Interglueball potential in lattice SU(N) gauge theories

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Pure Yang-Mills theory and glueballs

$$\mathcal{L}_{\rm YM} = -\frac{1}{4} F_a^{\mu\nu} F_{\mu\nu,a}$$

(a =1,…,N_c^2-1)

⇒ The simplest interacting gauge theory

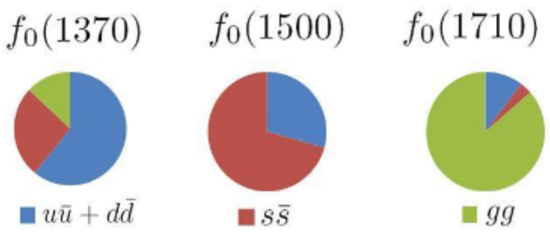
$\mathscr{L}_{\rm YM}$ does not have apparent scale, but the scale is dynamically generated (dimensional transmutation)

Lightest particles are 0^{++} glueballs !

Important applications:

- Hadron spectroscopy

There are several candidates of glueballs in experiments, but no firm evidence.



Cheng et al., Phys. Rev. D **74**, 094005 (2006).

- Glueballs of dark SU(N) gauge theory are good dark matter candidates

(DM represents the 27% of the energy of the Universe)

Interaction between dark glueballs may affect the structure of galactic halos, relic abundance, background gravitational waves, etc.

NY, H. Iida, A. Nakamura, M. Wakayama, Phys. Lett. B **813**, 136056 (2021);
Phys. Rev. D **102**, 054507 (2020) .

Object of study

In this work, we study the interglueball scattering on lattice which is the only way to quantify nonperturbative physics of nonabelian gauge theory.

The Yang-Mills theory depends only on the scale parameter Λ:
Λ is determined by hadronic experiments (case of QCD), or by astronomical observation (case of dark Yang-Mills theory).

Objective:

In this work, we study the interglueball scattering of SU(2,3,4) Yang-Mills theory on lattice

Setup

We consider the SU(2,3,4) pure Yang-Mills theory

- Standard SU(2) plaquette action :

  Lattice spacings : β = 2.1, 2.2, 2.3, 2.4, 2.5

  Volume : 10^3x12 ∼ 16^3x24

  Configurations generated with pseudo-heat-bath method

- Use SX-ACE (@RCNP, Osaka U.), vector machine

- Improvement of glueball operator : APE smearing

  We use all space-time translational and cubic rotational symmetries
  to effectively increase the statistics
  (like the all-mode average for meson and baryon observables)

  Reduction of the statistical error w/ cluster decomposition principle

  chiQCD Collaboration, PRD**97**, 034507 (2018)

Setup

We consider the SU(2,3,4) pure Yang-Mills theory

- Standard SU(3) plaquette action :

  Lattice spacings : β = 5.5, 5.7, 5.9, 6.1

  Volume : 12^3x12 ～ 20^3x24

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Setup

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  Lattice spacings : β = 10.789

  Volume : 16^3x24

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Scale determination

We leave the scale of YM theory as a free parameter Λ

(In QCD, Λ is around 200 MeV. In dark YMT, it is unknown)

⇒ We express all quantities in unit of Λ (and finally constrain Λ from exp.).

Relation between Λ and string tension:

$$\frac{\Lambda_{\overline{MS}}}{\sqrt{\sigma}} = 0.503(2)(40) + \frac{0.33(3)(3)}{N^2}$$

$$= 0.540(41) \qquad \text{(for SU(3))}$$

Fitted from the analysis of the running coupling

C. Allton et al., JHEP **0807** (2008) 021
M. Teper, Acta Phys. Polon. B **40** (2009) 3249

String tension in SU(3) YM :

β	$a\sqrt{\sigma}$
5.5	0.5830(130)
5.7	0.3879(39)
5.9	0.2613(28)
6.1	0.1876(12)

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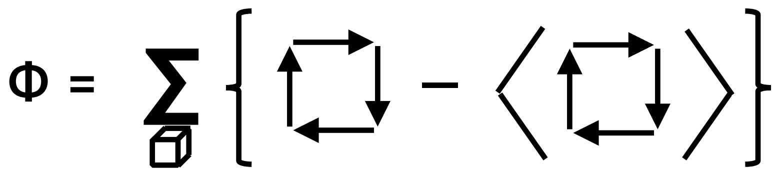
String tension in SU(3) YM :

β	$a\sqrt{\sigma}$	a (in unit of Λ^{-1})
5.5	0.5830(130)	0.315(24)
5.7	0.3879(39)	0.209(16)
5.9	0.2613(28)	0.141(11)
6.1	0.1876(12)	0.101(8)

$\Rightarrow$ Lattice spacing is now expressed in unit of Λ

Glueball operator and operator improvement

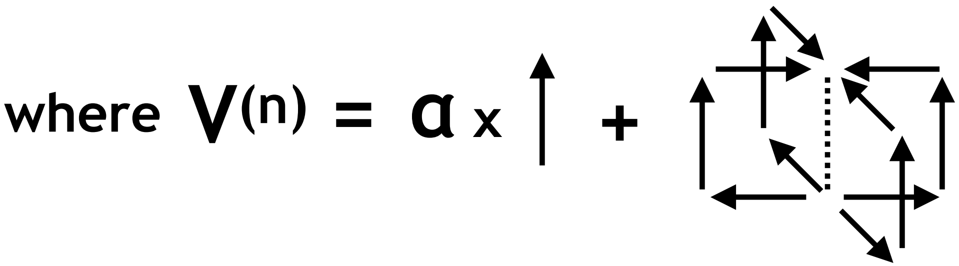
0++ glueball operator:



