (compared to MEM, BR, Kernel Ridge etc.) for determination of state widths.
- Exploiting Laplace shift transform reveals higher resolution, but at a cost (noise).
- Even with a refined shifting and noise subtraction routine, Backus-Gilbert is still a numerically inefficient method.

Thank you for your time.

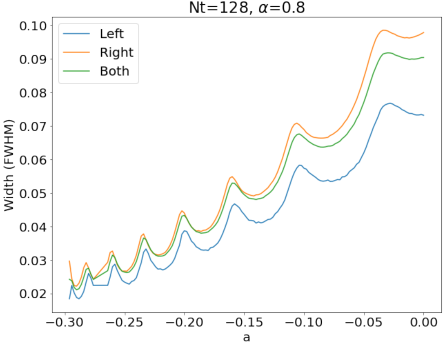
Any questions are welcome and I will be in Gather Town after this session.

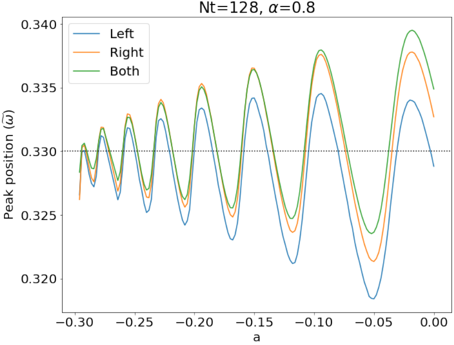
Technical plots overleaf...

Producing the localised average

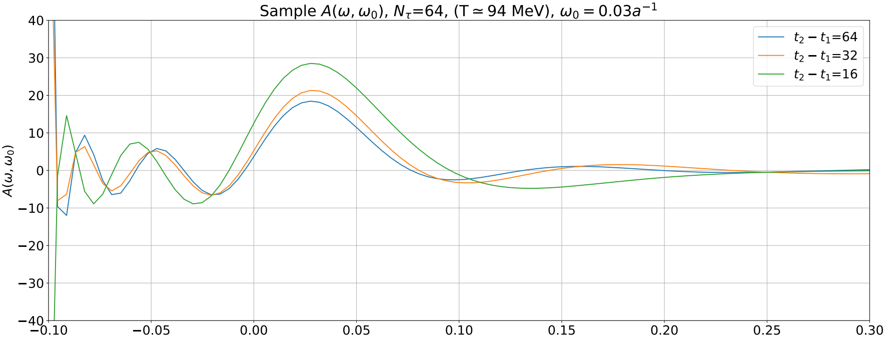


Masses and Widths from Laplace Shifting





Varying the Euclidean time range



Resolution limit