Tetraquark channels with $\bar{b}b$ pair in the static limit

The 38th International Symposium on Lattice Field Theory (28TH of July 2021)

LATTICE21

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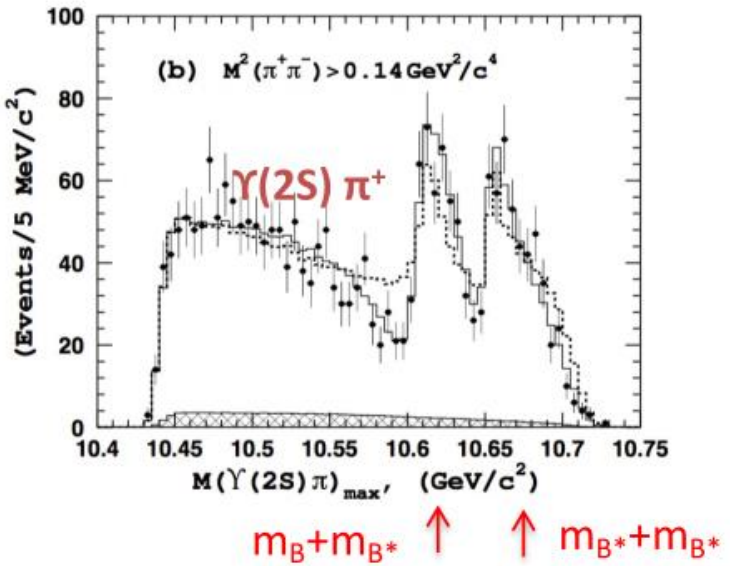
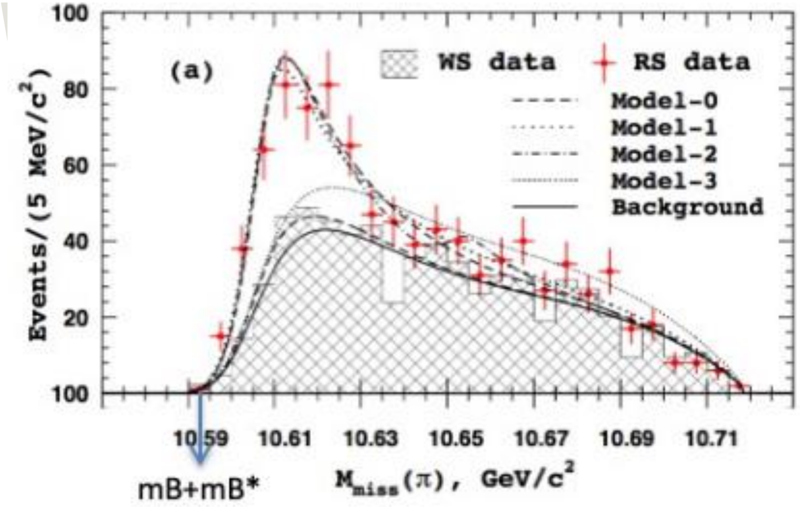
[*]speaker

Experimental evidence of Z_b

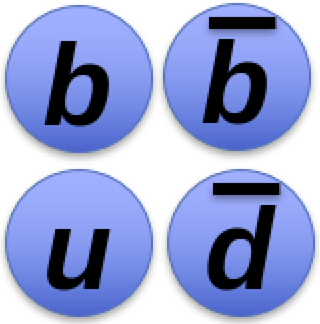


Belle, PRD **91**, 072003 (2015)

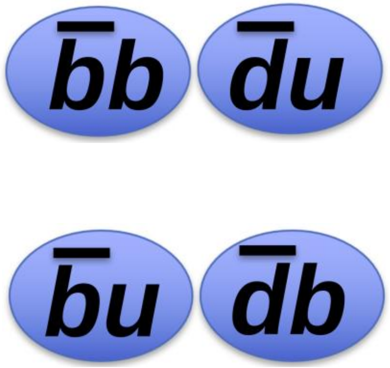


Belle, PRL **116**, 212001 (2016)

Z_b^+:



- Discovery by Belle in 2011
  - Belle, PRL **108**, 122001 (2012)
- I=1, $J^{PC}=1^{+-}$
- $Z_b(10610)$:
  - $B\bar{B}^*$ threshold
- $Z_b(10650)$:
  - $B^*\bar{B}^*$ threshold
- Decay modes:
  - $\Upsilon(1S)\pi$, $\Upsilon(2S)\pi$, $\Upsilon(3S)\pi$
  - $h_b(1P)\pi$, $h_b(2P)\pi$
  - $B\bar{B}^*$ ($Z_b(10610)$), $B^*\bar{B}^*$ ($Z_b(10650)$)



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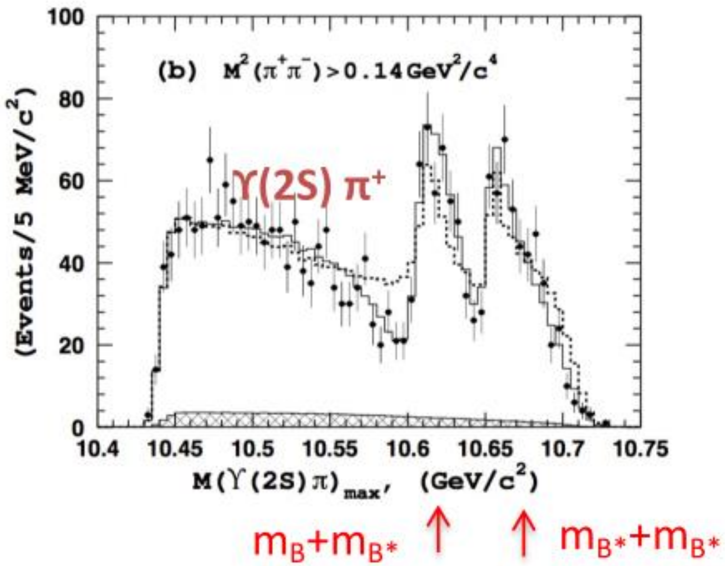

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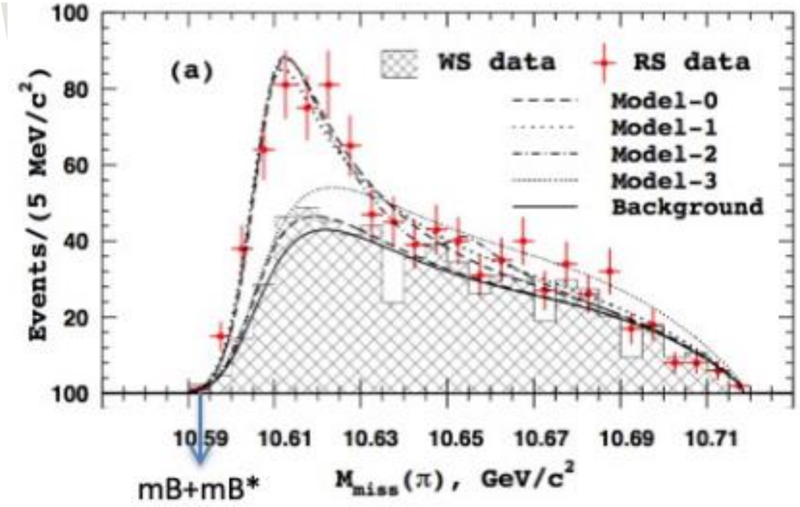