Glueball has vacuum expectation value
→ Subtract
Sum over cubic rotational invariance

APE smearing :

$U^{(n+1)}$ so as to maximize $\mathrm{Re\,Tr}\,[\,U^{(n+1)}\,V^{(n)\dagger}\,]$



⇒ Gaussian spread: $2\sqrt{\dfrac{n}{4+\alpha}}$

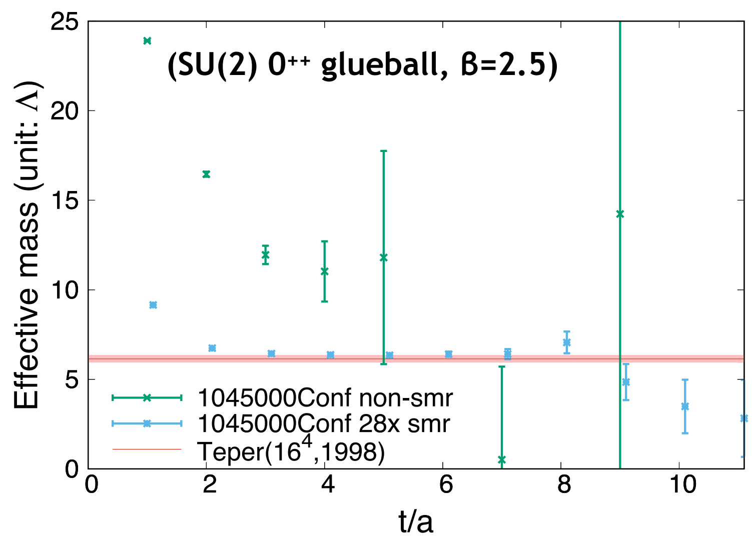
(in lattice unit)

Ape Collaboration, PLB **192** (1987) 163
N. Ishii et al., PRD **66**, 094506 (2002)

Optimal parameters
for SU(2), β=2.5:

n = 28

α = 2.0

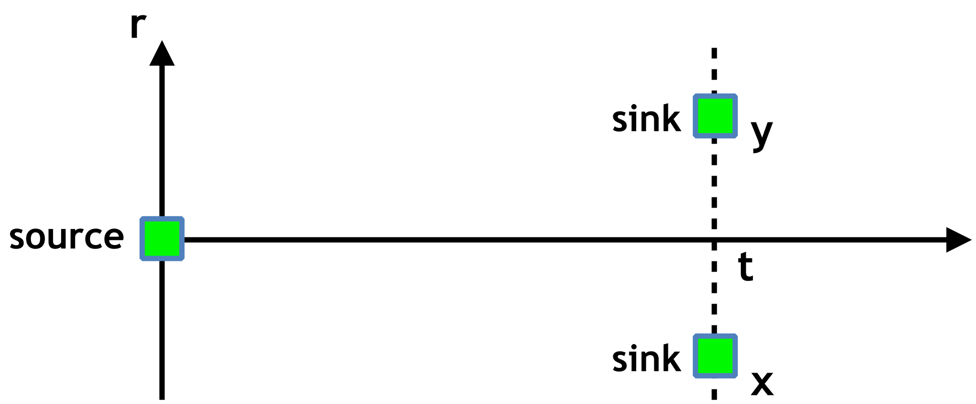


Nambu-Bethe-Salpeter amplitude

The information of the scattering is included in the following n-point correlator (Nambu-Bethe-Salpeter amplitude):

$$C_{\phi\phi}(t, \mathbf{x}-\mathbf{y}) \equiv \frac{1}{V}\sum_{\mathbf{r}} \langle 0\,|T[\phi(\mathbf{x}+\mathbf{r},t)\phi(\mathbf{y}+\mathbf{r},t)\cdot \mathcal{J}(0)]|\,0\rangle$$

$\mathcal{J}(0)$ **: source op.**



- **2-glueball (0^{++}) state mixes with all other multi-glueball states:**

  ⇒ The source may be chosen as 1-body, 2-body, etc, on convenience.

  We choose the smeared 1-body source, since the 0^{++} channel has as its lowest energy state the one-glueball state.

- **The NBS amplitude obeys the Schroedinger equation below inelastic threshold**

