Determination of the masses of hybrid charmonium mesons

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Hybrid Mesons - Intro

There are several observed resonances in the charmonium sector that do not correspond neatly with Quark model states. There are many competing explanations for these so-called XYZ states.

A hybrid meson is a meson with an excited gluonic component.

non-Quark model states are called exotic (glueballs, tetraquarks, pentaquarks etc). Nothing in principle prevents $\bar{q}qg$ states in QCD.

(relatively) recent studies of hybrid mesons include:

- The *MILC* collaboration [hep-lat/0301024]
- *Bali et al.* [1110.2381]
- *HadSpec* from their calculation of the excited charmonium spectrum [1610.01073]
- *Ma et al.* [1910.09819]

The PANDA experiment at FAIR will search for evidence of gluonic excitations in the hadron spectrum.

Hybrid operators

Since hybrids have an excited gluonic component the operators which we would expect to couple to them include the field strength tensor, F_{ij}^{ab}.
As we are using staggered fermions we replace γ matrices with phases and the operators now have a taste assignment.
We started by coding up the lightest hybrid multiplet (as determined theoretically by HadSpec).

$$
\begin{array}{lcl}
1^{-+} : \epsilon_{ijk}\bar{\psi}\gamma_j B_k\psi & \longrightarrow & \gamma_i \otimes \gamma_i : \bar{\chi}\epsilon_{ijk}(-1)^{x_j} B_k\chi \\
1_H^{--} : \bar{\psi}\gamma_5 B_i\psi & \longrightarrow & \gamma_5 \otimes \gamma_5 : \bar{\chi}B_i\chi \\
0_H^{-+} : \bar{\psi}\gamma_i B_i\psi & \longrightarrow & \gamma_i \otimes \gamma_i : \bar{\chi}(-1)^{x_i} B_i\chi \\
2_H^{-+} : |\epsilon_{ijk}|\bar{\psi}\gamma_j B_k\psi & \longrightarrow & \gamma_i \otimes \gamma_i : \bar{\chi}|\epsilon_{ijk}|(-1)^{x_j} B_k\chi
\end{array}
$$

$\frac{1}{2}\epsilon_{ijk}F_{jk} = B_i$

A state with 1^{-+} quantum numbers can't be formed from a quark-antiquark pair. We focus on this operator here.

Note: The previous MILC calculation used the taste singlet non-local ρ when constructing the hybrid operators. We use the local operator.

Simulation details

We use the Highly Improved Staggered Quark (HISQ) action with 2+1+1 dynamical flavours.
We use w_0 to set the scale (with values provided by MILC/HPQCD).[2005.01845]

name	size	a (fm)	m_l/m_s	$M_\pi L$	M_π (MeV)	configurations
very coarse	32^3x48	0.15088(79)	1/27	3.30	131.0(1)	1505
coarse	32^3x64	0.12225(65)	1/10	4.29	216.9(2)	1000
fine	32^3x96	0.09023(48)	1/5	4.50	312.7(6)	1008

Table 1: Ensemble specifications - these are provided courtesy of the MILC Collaboration

HPQCD has computed the conventional charmonium spectrum and found good agreement.

In this calculation we are not accounting for

- mixing between operators due to the reduced cubic symmetry of the lattice. (eg 1^{-+} with 4^{-+}),
- the fact that these states are really resonances,
- potential decays to $\bar{D}D$,...

We are focussing on getting a decent signal and reasonably precise masses for the hybrids

General fitting considerations

Our analysis is complicated by the fact that the hybrids are not the ground state in our correlators. To mitigate this we fit to matrices of correlators.

Correlators formed from different combinations of source and sink operators can be organised into a matrix of correlators $C(t)$.

Finding the energies of the states in a correlator matrix, $C(t)$, involves solving a **GEVP**. The GEVP is defined by,

$$C(t)v_n(t,t_0) = \lambda_n(t,t_0)C(t_0)v_n(t,t_0), \quad n = 1, ..., N, \quad t > t_0 \tag{1}$$

One can then show that the GEVP is solved by,

$$\lambda_n^{(0)}(t,t_0) = e^{-E_n(t-t_0)}, \qquad v_n(t,t_0) \propto u_n \tag{2}$$

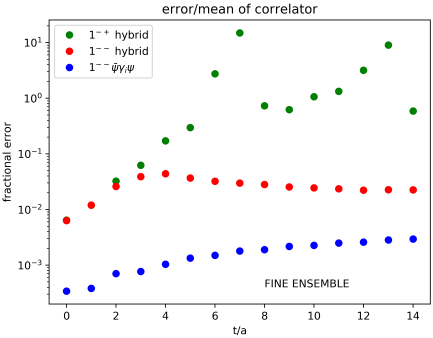
where E_n is the energy of state n.

With staggered fermions we modify this,

$$\lambda_n^{(0)}(t,t_0) = s_n(t-t_0)e^{-E_n(t-t_0)}, \qquad v_n(t,t_0) \propto u_n \tag{3}$$

where $s_n(t) = 1$ for a non-oscillating state and $s_n(t) = (-1)^t$ for an oscillating state. [1411.4676]

Correlators formed from hybrid meson operators are very noisy.



We employ a series of techniques to try and reduce the uncertainty in our results. These are well-established methods that are used in many lattice calculations.

- We average over multiple time sources and polarisations per configuration
- APE smearing on the gauge links - damp UV fluctuations
- Gaussian smearing on the quarks - $\left[1 + \frac{r^2\mathcal{D}}{4n}\right]^n$

Analysis - fitting procedure

We use Peter Lepage's `corrfitter` library, which incorporates the preceding formalism in a Bayesian framework.