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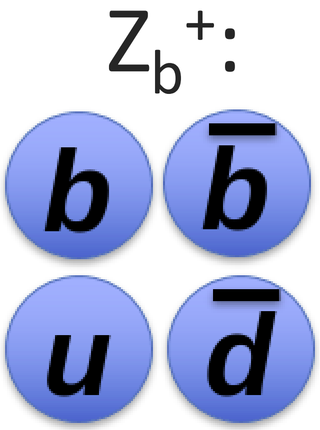
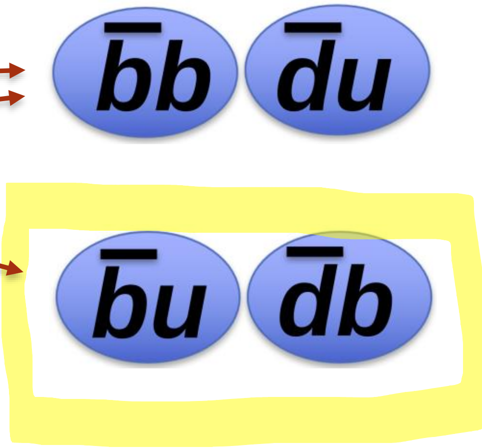
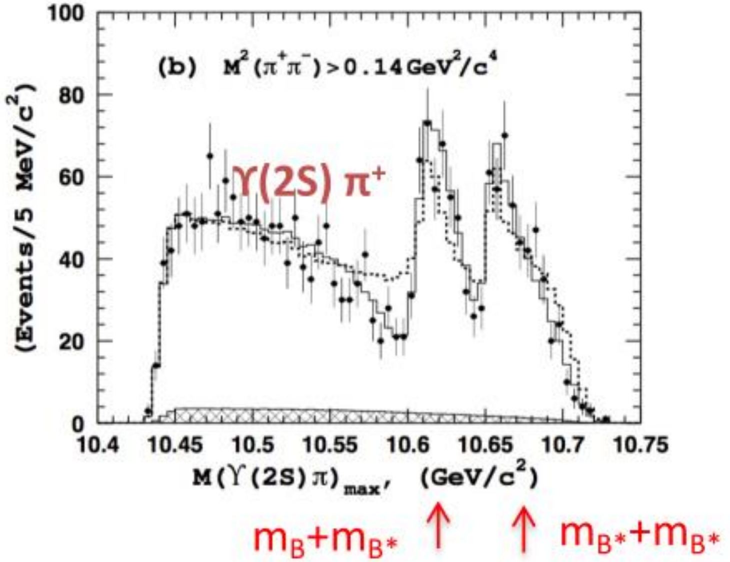
- Decay modes:
  - Υ(1S)π, Υ(2S)π, Υ(3S)π
  - $h_b(1P)\pi$, $h_b(2P)\pi$
  - **Dominant** ⟶ $B\bar{B}^*$ ($Z_b(10610)$), $B^*\bar{B}^*$ ($Z_b(10650)$)







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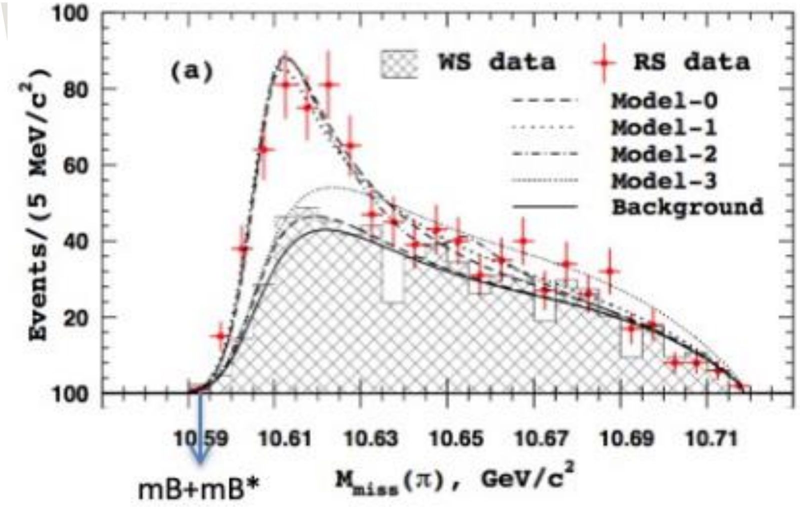

Belle, PRL **116**, 212001 (2016)

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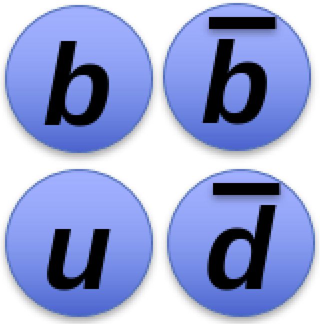
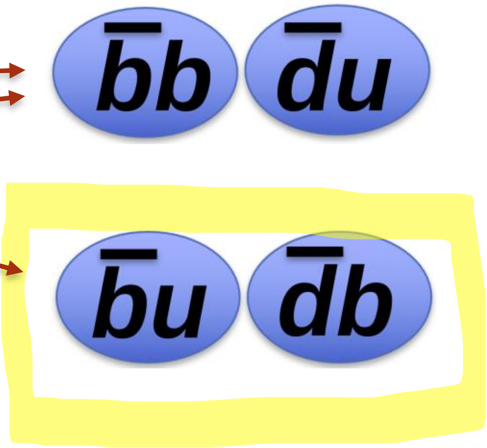
  - **Dominant** ⟶ $B\bar{B}^*$ ($Z_b(10610)$), $B^*\bar{B}^*$ ($Z_b(10650)$)
- Many decay channels:
  - challenging with Lüscher formalism

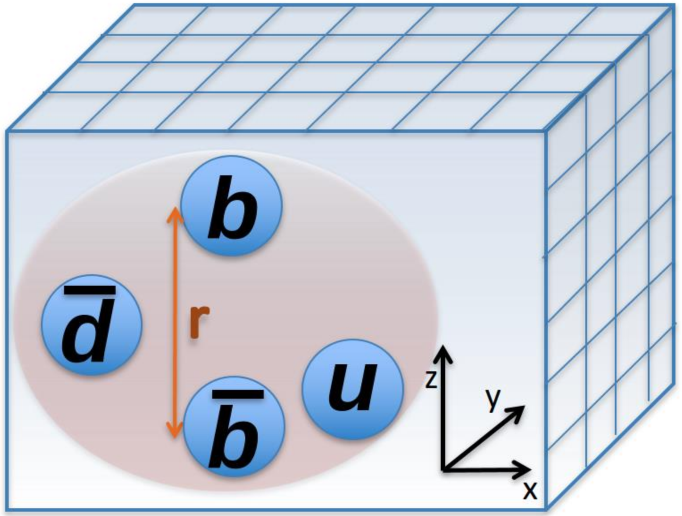
Z_b^+:





Static approximation

- Static $\bar{b}$ and b are at fixed positions at the distance r
- $m_b \to \infty$
- Extract eigen-energies $E_n(r)$ as a function of r
  - are needed for BO effective Lagrangian
    - yesterdays talk by N. Brambilla: "Effective Field Theories and Lattice QCD for the XYZ frontier"
- Studies done this way:



[1] S. Prelovsek, H. Bahtiyar and J. Petkovic, Phys. Lett. B **805**, 135467 (2020) [arXiv:1912.02656 [hep-lat]].