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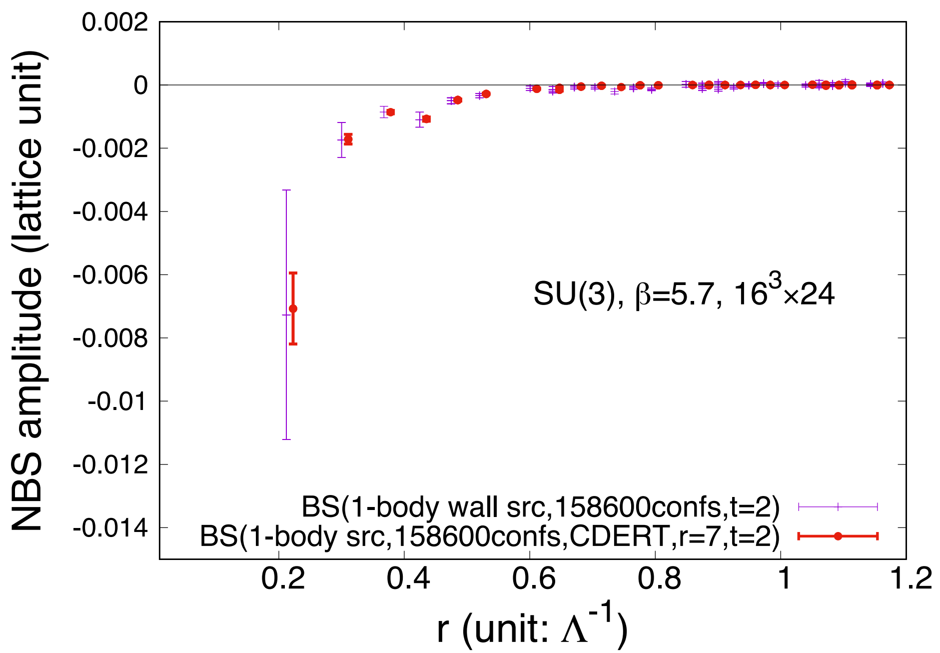
Result of NBS amplitude calculation:



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  ⇒ The sou ... **onvenience.**
  We ch ... **channel has**
  as its
- **The NBS a** ... **inelastic**
  threshold

HALQCD method

Extract the interglueball potential from the NBS amplitude by inversely solving Schroedinger equation

$$\left[\frac{1}{4m_\phi}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} + \frac{1}{m_\phi}\nabla^2 - \frac{(\mathbf{r}\times\nabla)^2}{m_\phi r^2}\right] R(t,\mathbf{r}) = \int d^3\mathbf{r}' U(\mathbf{r},\mathbf{r}') R(t,\mathbf{r}')$$

$$R(t,\mathbf{r}) \equiv \frac{C_{\phi\phi}(t,\mathbf{r})}{e^{-2m_\phi t}}$$

N. Ishii et al., PLB **712** (2012) 437.

- Crucial advantage : No ground state saturation is needed

➡ Almost mandatory to use time-dependent HAL method for the glueball analysis, since the glueball correlator becomes very noisy before ground state saturation

- Inelastic threshold for glueball = $3m_\varphi$: high enough to use low t

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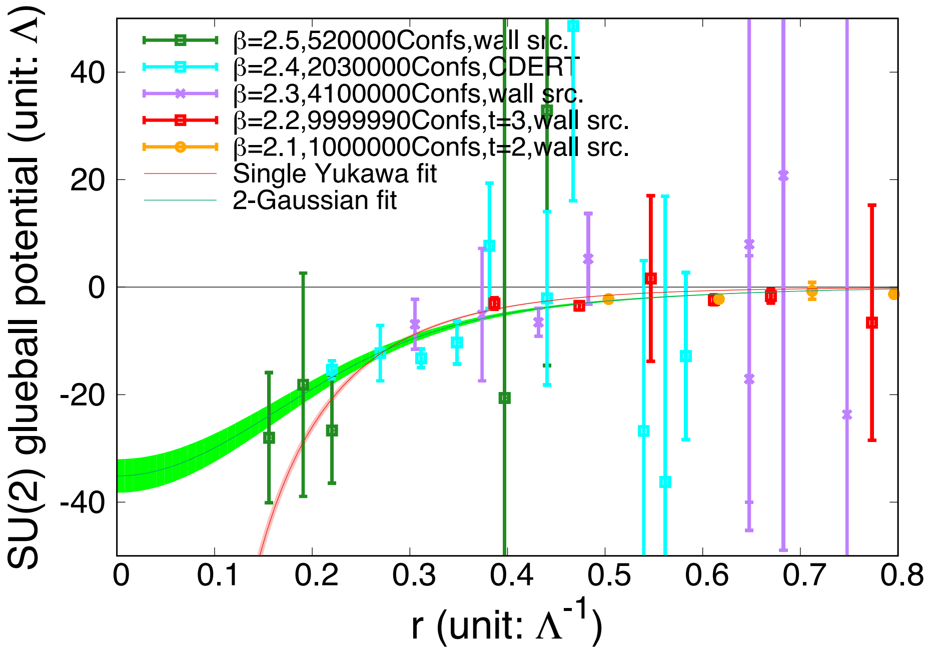
- Inelastic threshold for glueball = $3m_\varphi$: high enough to use low t

- Subtract centrifugal force for removing higher angular momenta

SU(2) result



We test two fitting forms:

- **Yukawa fit:**

$$V_Y(r) = V_1 \frac{e^{-m_\phi r}}{4\pi r}$$

V_1 = -231 ± 8 **x^2 d.o.f. = 1.3**

- **2-Gaussian fit:**

$$V(r) = V_1 e^{-\frac{(m_\phi r)^2}{8}} + V_2 e^{-\frac{(m_\phi r)^2}{2}}$$

V_1 = (-8.5 ± 0.5)Λ
V_2 = (-26.6 ± 2.6)Λ **x^2 d.o.f. = 0.9**

DM cross section is derived from phase shift calculated with the potentials

$$\sigma_{\text{tot}} = \frac{4\pi}{k^2} \sin^2[\delta(k \to 0)]$$

Yukawa: **σ_{tot} = (2.5 - 4.7)Λ^{-2} (stat.)**

2-Gaussian: **σ_{tot} = (14 - 51)Λ^{-2} (stat.)**

σ_{tot} = (2 - 51) Λ^{-2} (stat.and sys.)

