

$\bar{b}\bar{c}q_1q_2$ fourquarks from lattice QCD

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Lattice 2021, MIT

Exotic Multiquark States

Four and five quark states	Six quark states
<ul style="list-style-type: none">• Multiple states, with two heavy quarks (mainly charm), having four quark configurations, have been discovered.• Charmed pentaquark states have been discovered.	<ul style="list-style-type: none">• Beside deuteron, only one more state with six quark configurations ($d^*(2380)$) have been observed in one experiment.• A few hypernuclei with very shallow binding have been observed.

Exotic Multiquark States

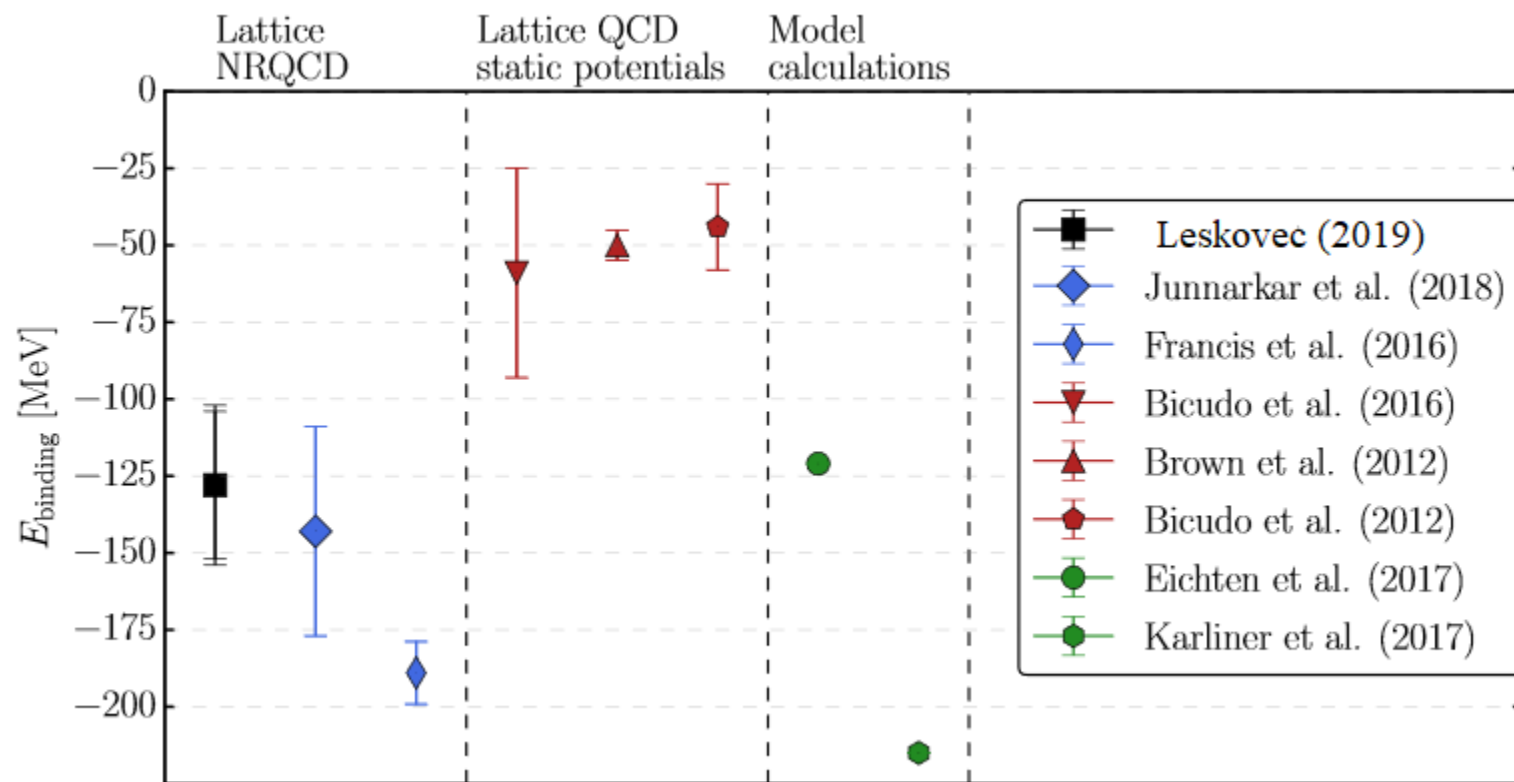
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- Is their possibility to find more of those?
- What can lattice studies do?

