HAL QCD potentials with non-zero total momentum and application to the I=2 $\pi\pi$ scattering

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0. Introduction

Necessity of boosted systems
for the HAL QCD method

HAL QCD method for resonances

σ resonance from $\pi\pi$ scattering in the center of mass system

$$\langle 0|\pi(t)\pi(t)\sigma(0)|0\rangle \simeq \underline{\langle 0|\pi(t)\pi(t)|0\rangle}\langle 0|\sigma(0)|0\rangle + e^{-E_{\pi\pi}t}\langle 0|\pi(t)\pi(t)|\pi\pi\rangle\langle\pi\pi|\sigma(0)|0\rangle$$

vacuum states dominates signals

non-zero total momentum (boosted system)

$$\langle 0|\pi(t)\pi(t)\sigma(0)|0\rangle \simeq e^{-E_{\pi\pi}t}\langle 0|\pi(t)\pi(t)|\pi\pi\rangle\langle\pi\pi|\sigma(0)|0\rangle + \cdots$$

vacuum contribution is absent

HAL QCD method was formulated for a boosted system. S. Aoki, Lattice 2019.

But no numerical test has been performed so far.

This talk for I=2 $\pi\pi$ scattering.

I. The HAL QCD potential from the moving system (Theory)

S. Aoki, Lattice 2019.

Setup

Center of mass (CM)

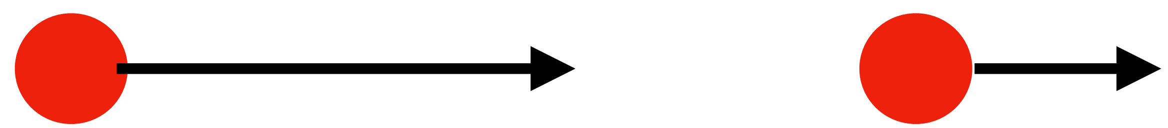
$(E_k, \mathbf{k})$ $(E_k, -\mathbf{k})$

$$\mathbf{P}^* = 0$$

$$E_k = \sqrt{\mathbf{k}^2 + m^2}$$

Moving

x_1 x_2



$$\mathbf{P} = \mathbf{k}_1 + \mathbf{k}_2$$

$(E_{k_1}, \mathbf{k}_1)$ $(E_{k_2}, \mathbf{k}_2)$

Lorentz transformation

$$\mathbf{P}^* = \gamma(\mathbf{P} - \mathbf{v}W) = 0 \qquad W = \sqrt{\mathbf{k}_1^2 + m^2} + \sqrt{\mathbf{k}_2^2 + m^2} \qquad \gamma := \frac{1}{\sqrt{1 - \mathbf{v}^2}}$$

$$P \cdot X = P^* \cdot X^* \qquad X := \frac{x_1 + x_2}{2}, x := x_1 - x_2$$

HAL QCD potential from boosted NBS wave function

Leading order HAL QCD potential $$V^{\rm LO}_{x^{*4}}(\mathbf{x}^*) = \frac{(\nabla^{*2} + k^{*2})\varphi_{k_1^*,k_2^*}(\mathbf{x}^*, x^{*4})}{2\mu\varphi_{k_1^*,k_2^*}(\mathbf{x}^*, x^{*4})}.$$ CM

NBS wave function $$e^{iP\cdot X}\varphi_{k_1,k_2}(x) = e^{iP^*\cdot X^*}\varphi_{k_1^*,k_2^*}(x^*)$$

Moving CM

$$x^{*4} = \gamma(x^4 - i\mathbf{v}\cdot\mathbf{x}_\parallel), \quad \mathbf{x}^*_\parallel = \gamma(\mathbf{x}_\parallel + i\mathbf{v}x^4), \quad \mathbf{x}^*_\perp = \mathbf{x}_\perp.$$

LO potential $$x^4 = \mathbf{x}_\parallel = 0$$

Moving

$$V^{\rm LO}_{x^{*4}=0}(\mathbf{x}^*_\perp) = \left.\frac{(\nabla^2_\perp + \gamma^2(\nabla_\parallel + i\mathbf{v}\partial_{x^4})^2 + k^{*2})\varphi_{k_1,k_2}(\mathbf{x}, x^4)}{2\mu\varphi_{k_1,k_2}(\mathbf{x}, x^4)}\right|_{x^4=0,\mathbf{x}_\parallel=0}$$

CM Moving

Time dependent method

CM

$$\left(-H_0 - \partial_{X^{*4}} + \frac{1}{4m}\partial^2_{X^{*4}}\right) R(\mathbf{x}^*, x^{*4}, X^{*4}) = V^{\mathrm{LO}}_{x^{*4}}(\mathbf{x}^*) R(\mathbf{x}^*, x^{*4}, X^{*4})$$

$$R(\mathbf{x}^*, x^{*4}, X^{*4}) := \sum_n B_n \varphi_{W_n^*}(x^*) e^{-\underline{(W_n^* - 2m)}X^{*4}} + \cdots$$

NBS

Moving

$$R(\mathbf{x}, x^4, X^4) \simeq \sum_n B_n \varphi_{W_n}(x) e^{-(W_n - 2m)X^4}$$

$$V^{\mathrm{LO}}_{x^{*4}=0}(\mathbf{x}_\perp) = \left. \frac{\left(L_\perp + L_\parallel + mE\right)(\mathbf{x}, x^4, X^4)}{mG(\mathbf{x}, x^4, X^4)} \right|_{x^4=0, \mathbf{x}_\parallel=0}$$

$$G(\mathbf{x}, x^4, X^4) = \left((\partial_{X^4} - 2m)^2 - \mathbf{P}^2\right) R(\mathbf{x}, x^4, X^4),$$

$$E(\mathbf{x}, x^4, X^4) = \left[\partial^2_{X^4}/4m - \partial_{X^4} - \mathbf{P}^2/4m\right] G(\mathbf{x}, x^4, X^4),$$

$$L_\perp(\mathbf{x}, x^4, X^4) = \nabla^2_\perp G(\mathbf{x}, x^4, X^4),$$

$$L_\parallel(\mathbf{x}, x^4, X^4) = \left(-(\partial_{X^4} - 2m)\nabla_\parallel + i\mathbf{P}\partial_{x^4}\right)^2 R(\mathbf{x}, x^4, X^4).$$

II. Numerical results

$I = 2$ $\pi\pi$ potential

Akahoshi and Aoki, in preparation.