[2] A. Peters, P. Bicudo, K. Cichy and M. Wagner, J. Phys. Conf. Ser. **742**, 012006 (2016), [arXiv:1602.07621].

Quantum numbers in static approximation

- J_z^l - z-component of the angular momentum of the light degrees of freedom
- $C \cdot P$ - combined parity and charge conjugation
- ϵ - reflection over the yz-plane

quantum numbers						
I	I_3	J_z^l	$C \cdot P$	ϵ	Λ_η^ϵ convention	
1	0	0	-1	-1	Σ_u^-	[1]
			$+1$	$+1$	Σ_g^+	this work
			$+1$	-1	Σ_g^-	
			-1	$+1$	Σ_u^+	

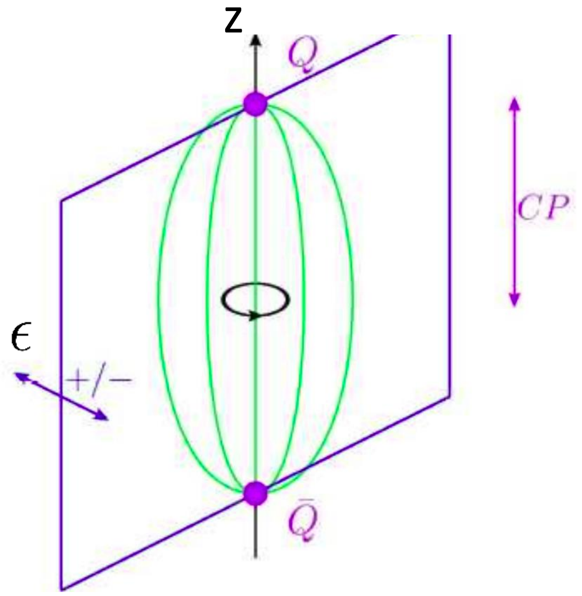


Figure adapted from yesterdays talk by N. Brambilla: "Effective Field Theories and Lattice QCD for the XYZ frontier"

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Procedure

- Extract eigen-energies $E_n(r)$
- Check which operator dominates the eigenstate with overlaps
  - $C_{ij}(t) = \sum_n \langle O_i|n\rangle e^{-E_n t} \langle n|O_j^\dagger\rangle$
- Compare $E_n(r)$ with the non-interacting energies $E^{n.i.}$

Lattice ensemble

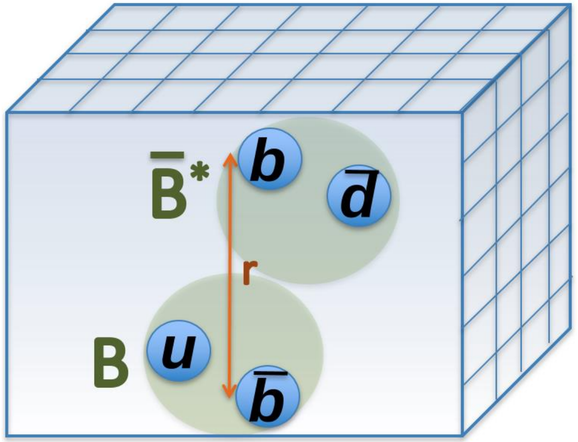
- $a \simeq 0.1239(13)\,\text{fm}$
- $m_\pi \simeq 266(5)$
- $N_L = 16\,,\ L \simeq 2\,\text{fm}$
  - $\vec{p} = \vec{n}\frac{2\pi}{L}$
  - smaller L gives less populated spectrum
- $N_T = 32$
- 281 configurations
- Dynamical Wilson-clover u/d quarks
- A. Hasenfratz *et al.*, PRD **78**, 054511 (2008)

CP=+1, ϵ=+1

$$O_1 = O_{B\bar{B}^*} \propto \sum_{a,b} \sum_{A,B,C,D} (P_-\gamma_z)_{BA} \tilde{\Gamma}_{CD}\ \bar{b}^a_C(0) q^a_A(0)\ \bar{q}^b_B(r) b^b_D(r)$$

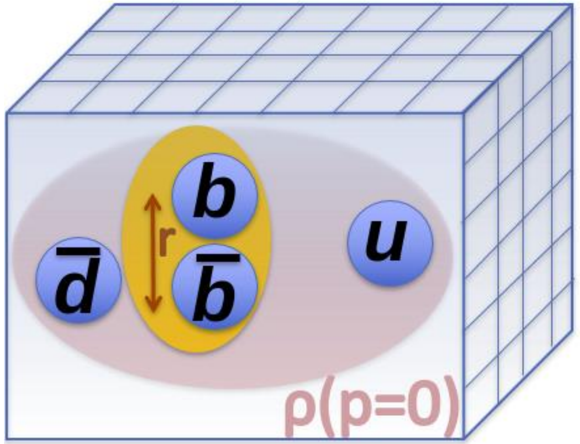
$$\propto \left([\bar{b}(0) P_- \gamma_5 q(0)]\ [\bar{q}(r)\gamma_z P_+ b(r)] + \{\gamma_5 \leftrightarrow \gamma_z\} \right)$$

$$- \left([\bar{b}(0) P_- \gamma_y q(0)]\ [\bar{q}(r)\gamma_x P_+ b(r)] + \{\gamma_y \leftrightarrow \gamma_x\} \right)$$



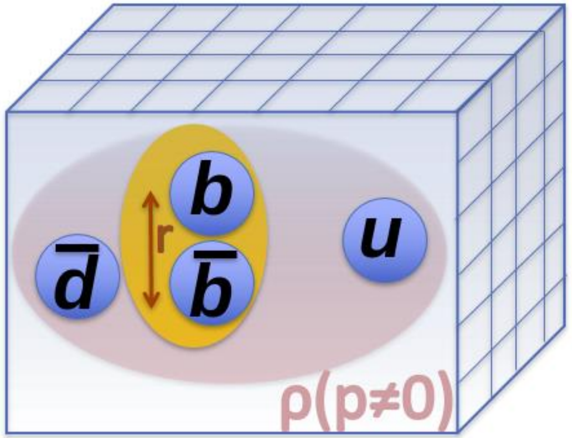
$$O_2 = O_{(B\bar{B}^*)'} \propto \sum_{a,b} \sum_{A,B,C,D} (P_-\gamma_z)_{BA} \tilde{\Gamma}_{CD}\ \bar{b}^a_C(0) \nabla^2 q^a_A(0)\ \nabla^2 \bar{q}^b_B(r) b^b_D(r)$$