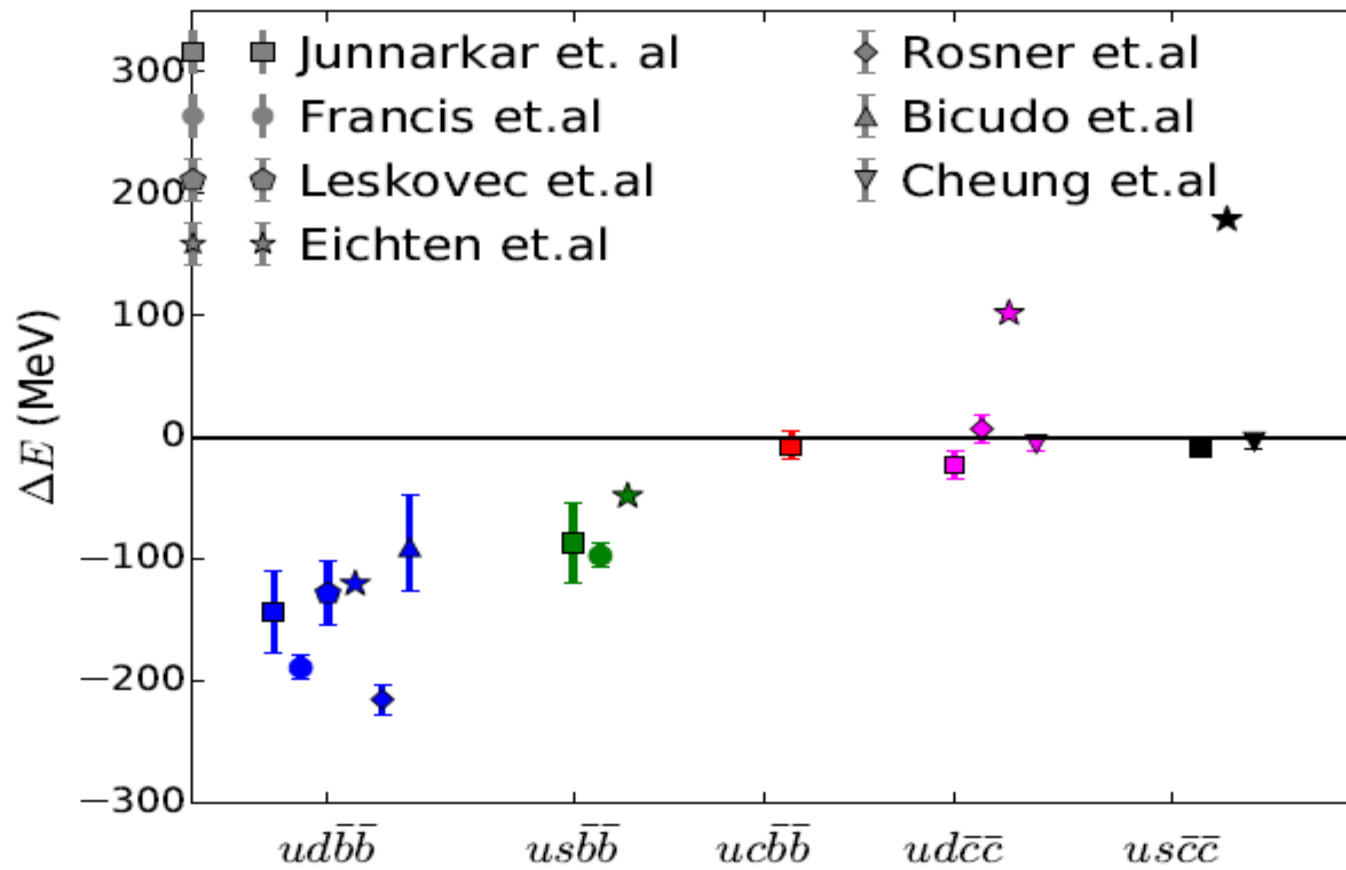
Lattice study of heavy exotics

- Nonrelativistic b quark with relativistic other quarks (calculations with relativistic b quarks are starting.)
- Potential:
 - Static quark potential (Born-Oppenheimer)...
P. Bicudo et al, Phys. Rev. D95, 034502(2017)
 - HALQCD lattice potential ..HAL QCD ('16,'18)