We do the following:

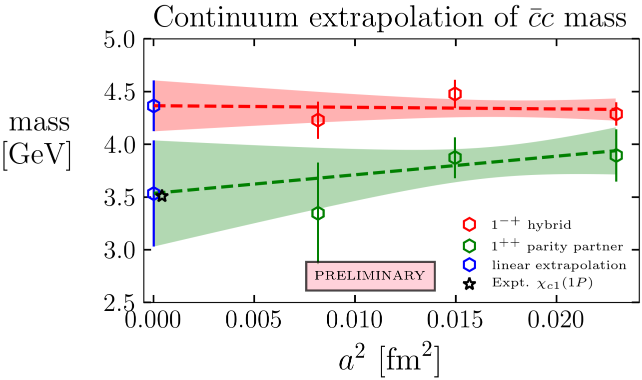
- Form a 2 operator basis - with and without smearing
- Choose t_0 - this generates the priors
- Set the number of exponentials to 10 + 10
- vary the fit range until goodness of fit parameters optimised

Ensemble	# tsrc	t_0	range	svdcut	χ^2 per dof	Q value	Mass (GeV)	PP* Mass (GeV)
very coarse	16	1	1-6	3×10^{-7}	1.1	0.31	4.29(11)	3.89(25)
coarse	16	1	1-5	3×10^{-5}	1.3	0.22	4.575(66)	3.58(28)
fine	16	2	2-8	7×10^{-4}	0.93	0.55	4.23(18)	3.35(48)

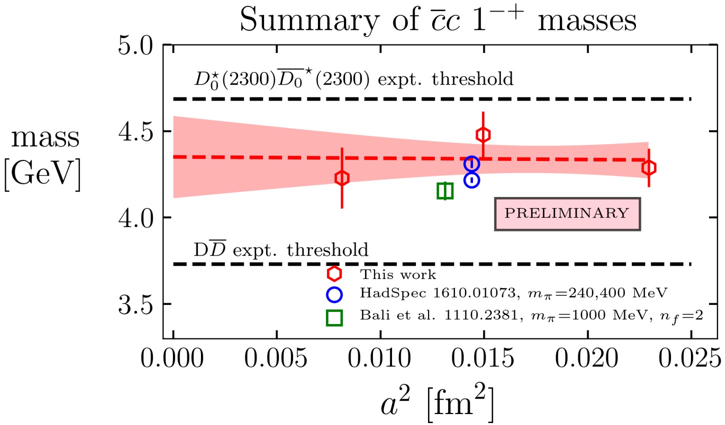
Table 2: Summary of 1^{-+} fits

PP = Parity Partner (1^{++}) - from the oscillating component of the correlator

Analysis - Continuum extrapolation



Here we assume no dependence on light quark mass.



Summary of $\overline{c}c$ 1^{-+} masses
5.0
4.5
4.0
3.5
mass [GeV]
$D_0^\star(2300)\overline{D_0}^\star(2300)$ expt. threshold
PRELIMINARY
$D\overline{D}$ expt. threshold
This work
HadSpec 1610.01073, m_π=240,400 MeV
Bali et al. 1110.2381, m_π=1000 MeV, n_f=2
0.000
0.005
0.010
0.015
0.020
0.025
a^2 [fm^2]

4x4 fits with the J/ψ and 1^{--} hybrid meson

Here the hybrid state is the *second* excited state.
We perform a GEVP analysis with a 4 operator basis. These are preliminary results on the fine ensemble with just the z polarisation.

state	am	m/GeV	amplitude	f/MeV	comment
J/ψ	1.41609(37)	3.096(16)	0.16381(54)	416(3)	2x2 matrix fit
$\psi(2S)$	1.712(29)	3.744(67)	0.169(25)	390(60)	2x2 matrix fit
$h_c(1P)$	1.6170(66)	3.536(24)	0.0666(33)	–	2x2 matrix fit
J/ψ	1.3659(49)	2.987(19)	0.1895(54)	490(14)	4x4 matrix fit
Hybrid	1.815(36)	3.969(82)	0.0472(41)	–	4x4 matrix fit

Defining and calculating hybrid decay constants can be useful in characterising hybrid states. Even very small decay constants can be useful in constraining leptonic decay widths.

$$\langle 0 | (\hat{1}^{--})_i | H \rangle = f_{(1^{--})} M_H \epsilon_i \tag{4}$$

Our amplitude for the vector operator into the 1^{--} hybrid state is very small and = 0.005(19)

We note that decay constants defined through the 1^{-+} operator would involve complications wrt renormalisation

Summary & Outlook

We have presented:

- **A first unquenched continuum extrapolation of the 1^{-+} charmonium hybrid meson mass**
- An early look at the 1^{--} hybrid mass at a single (0.09fm) lattice spacing

Future work:

- Run with the other ops in the lightest hybrid multiplet - $0^{-+}, 2^{-+}$
- Run on physical ensembles and at a finer lattice spacing (0.06 fm)
- Similiar analysis targeting the bottomonium sector
- Look at other exotic quantum number hybrids like the 0^{+-}

We would like to resolve the order of the hybrid multiplet in the continuum limit.

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Thank you for listening.

Backup slide(s)

- Leptonic decay width of the hybrid vector charmonium [1604.03401]
  $\Gamma(X \longrightarrow e^+e^-) < 40\text{eV}$
- Recent pheno estimate of the 1^{-+} hybrid charmonium leptonic width $\sim$ 90 eV [2002.09566]. [uses lattice data as input to potential]
- A recent HadSpec paper attempted to calculate the width of the hadronic decays of the 1^{-+} hybrid with heavy (700 MeV) pions and exact SU(3) flavour symmetry ($m_u = m_d = m_s$). Focusing on the light sector they find a broad state, π_1, decaying primarily to $b_1\pi$. [2009.10034]