(sys. due to fitting forms)

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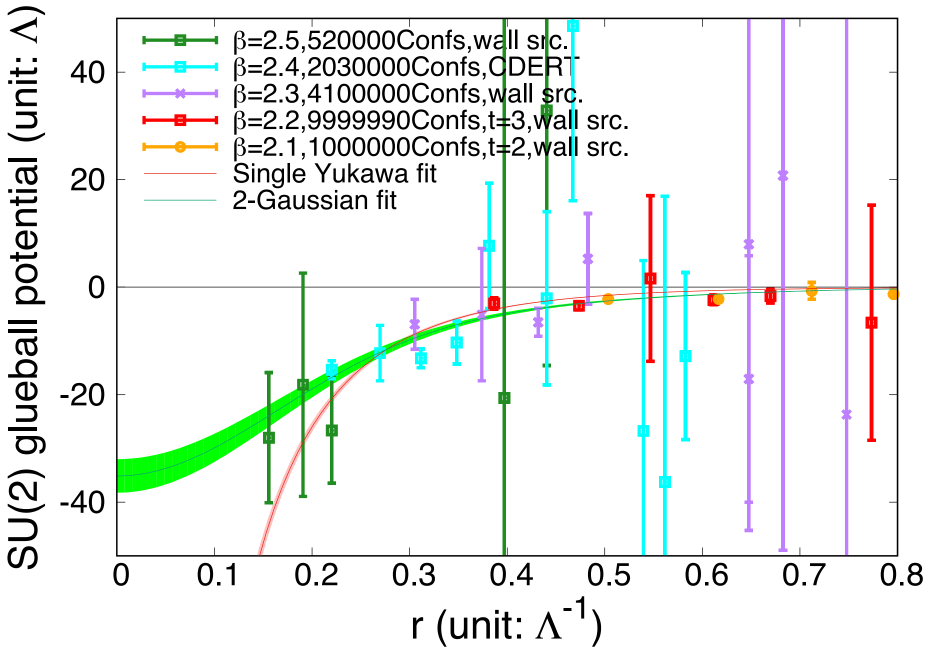

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From galactic cluster shape and collisions

$$\frac{\sigma_{tot}}{m_\Phi} < 1.0\ \mathrm{cm}^2/\mathrm{g}$$

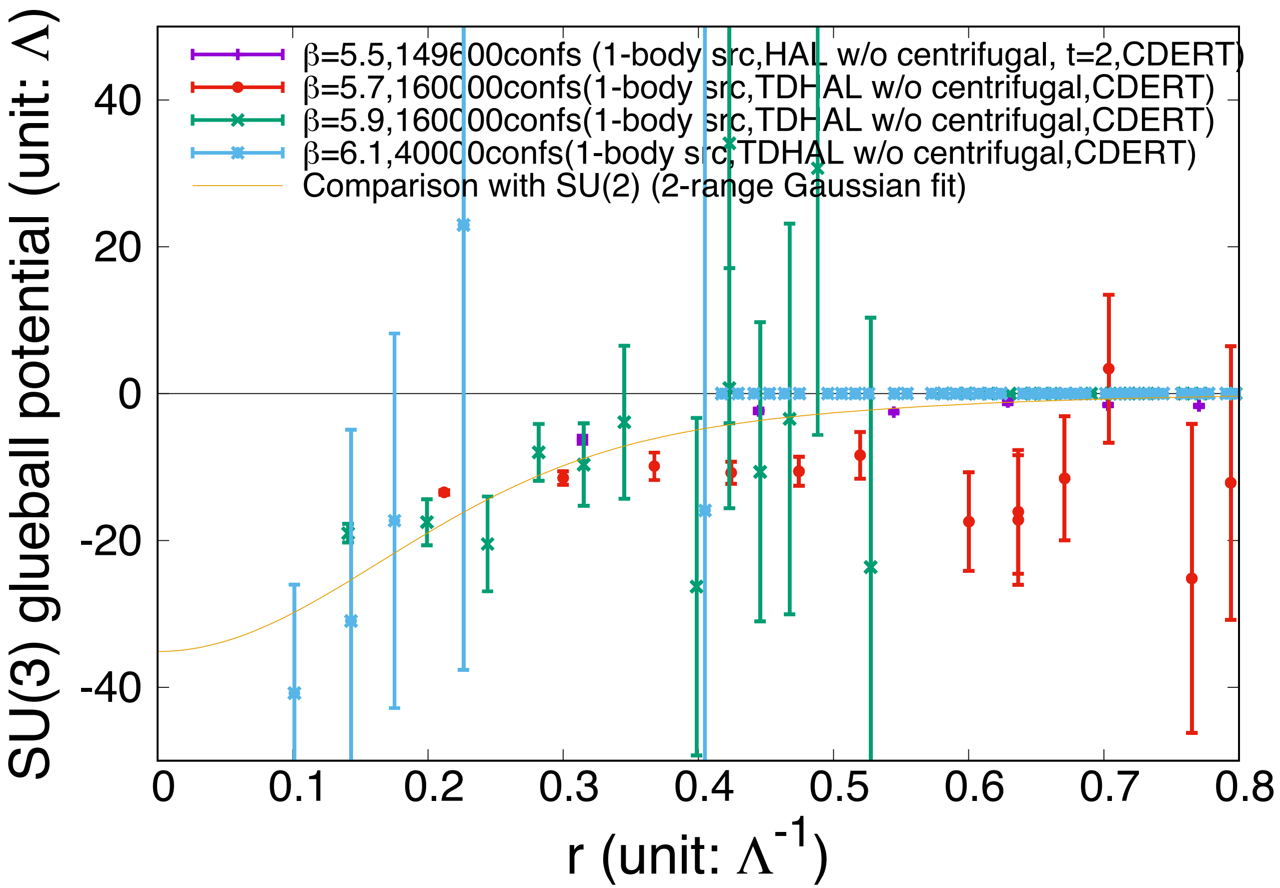
A. H. Peter et al., MNRAS **430**, 81 (2013), **430**, 105 (2013);
B. S. W. Randall et al., APJ **679**, 1173 (2008).