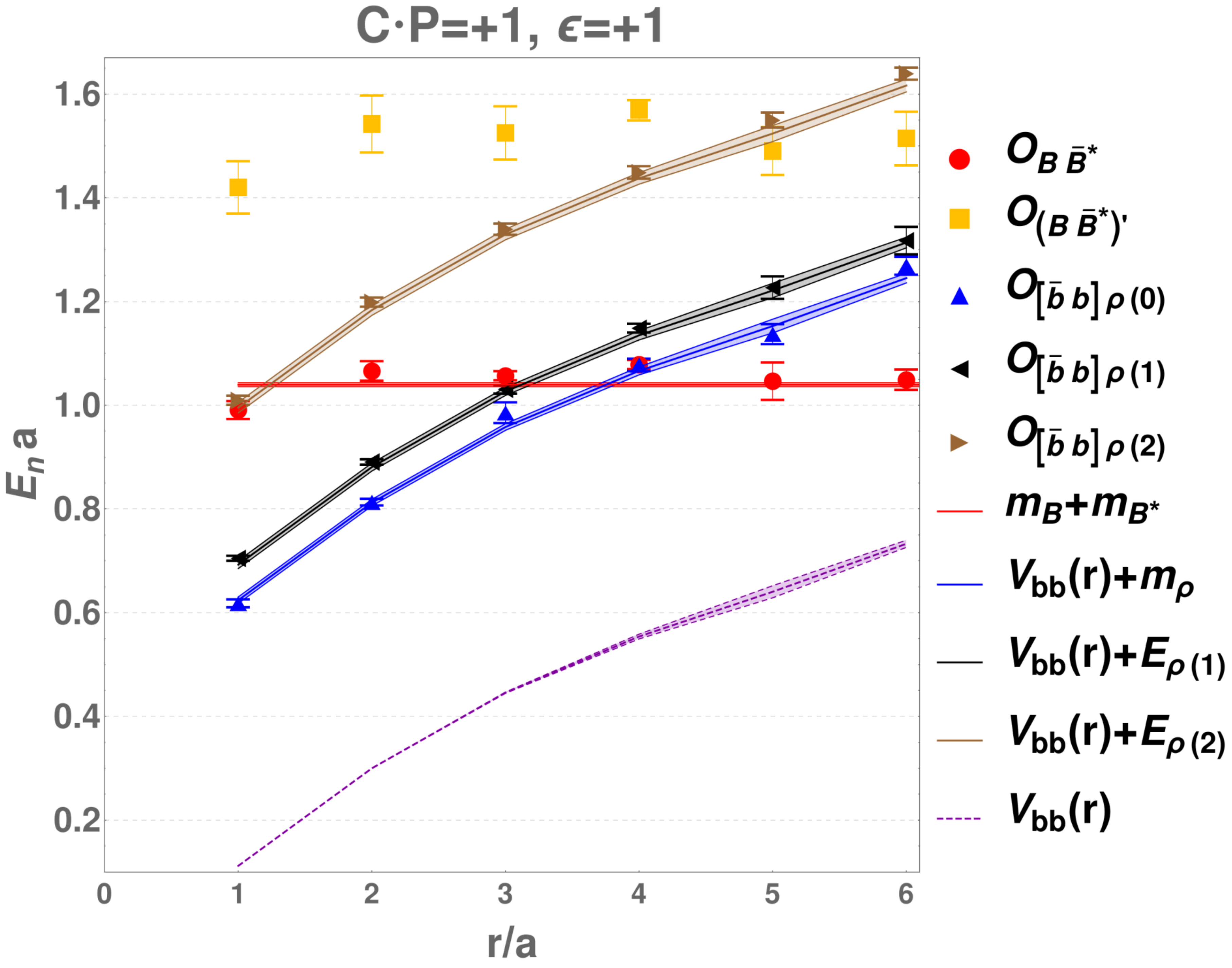
$$O_3 = O_{[\bar{b}b]\rho(0)} \propto [\bar{b}(0) U \Gamma^{(\mathrm{H})} b(r)]\ [\bar{q}\gamma_z q]_{\vec{p}=\vec{0}}$$

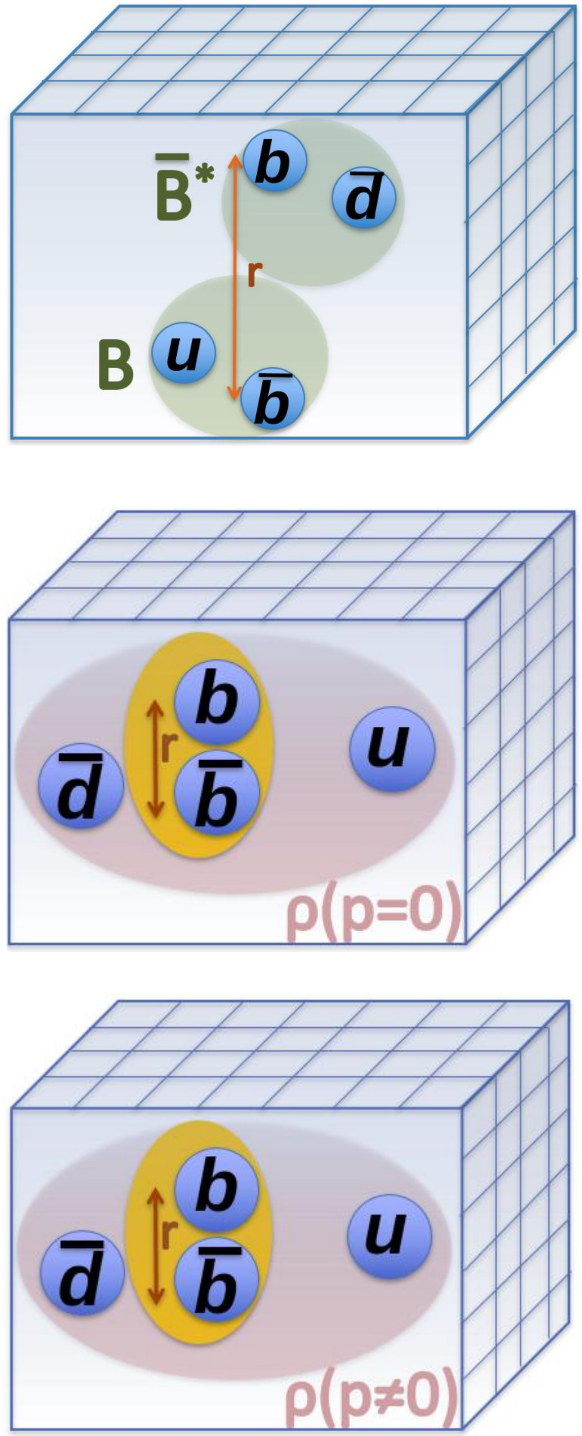


$$O_4 = O_{[\bar{b}b]\rho(1)} \propto [\bar{b}(0) U \Gamma^{(\mathrm{H})} b(r)]\ \left([\bar{q}\gamma_z q]_{\vec{p}=\vec{e}_z} + [\bar{q}\gamma_z q]_{\vec{p}=-\vec{e}_z} \right)$$

$$O_5 = O_{[\bar{b}b]\rho(2)} \propto [\bar{b}(0) U \Gamma^{(\mathrm{H})} b(r)]\ \left([\bar{q}\gamma_z q]_{\vec{p}=2\vec{e}_z} + [\bar{q}\gamma_z q]_{\vec{p}=-2\vec{e}_z} \right)$$