Numerical setup

$2+1$ flavor CP-PACS configurations on a $32^3 \times 64$ latttice

Iwasaki gauge action and non-perturbatively improved Wilson quark action

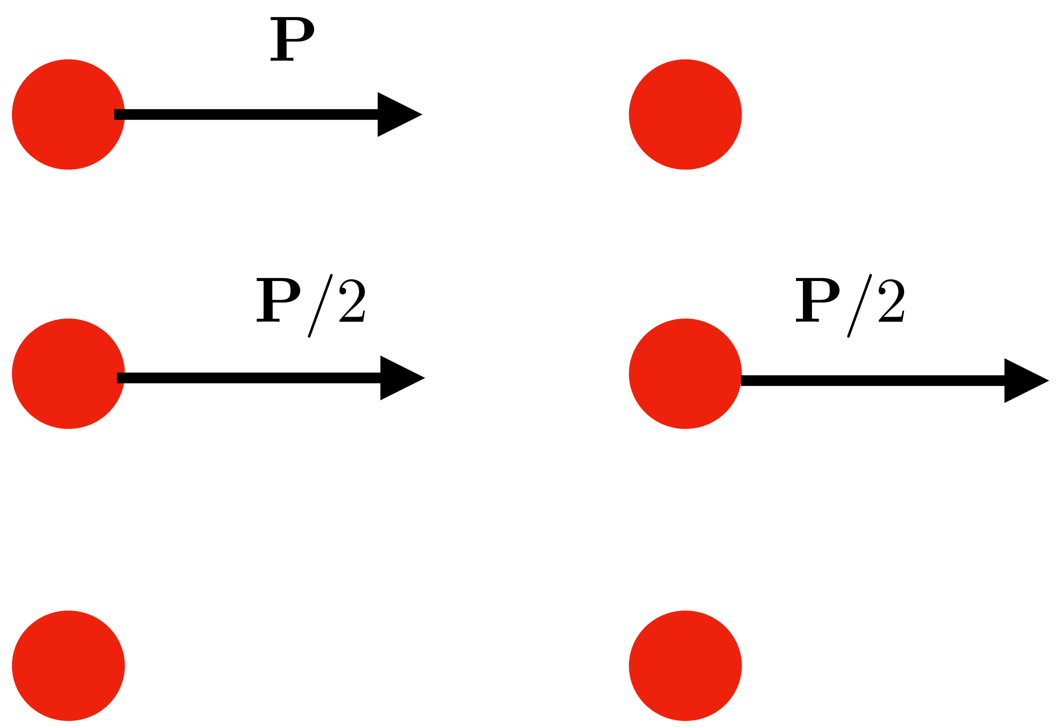
$a \simeq 0.0907$ fm, $m_\pi \simeq 700$ MeV

smeared quark source

case1: $\mathbf{P} = (0, 0, 2\pi/L)$,

case2: $\mathbf{P} = (0, 0, 4\pi/L)$,

CM: $\mathbf{P} = (0, 0, 0)$,



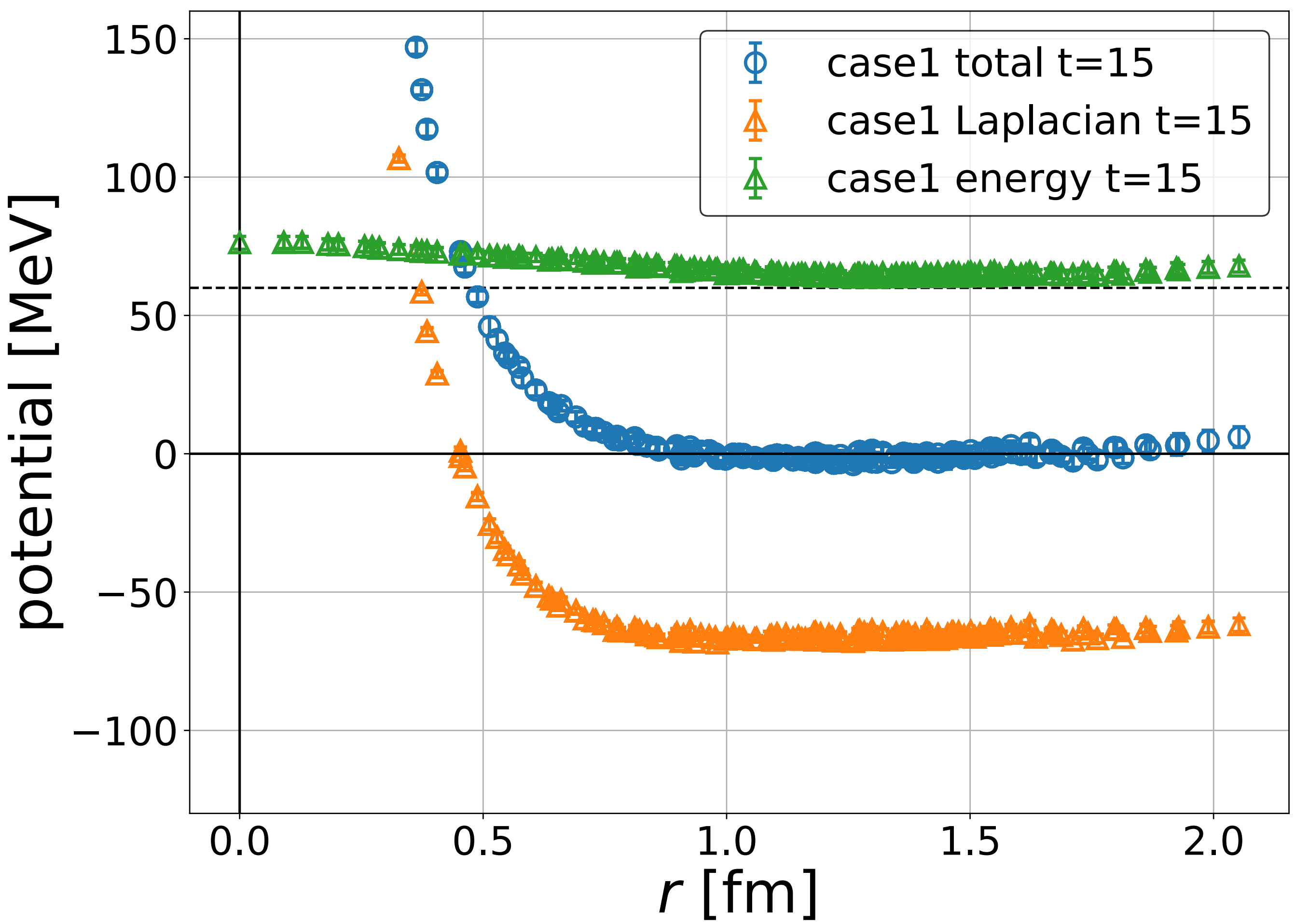
Potentials (breakup)