we may obtain to the following constraint on the scale

$$\Lambda_{N=2} > 60\ \mathrm{MeV}$$

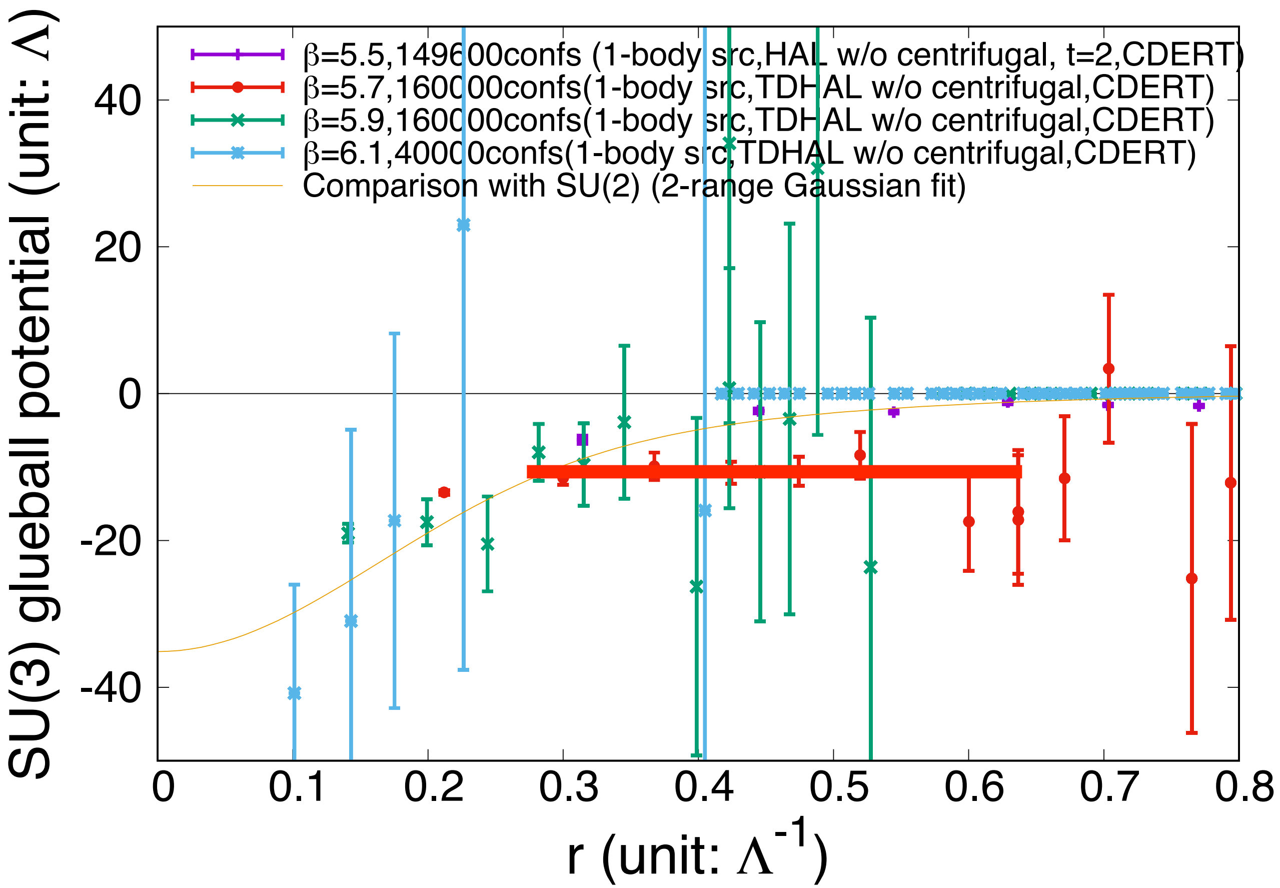
NY, H. Iida, A. Nakamura, M. Wakayama, Phys. Lett. B **813**, 136056 (2021); Phys. Rev. D **102**, 054507 (2020).