C·P=+1, ϵ=+1
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
$E_n a$
0
1
2
3
4
5
6
r/a
$O_{B\bar{B}^*}$
$O_{(B\bar{B}^*)'}$
$O_{[\bar{b}b]\rho(0)}$
$O_{[\bar{b}b]\rho(1)}$
$O_{[\bar{b}b]\rho(2)}$
$m_B+m_{B^*}$
$V_{bb}(r)+m_\rho$
$V_{bb}(r)+E_{\rho(1)}$
$V_{bb}(r)+E_{\rho(2)}$
$V_{bb}(r)$


$\bar{B}^*$
b
$\bar{d}$
r
B
u
$\bar{b}$
b
$\bar{d}$
r
$\bar{b}$
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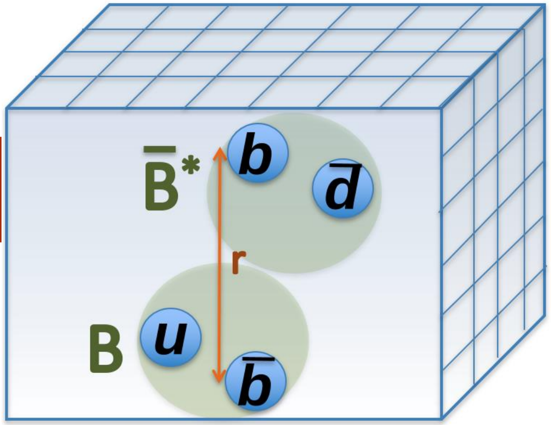
ρ(p=0)
b
$\bar{d}$
r
$\bar{b}$
u
ρ(p≠0)

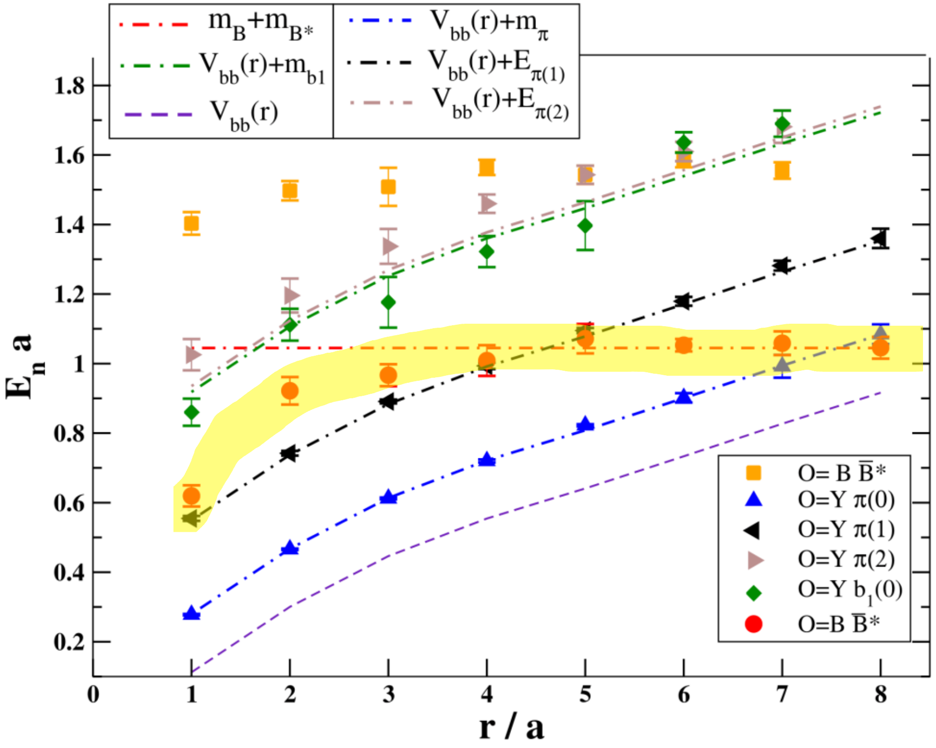


Comparison

CP=-1, ϵ=-1 from

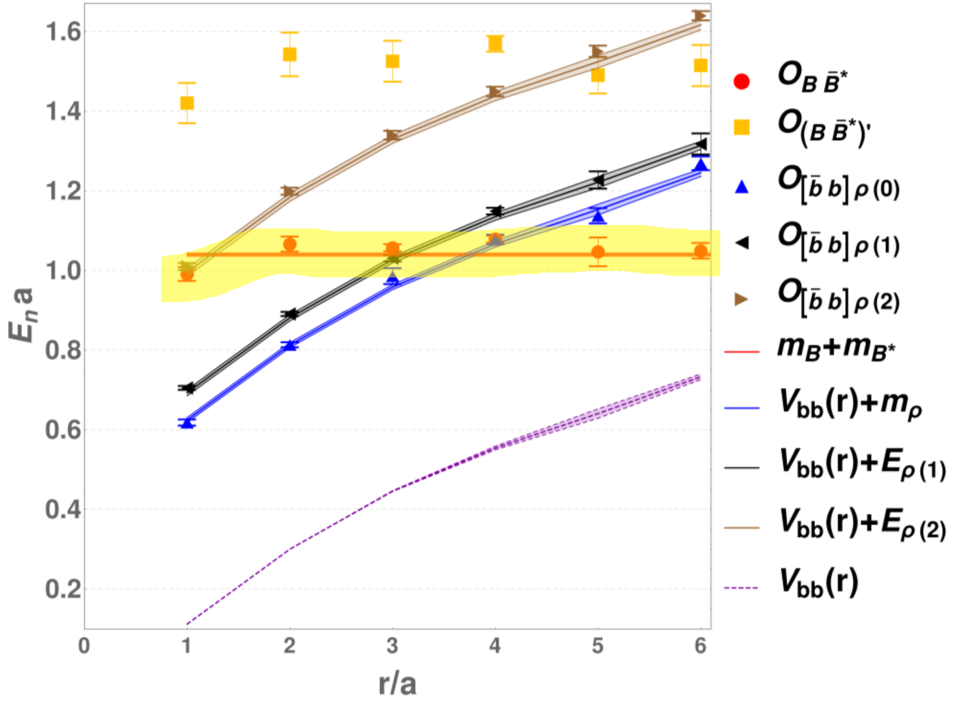
[1] S. Prelovsek, H. Bahtiyar and J. Petkovic, Phys. Lett. B **805**, 135467 (2020) [arXiv:1912.02656 [hep-lat]].