$$V^{\rm LO}_{x^{*4}=0}(\mathbf{x}_\perp) = \left.\frac{(L_\perp + L_\parallel)(\mathbf{x}, x^4, X^4)}{mG(\mathbf{x}, x^4, X^4)}\right|_{x^4=0,\mathbf{x}_\parallel=0} + \left.\frac{E(\mathbf{x}, x^4, X^4)}{G(\mathbf{x}, x^4, X^4)}\right|_{x^4=0,\mathbf{x}_\parallel=0}$$

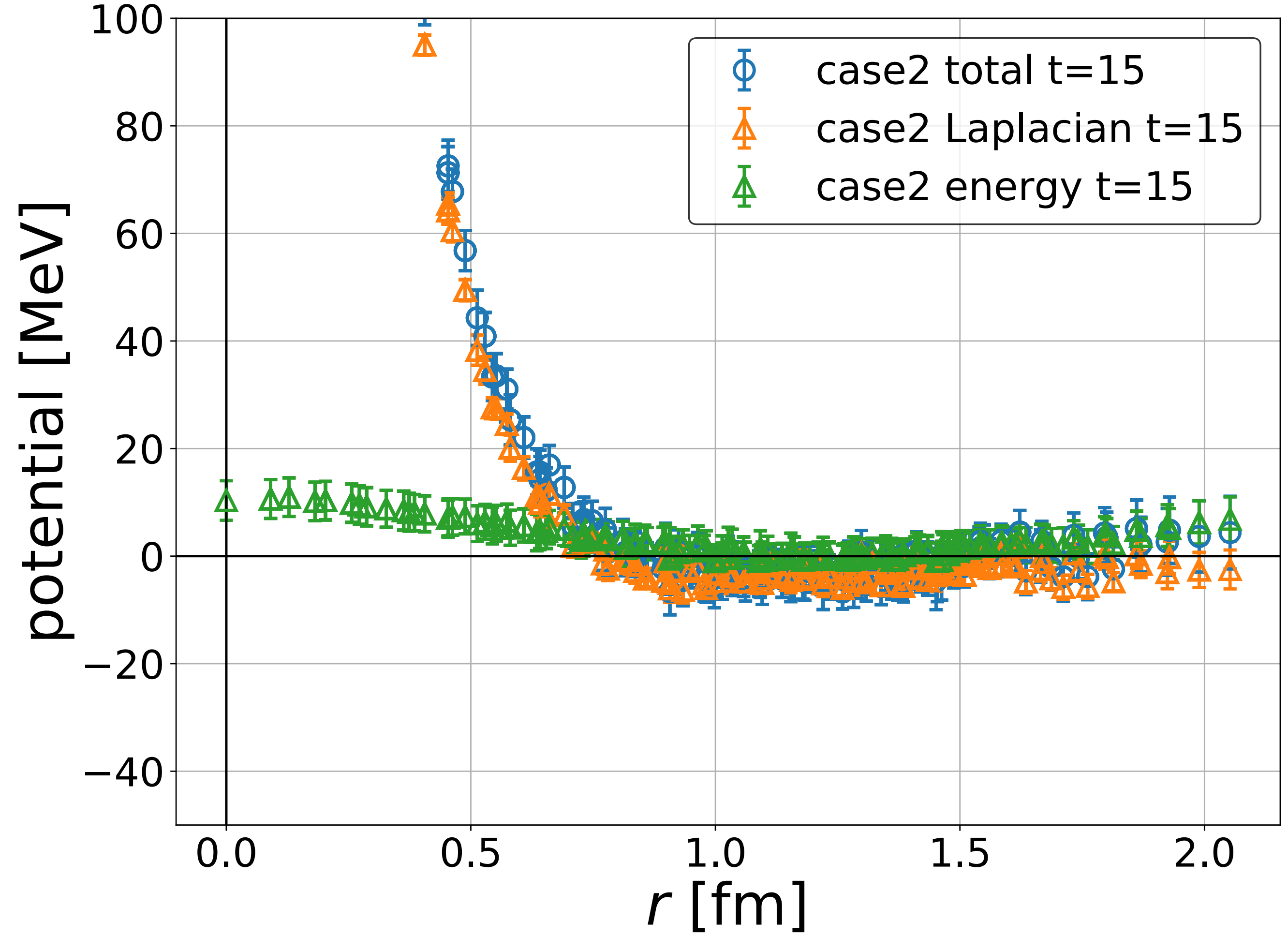
Laplacian

energy

case1

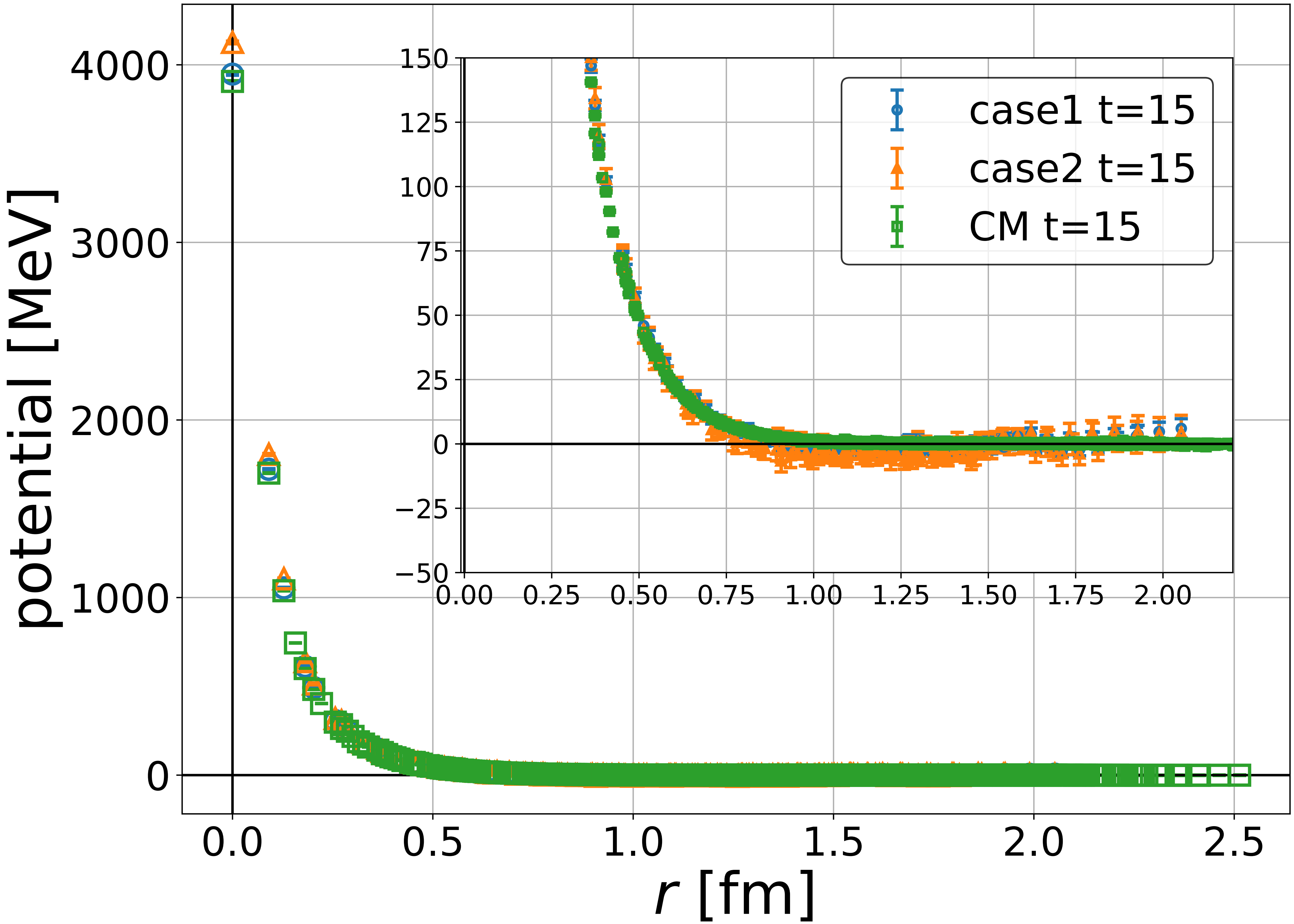


case2