$ud\bar{b}\bar{b} (1^+)$



Leskovec et al, Phys. Rev. D 100, 014503 (2019)



Junnarkar et al,
Phys. Rev. D99 034507 (2019)

State	ΔE^1 [MeV]	State	ΔE^1 [MeV]
$ud\bar{b}\bar{b}$	-143(34)	$us\bar{b}\bar{b}$	-87(32)
$uc\bar{b}\bar{b}$	-6(11)	$sc\bar{b}\bar{b}$	-8(3)
$ud\bar{c}\bar{c}$	-23(11)	$us\bar{c}\bar{c}$	-8(8)

State	ΔE^0 [MeV]	State	ΔE^0 [MeV]
$uu\bar{b}\bar{b}$	-5(18)	$uu\bar{c}\bar{c}$	26(11)
$ss\bar{b}\bar{b}$	3(9)	$ss\bar{c}\bar{c}$	14(4)
$cc\bar{b}\bar{b}$	16(1)		



Observation of T_{cc}^+ state



NEW

LHCb-PAPER-2021-031

- **First observation of a same-sign doubly charmed tetraquark T_{cc}^+**

- Very narrow state in $D^0 D^0 \pi^+$ mass spectrum
- Consistent with $cc\bar{u}\bar{d}$ tetraquark
- Mass very close to $D^{*+}D^0$ mass thresholds
- Manifestly exotic