- Conclusions from this study:
  - for small r $B\bar{B}^*$ dominated eigenstate has energy below the non-interacting energy $E^{n.i.}$
  - attractive static potential
  - bound state below $B\bar{B}^*$ threshold



CP=+1, ϵ=+1, present work

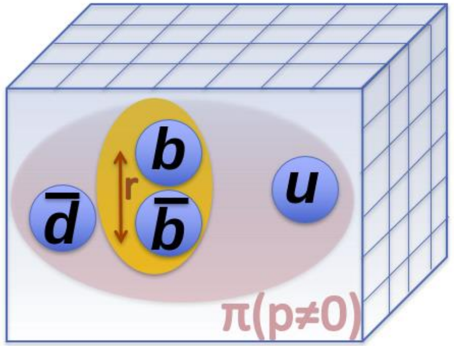
- No attractive static potential



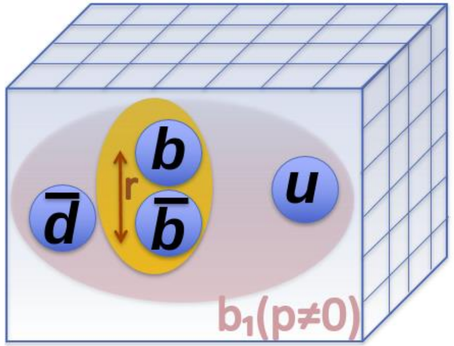
CP=+1, ϵ=-1

$$O_1 = O_{[\bar{b}b]\pi(1)} \propto [\bar{b}(0)U\Gamma^{(\mathrm{H})}b(r)] \ \left([\bar{q}\gamma_5 q]_{\vec{p}=\vec{e}_z} - [\bar{q}\gamma_5 q]_{\vec{p}=-\vec{e}_z}\right)$$

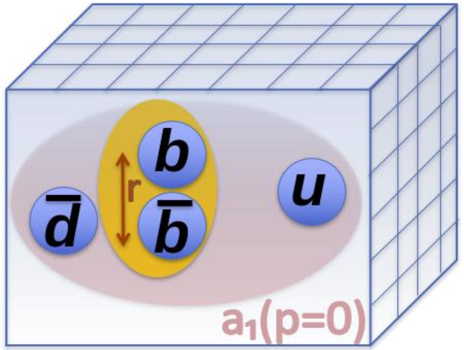
$$O_2 = O_{[\bar{b}b]\pi(2)} \propto [\bar{b}(0)U\Gamma^{(\mathrm{H})}b(r)] \ \left([\bar{q}\gamma_5 q]_{\vec{p}=2\vec{e}_z} - [\bar{q}\gamma_5 q]_{\vec{p}=-2\vec{e}_z}\right)$$



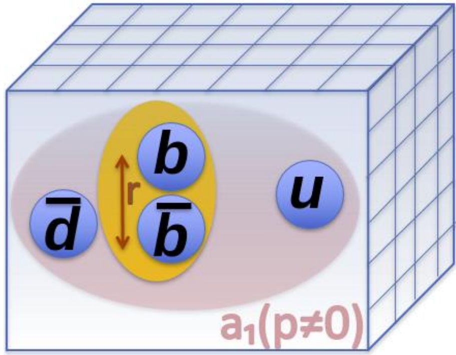
$$O_3 = O_{[\bar{b}b]b_1(1)} \propto [\bar{b}(0)U\Gamma^{(\mathrm{H})}b(r)] \ \left([\bar{q}\gamma_x\gamma_y q]_{\vec{p}=\vec{e}_z} - [\bar{q}\gamma_x\gamma_y q]_{\vec{p}=-\vec{e}_z}\right)$$

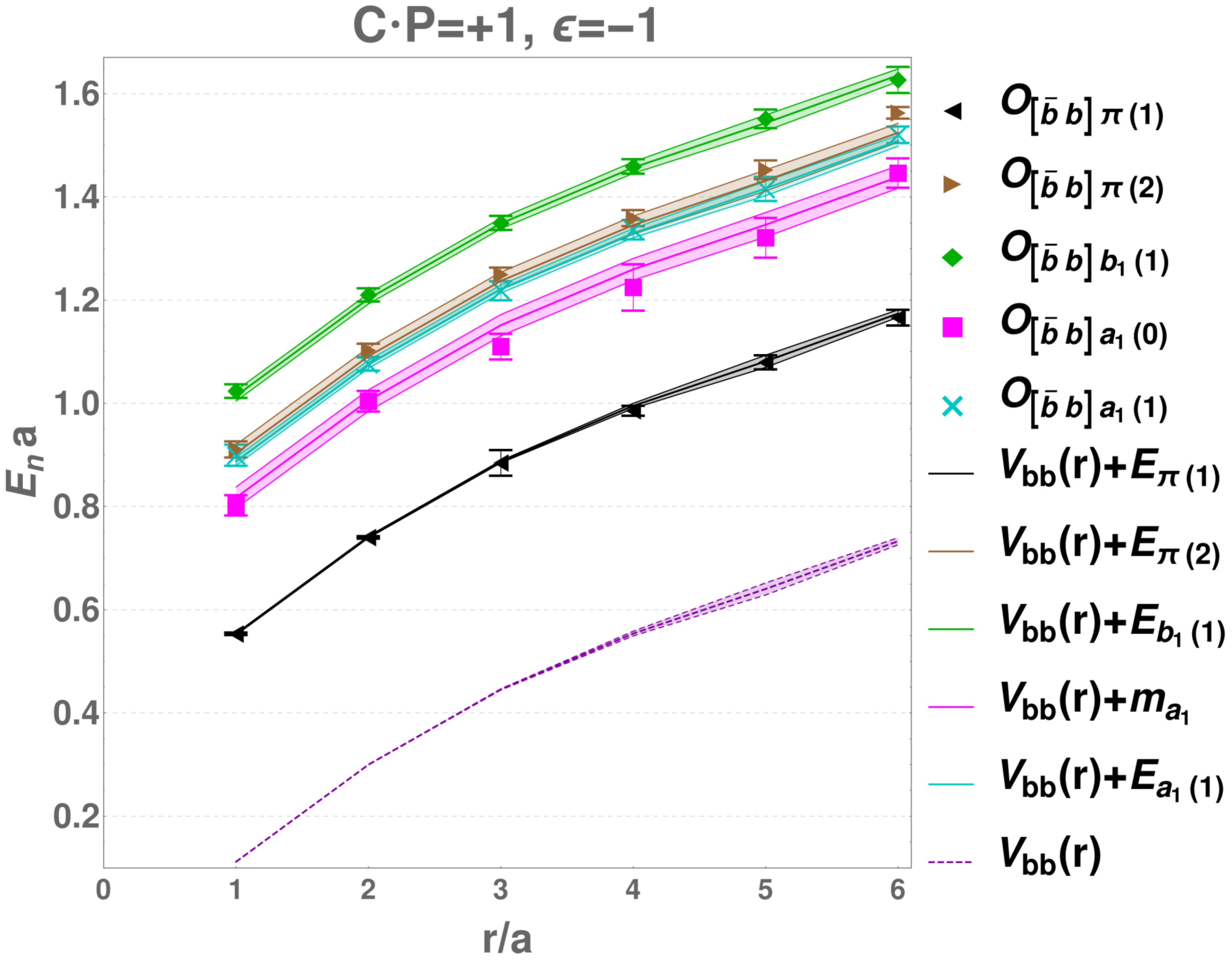


$$O_4 = O_{[\bar{b}b]a_1(0)} \propto [\bar{b}(0)U\Gamma^{(\mathrm{H})}b(r)] \ [\bar{q}\gamma_5\gamma_z q]_{\vec{p}=\vec{0}}$$



$$O_5 = O_{[\bar{b}b]a_1(1)} \propto [\bar{b}(0)U\Gamma^{(\mathrm{H})}b(r)] \ \left([\bar{q}\gamma_5\gamma_z q]_{\vec{p}=\vec{e}_z} + [\bar{q}\gamma_5\gamma_z q]_{\vec{p}=-\vec{e}_z}\right)$$