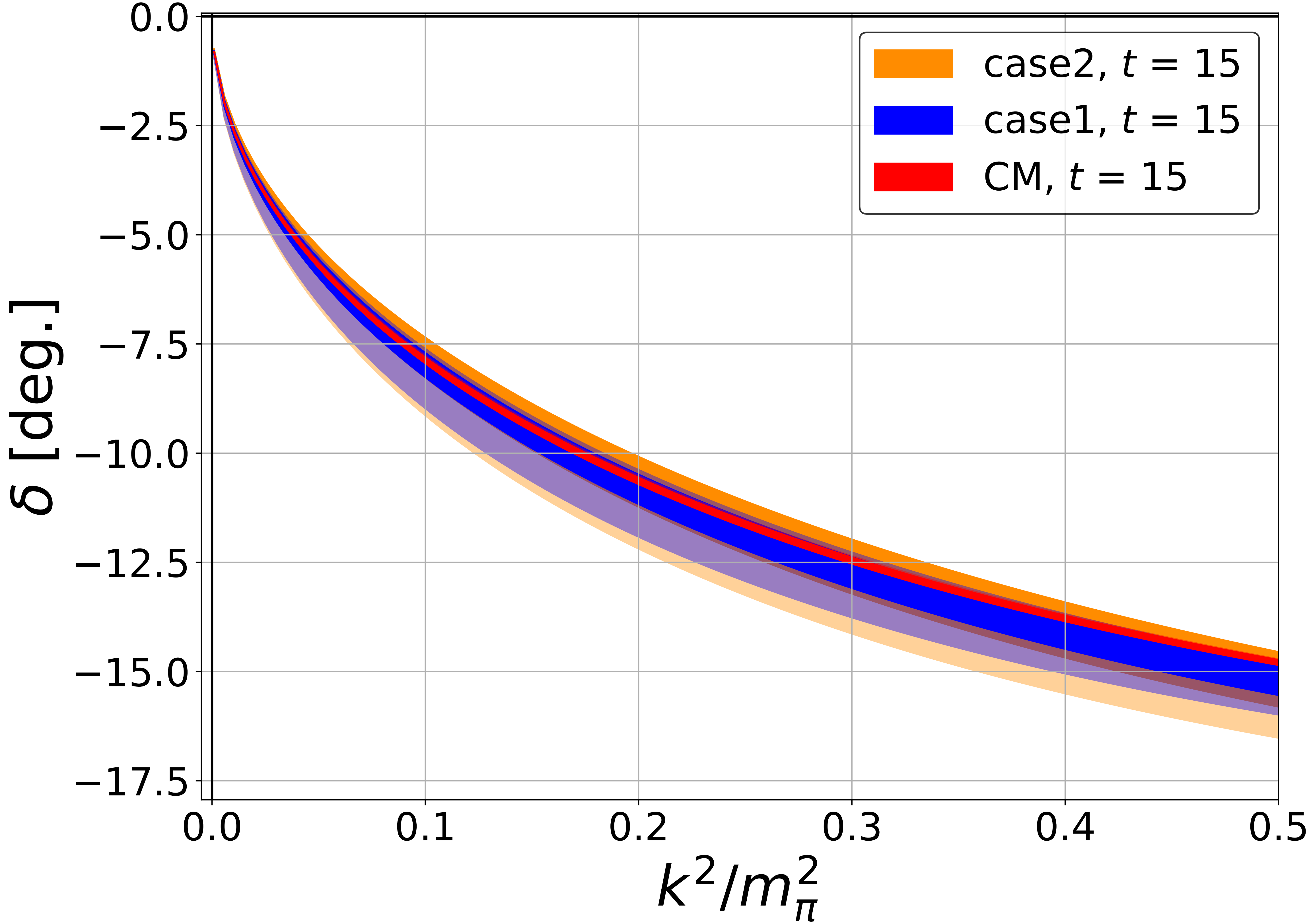
$\simeq$ CM

Potentials (comparison)



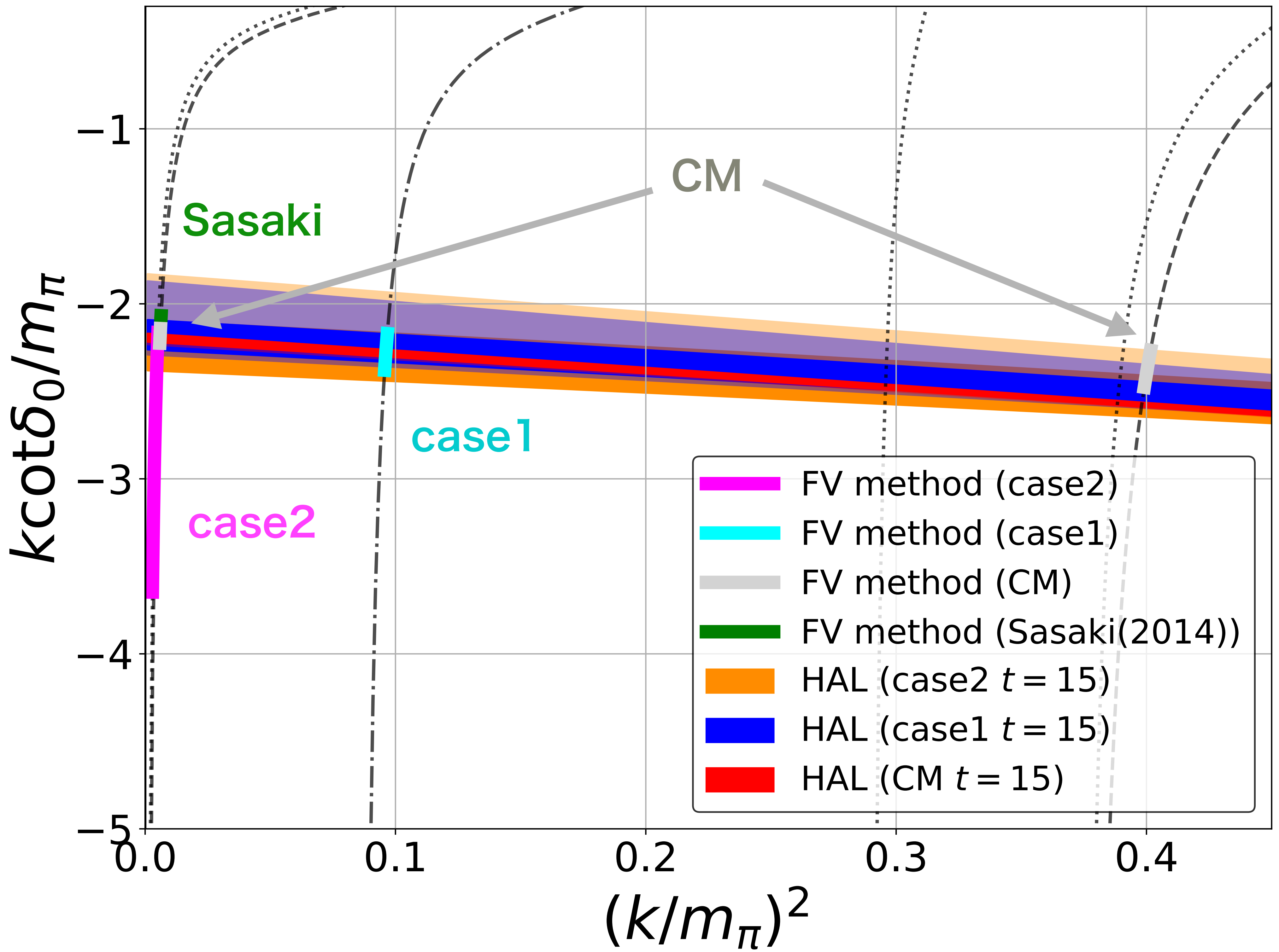
They are consistent except at short distances, though boosted ones are noisier.