SU(3) result



- **SU(3) interglueball potential resembles the SU(2) result**

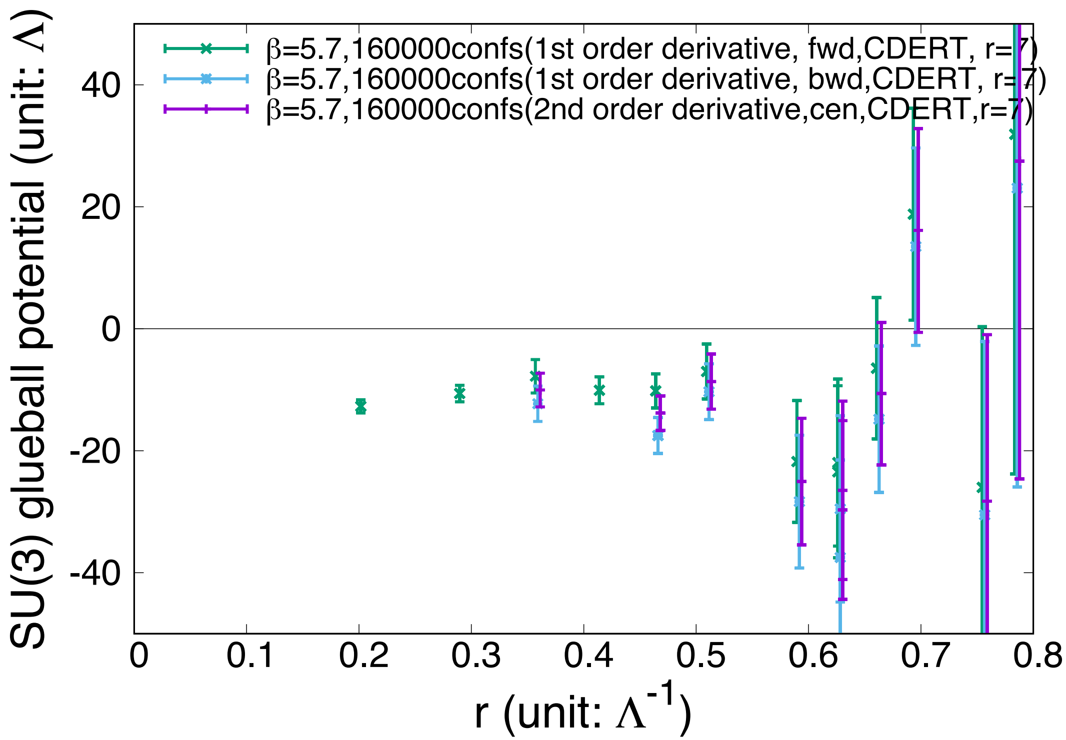
SU(3) result



- SU(3) interglueball potential resembles the SU(2) result
- However, we see some non-zero almost constant potential around r = 0.4 Λ^{-1}

Centrifugal force : systematics from finite difference

We inspect the systematics due to the subtraction of the centrifugal force by changing the finite difference form



Three types of derivatives:

- **1st order forward**

$$\frac{f(x+a)-f(x)}{a}$$

- **1st order backward**

$$\frac{f(x)-f(x-a)}{a}$$

- **2nd order central**

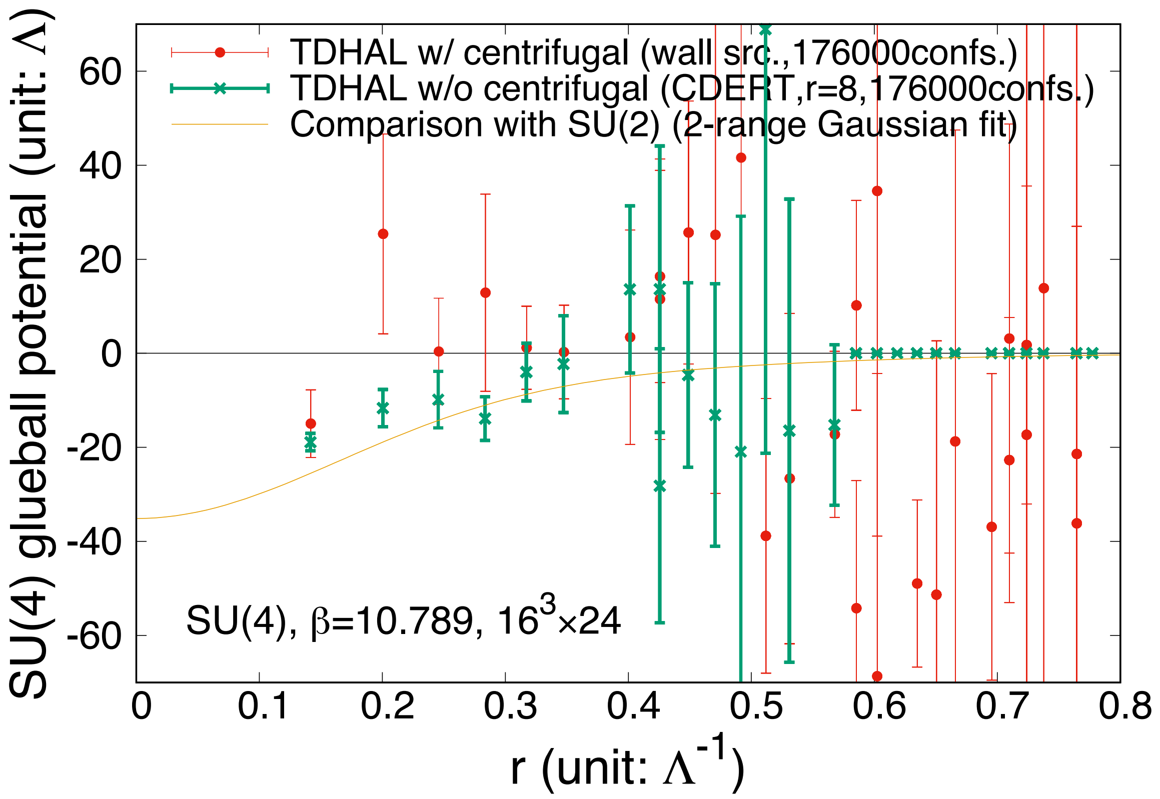
$$\frac{f(x+a)-f(x-a)}{2a}$$

Change of differential forms reveals sizable systematics

Non-zero potential near r = 0.4 Λ^{-1} is explained by this uncertainty?