C·P=+1, ϵ=−1
$E_n a$
r/a
0
1
2
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0.2
0.4
0.6
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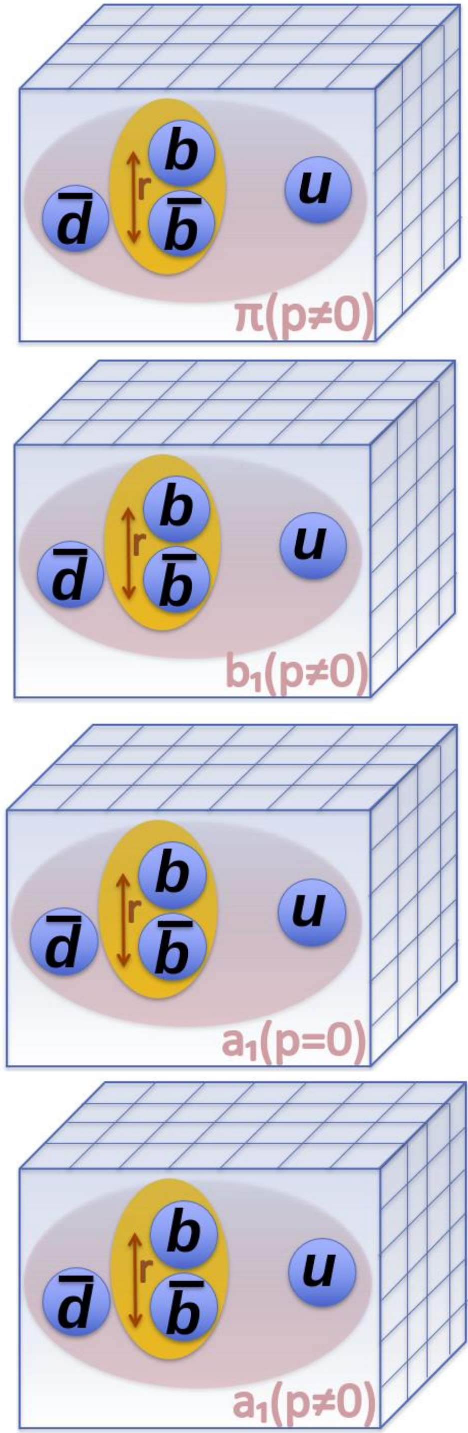
1.0
1.2
1.4
1.6
$O_{[\bar{b}b]\pi(1)}$
$O_{[\bar{b}b]\pi(2)}$
$O_{[\bar{b}b]b_1(1)}$
$O_{[\bar{b}b]a_1(0)}$
$O_{[\bar{b}b]a_1(1)}$
$V_{bb}(r)+E_{\pi(1)}$
$V_{bb}(r)+E_{\pi(2)}$
$V_{bb}(r)+E_{b_1(1)}$
$V_{bb}(r)+m_{a_1}$
$V_{bb}(r)+E_{a_1(1)}$
$V_{bb}(r)$



b
$\bar{b}$
$\bar{d}$
u
r
π(p≠0)
b
$\bar{b}$
$\bar{d}$
u
r
b_1(p≠0)
b
$\bar{b}$
$\bar{d}$
u
r
a_1(p=0)
b
$\bar{b}$
$\bar{d}$
u
r
a_1(p≠0)

CP=-1, ϵ=+1

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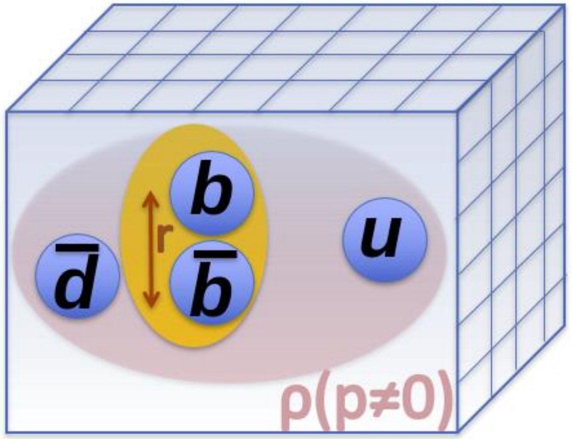