Scattering phase shifts



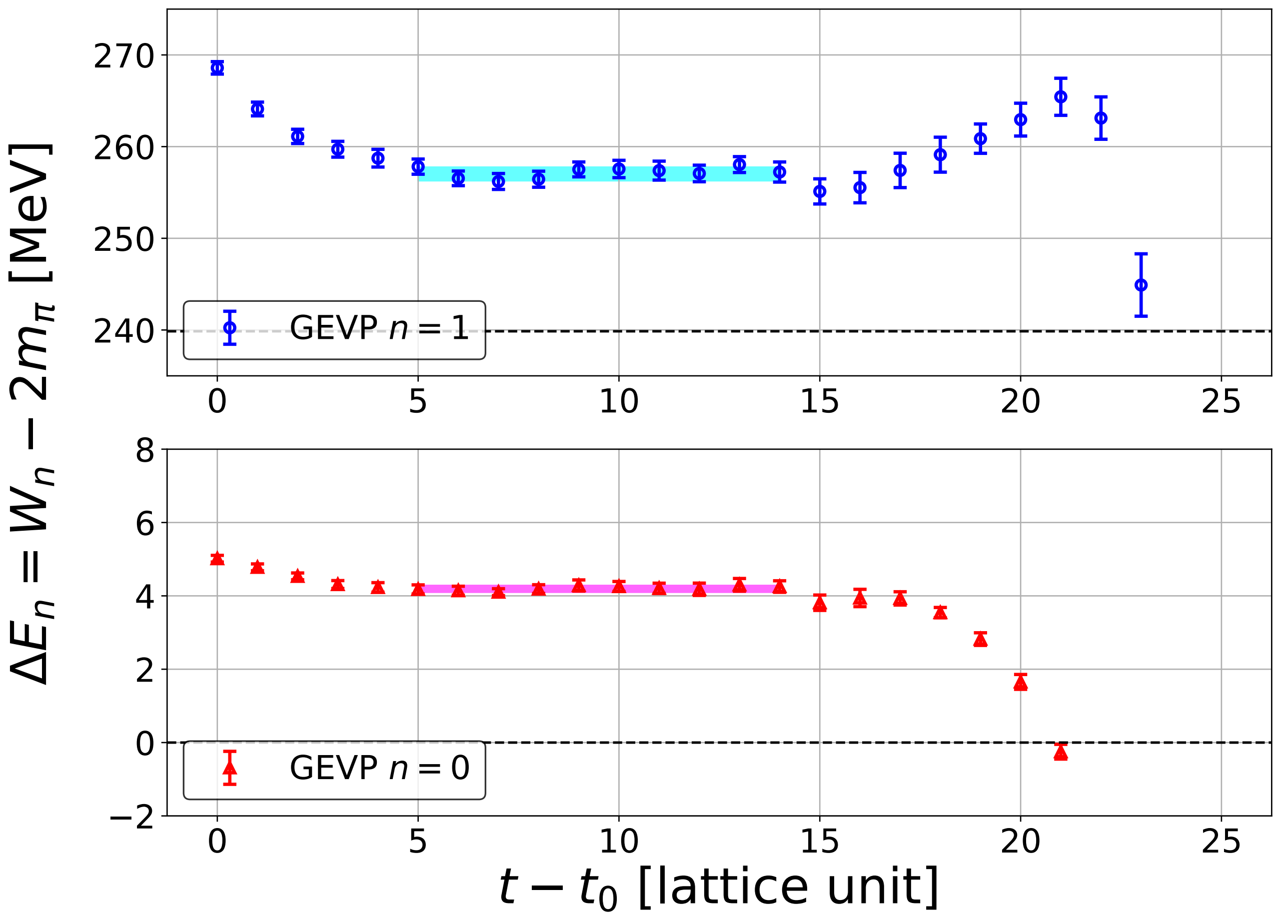
All three cases gives consistent results.

Comparison with finite volume method



HAL QCD potentials with non-zero momentum work !