Use improved action ??

SU(4) result



- **The SU(4) interglueball potential is close to the SU(2), SU(3) ones**
- **The large N_c scaling is not clear (the potential scales as $1/N_c^2$)**

Summary

- Unveiling the dynamics of glueballs is important in the context of QCD and dark matter physics.
- We calculated the interglueball potential in the SU(2,3,4) YMT with the Time-dependent HALQCD method.
- We used the cluster decomposition principle to reduce the statistical noise.
- We removed the centrifugal force : the interglueball potential has sizable systematics.
- SU(2) YMT: observational constraint on dark matter scattering implies $\Lambda > 60$ MeV.
- Preliminary results of SU(3) and SU(4) YMTs look consistent with SU(2) glueball : large N_c is correct?

Homeworks:

- Systematics due to the discretization to be studied : evaluation with the improved action.
- Check of large N_c scaling : Simulation of YMT with $N_c > 4$.

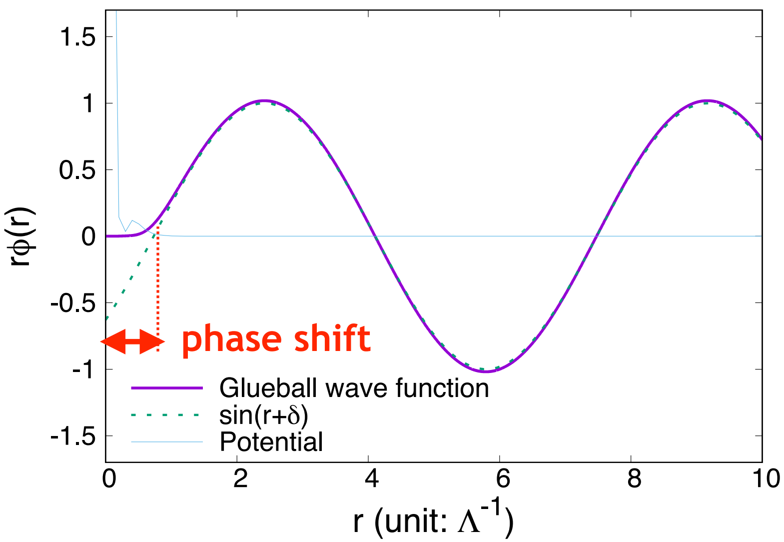
Backup slides

From potential to scattering cross section

Potential ⇒ Scattering phase shift:

Solve $$\left(\frac{\partial^2}{\partial r^2} + k^2 + U(r)\right)\phi(r) = 0$$

➡ $$\phi(r) \propto \sin[r + \delta(k)] \quad (r \to \infty)$$



Scattering phase shift ⇒ Cross section:

We are interested in low energy DM cross section, s-wave dominant :

➡ $$\sigma_{\rm tot} = \frac{4\pi}{k^2}\sin^2[\delta(k \to 0)]$$

Yukawa: σ_{tot} = (2.5 - 4.7)Λ^{-2} (stat.)

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We do not know the scale of the YM theory, so we leave it as a free parameter Λ

Nevertheless, all quantities calculated on lattice depend on Λ

⇒ We express all quantities in unit of Λ (and finally constrain Λ from other data).

Relation between Λ and string tension:

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String tension in SU(2) YM :

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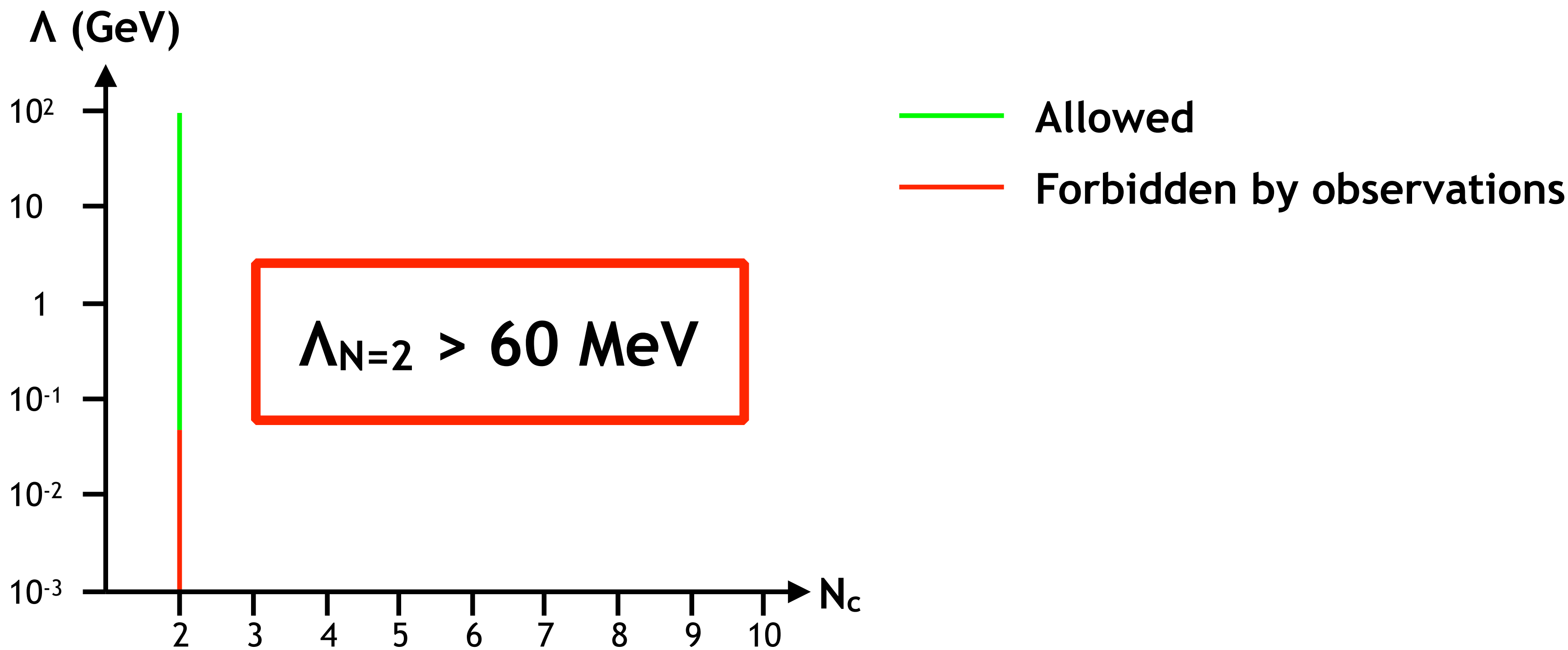
⇒ Lattice spacing is now expressed in unit of Λ