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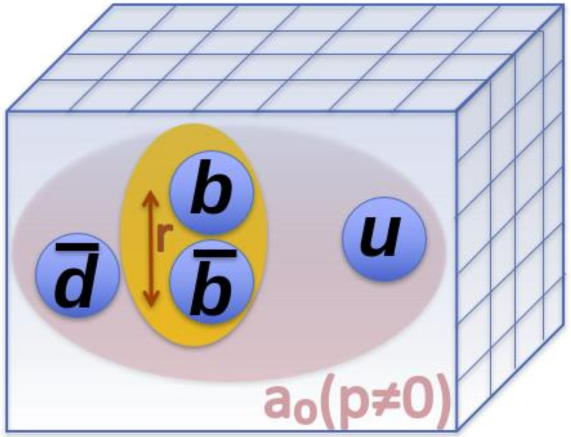
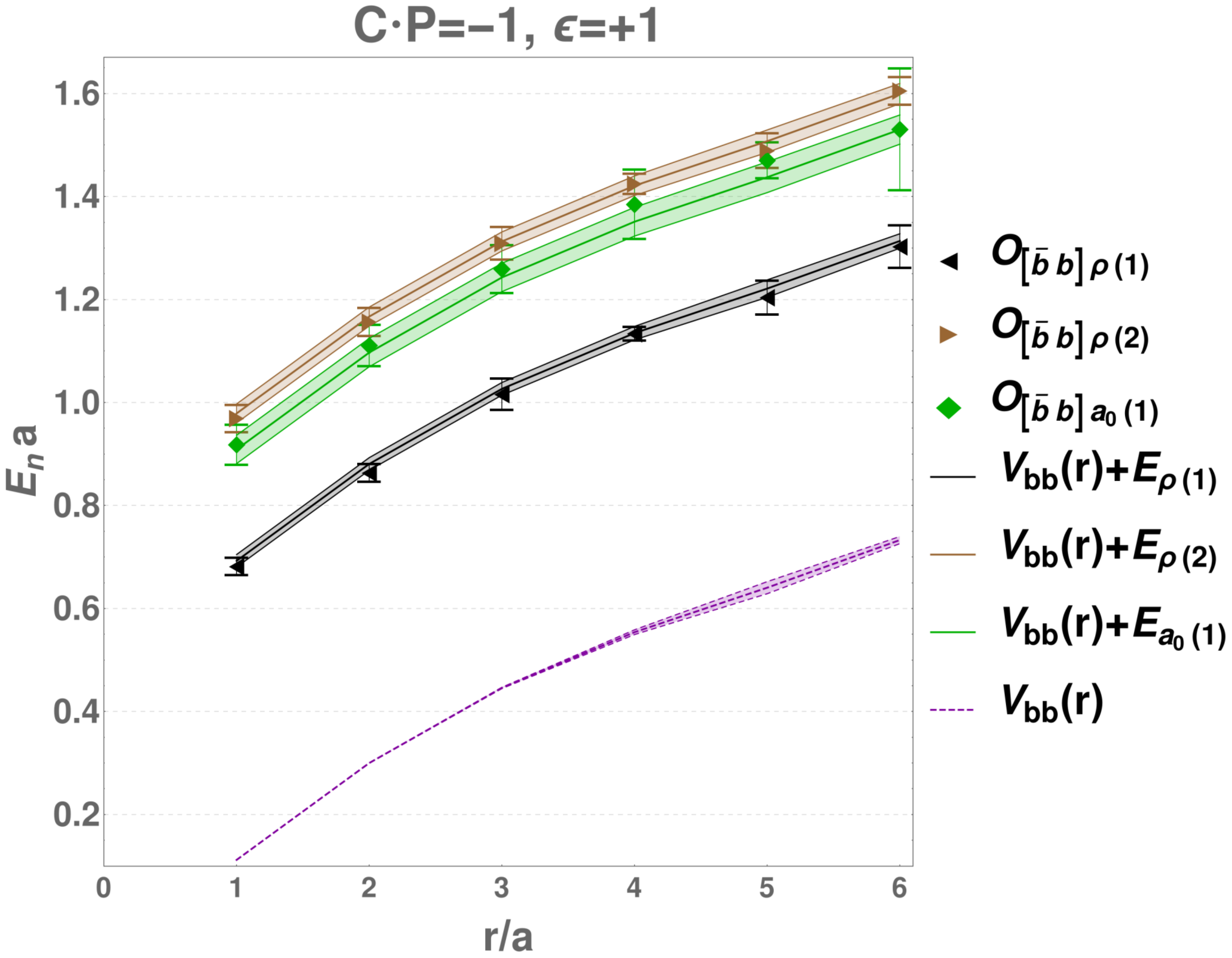
$$O_3 = O_{[\bar{b}b]a_0(1)} \propto [\bar{b}(0) U \Gamma^{(\mathrm{H})} b(r)] \left([\bar{q}\mathbb{1} q]_{\vec{p}=\vec{e}_z} - [\bar{q}\mathbb{1} q]_{\vec{p}=-\vec{e}_z} \right)$$

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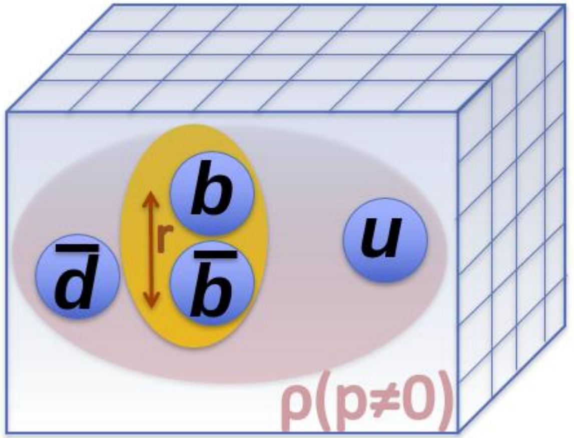
$O_{[\bar{b}\,b]\,\rho\,(1)}$
$O_{[\bar{b}\,b]\,\rho\,(2)}$
$O_{[\bar{b}\,b]\,a_0\,(1)}$
$V_{bb}(r)+E_{\rho\,(1)}$
$V_{bb}(r)+E_{\rho\,(2)}$
$V_{bb}(r)+E_{a_0\,(1)}$
$V_{bb}(r)$


b
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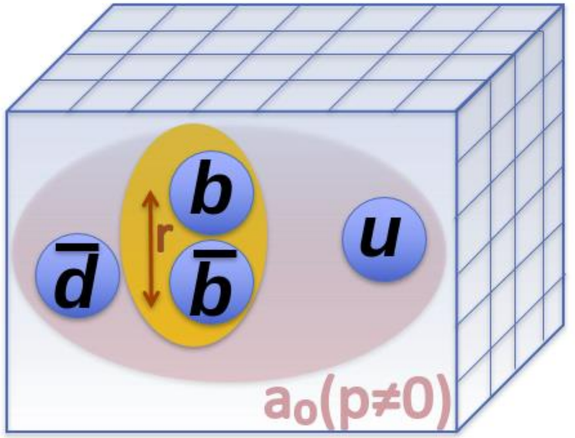
$\bar{d}$
$\bar{b}$
u
ρ(p≠0)


b
r
$\bar{d}$
$\bar{b}$
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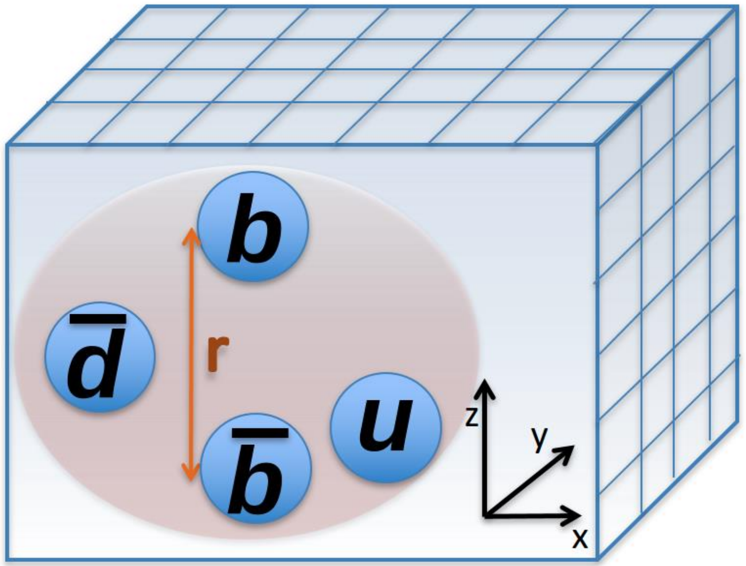
a_0(p≠0)

Conclusions



- We extracted eigen-energies $E_n(r)$ for the system $\bar{b}b\bar{q}q$ where $\bar{b}$ and b are static and consider different quantum numbers:

quantum numbers							conclusion
I	I_3	J_z^1	$C \cdot P$	ϵ	Λ_η^ϵ convention		
			-1	-1	Σ_u^-	[1]	significant attraction for small r
			$+1$	$+1$	Σ_g^+	this work	small attraction for small r
1	0	0	$+1$	-1	Σ_g^-	this work	no energy shift
			-1	$+1$	Σ_u^+	this work	no energy shift

[1] S. Prelovsek, H. Bahtiyar and J. Petkovic, Phys. Lett. B **805**, 135467 (2020) [arXiv:1912.02656 [hep-lat]].

Thank you for your attention