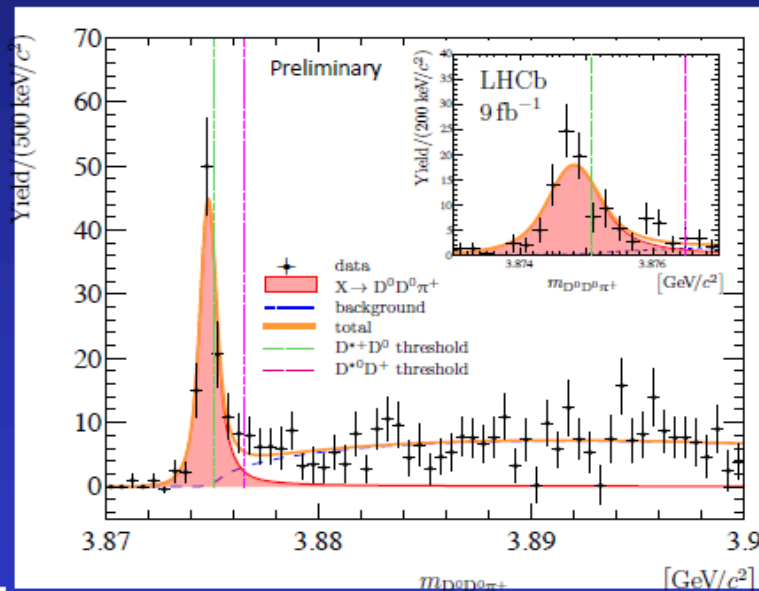
- **Parameters of T_{cc}^+**

- Fit structure with P-wave relativistic Breit-Wigner

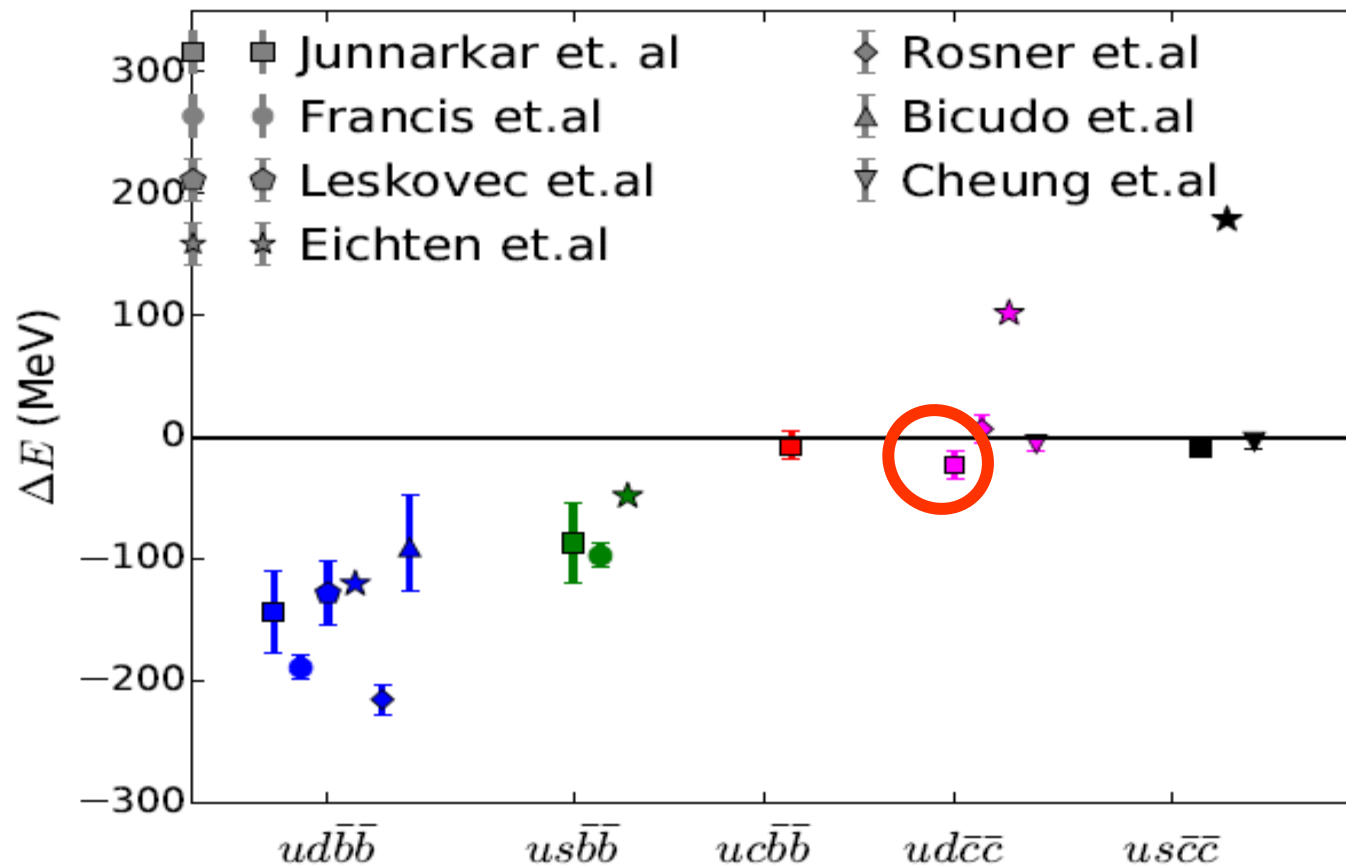
$$\delta m_{BW} = -273 \pm 61 \pm 5 \begin{matrix} +11 \\ -14 \end{matrix} \text{ keV}/c^2,$$

$$\Gamma_{BW} = 410 \pm 165 \pm 43 \begin{matrix} +18 \\ -38 \end{matrix} \text{ keV},$$

- Uncertainties stat, syst and due $J^P = 1^+$ assumption
- Significance for signal $> 10 \sigma$
- Significance for $\delta m_{BW} < 0$ 4.3σ



Recent LHCb results on exotic meson candidates
Ivan Polyakov



Junnarkar et al,
Phys. Rev. D99 034507 (2019)

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What about $\bar{b}\bar{c}q_1q_2$?

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Various models predicted mixed results for $ud\bar{b}\bar{c} (1^+)$:

- HQ-symmetry inspired and non-chiral models: mostly unbound or very weakly bound
- QCD sum rule, chiral models: a bound state (both for 0 and 1-isospins) with binding over a wide range $\sim 20\text{-}400$ MeV !