Constraint on SU(N) YM scale parameter from DM X section

Observational constraints: $\frac{\sigma_{tot}}{m_\Phi} < 1.0\ \mathrm{cm}^2/\mathrm{g}$

Robust constraint from galactic cluster shape, collisions (upper limit)

A. H. Peter et al., MNRAS **430**, 81 (2013), **430**, 105 (2013); S. W. Randall et al., APJ **679**, 1173 (2008).



N_c vs. scale parameter (Λ) diagram

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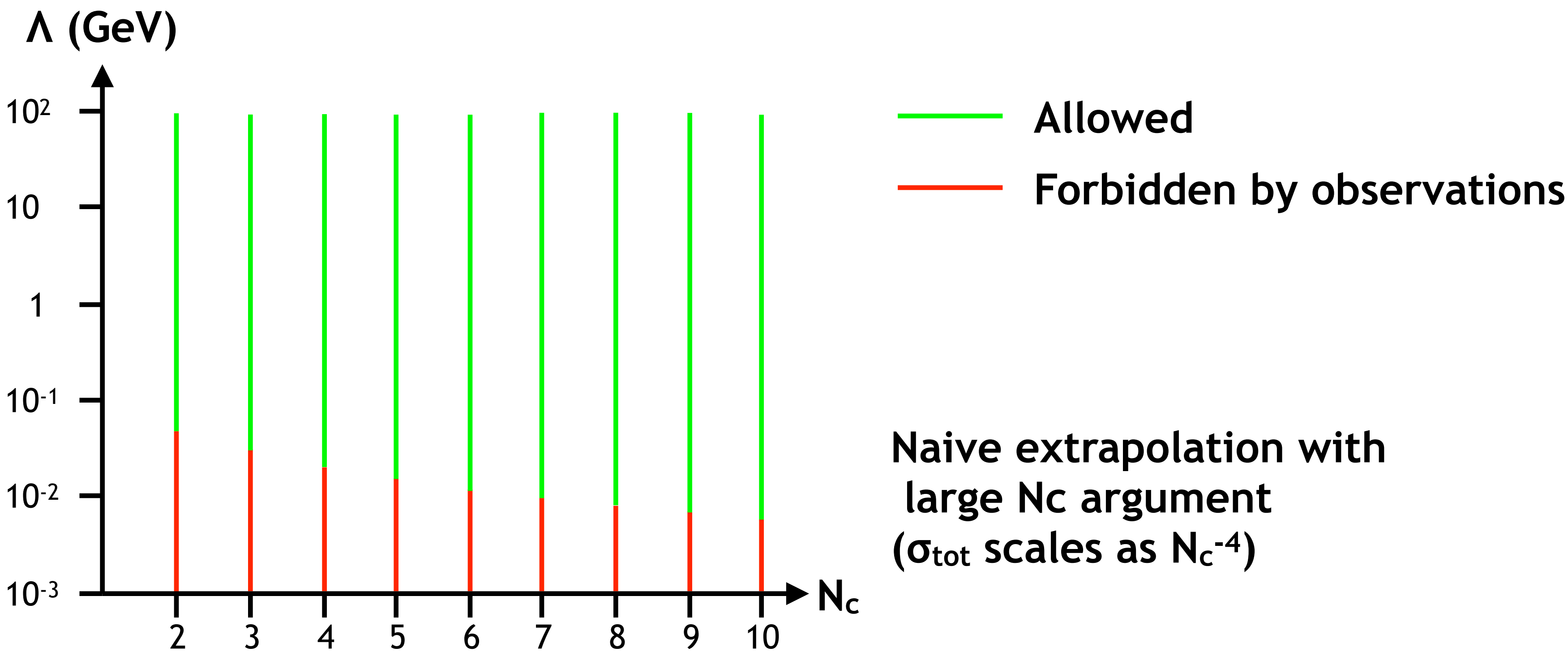

Naive extrapolation with large Nc argument (σ_{tot} scales as N_c^{-4})

N_c vs. scale parameter (Λ) diagram