Finite volume spectra

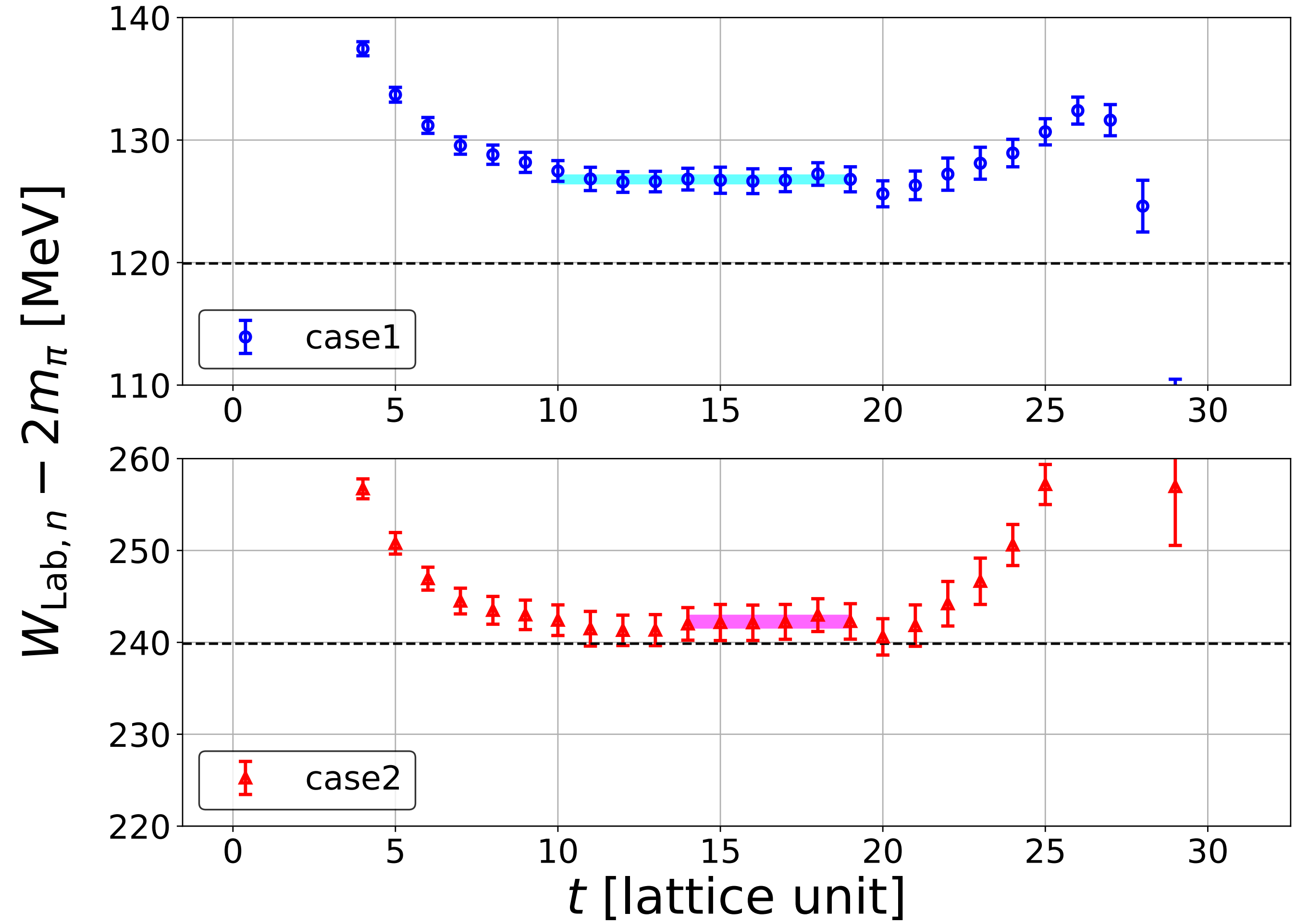


CM (1st excited)

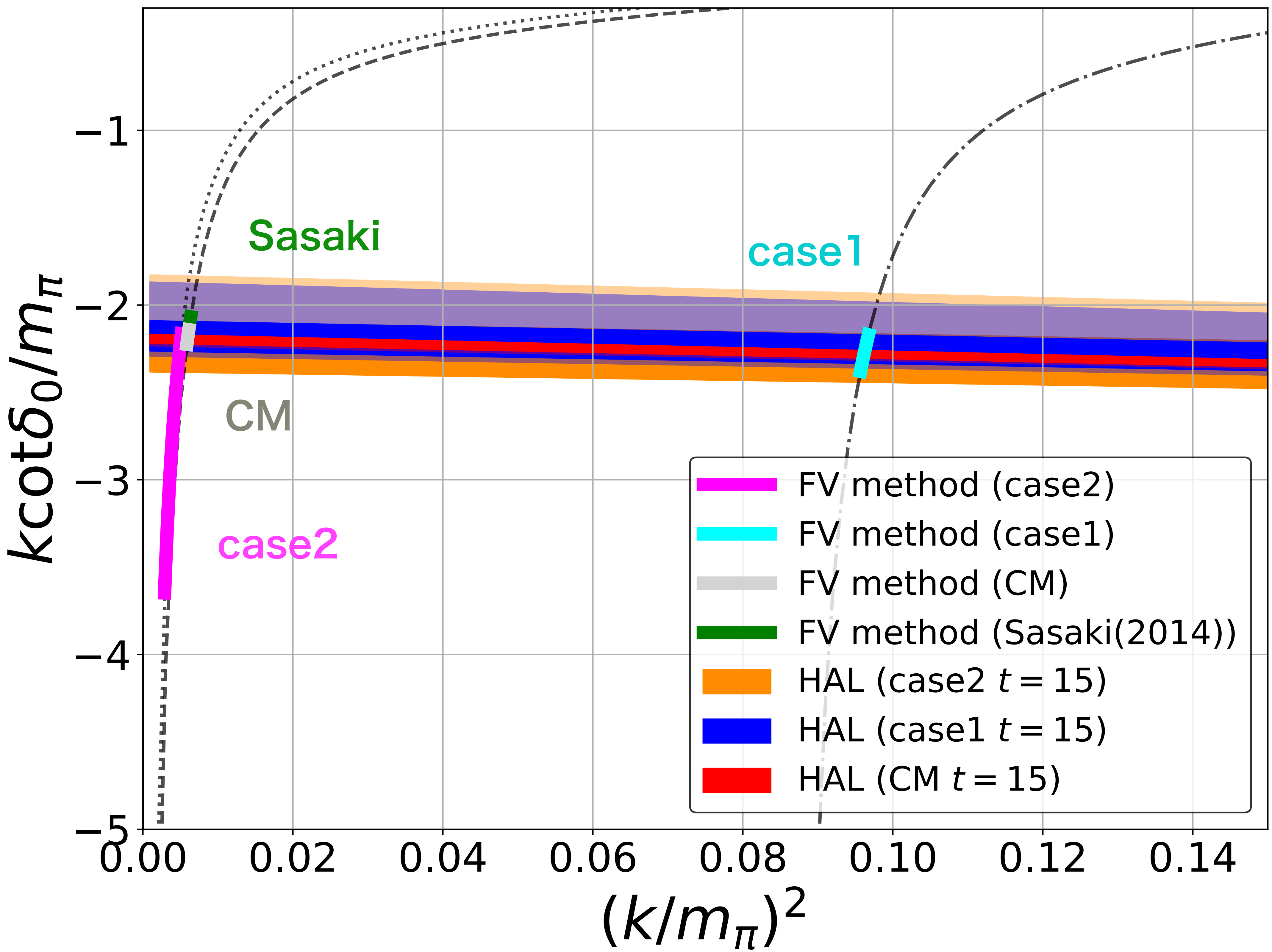
CM (Lowest)

case1

case2



Comparison at low energies



Conclusion

HAL QCD method with non-zero total momentum is formulated.

Application to I=2 $\pi\pi$ system is presented.

The HAL QCD potential works with non-zero total momentum !

Studies in future.

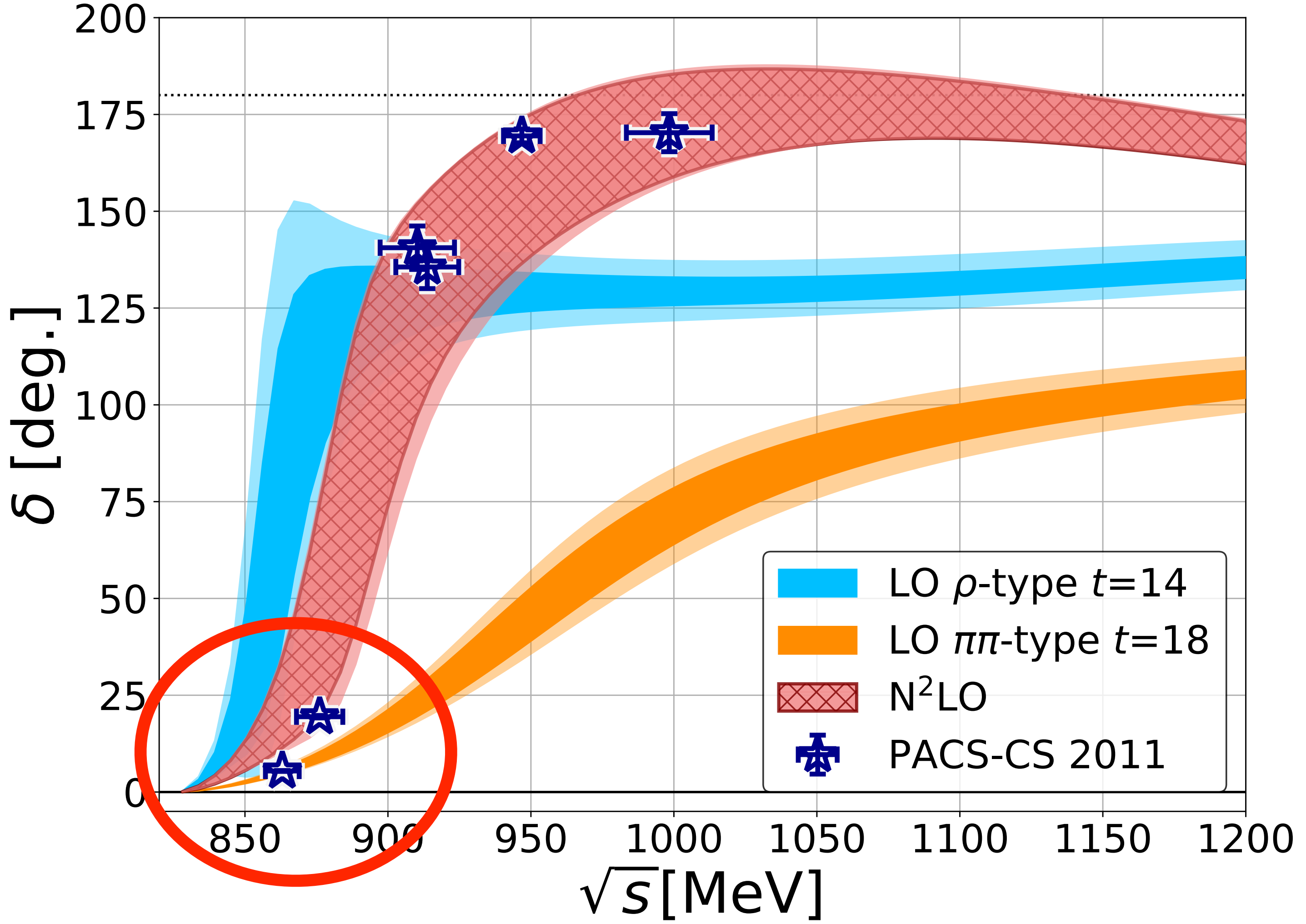
1. resonance in the HAL QCD potential

σ resonance in $I = 0$ $\pi\pi$ moving system

ρ resonance in moving system

2. extension to 3 body interactions ?

CM result



moving