Lattice QCD: $ud\bar{b}\bar{c}$ $0(1^+)$

$$-61 \text{ MeV} < \Delta E_{ud\bar{b}\bar{c}} \sim -15 \text{ MeV}$$

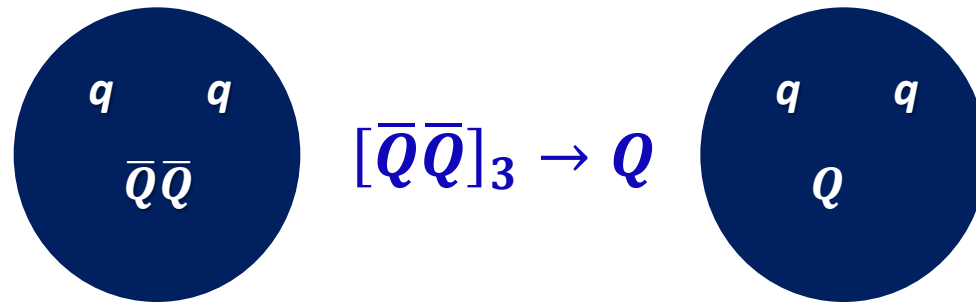
$\bar{D}B^*$ threshold

A. Francis et al:

Phys. Rev. D99 (2019) no.5, 054505

No state below thresholds: Hudspith et al, Phys. Rev. D102, 114506 (2020), Lattice'21.
Pflaumer, Lattice'21

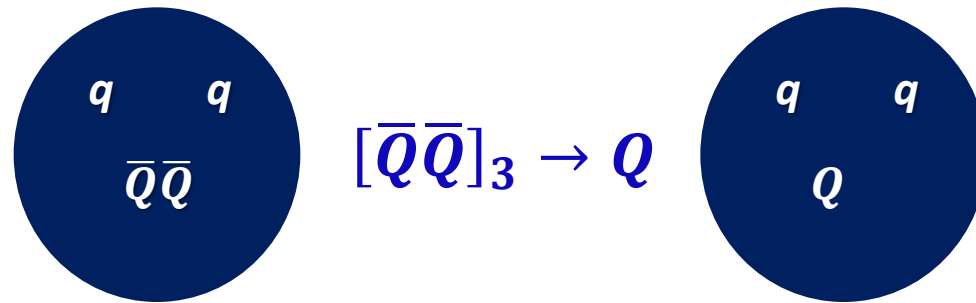
Heavy four-quark interpolating fields



Diquark properties:

- $[\bar{q}q]_3^{m_q \rightarrow \infty} \longrightarrow$ compact
- $\{qq'\}_{\bar{3}}$ attractive
- $\{qq'\} \equiv (qC\gamma_5q')$ lightest
- $m(\{ud\}) < m(\{us\})$

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A possible structure:

$$\begin{array}{cc}
 (qC\gamma_5 q') & (\bar{Q}C\gamma_i \bar{Q}') \\
 \downarrow & \downarrow \\
 \{qq'\} & \{\bar{Q}\bar{Q}'\}
 \end{array}$$

Heavy four-quark interpolating fields

A possible structure:

How about ?

$$(qC\gamma_5 q')(\bar{Q}C\gamma_i \bar{Q}')$$

\downarrow \downarrow

$$\{qq'\} \qquad \qquad \{ \bar{Q}\bar{Q}' \}$$

Why not?

$$(qC\gamma_i q')(\bar{Q}C\gamma_5 \bar{Q}')$$

\downarrow \downarrow

$$\{qq'\} \qquad \qquad \{ \bar{Q}\bar{Q}' \}$$

When Q or Q' is not heavy enough?

Similarly,

$$\varepsilon_{ijk}(qC\gamma_i q')(\bar{Q}C\gamma_k \bar{Q}')$$

Inclusion of these diquark structures are allowed in these cases since charm quark is not too heavy in mass (unlike bottom quark mass).

Interpolating fields for $ud\bar{b}\bar{c} 0(1^+)$

Diquark-Anti-diquark type

$$[uC\gamma_5 d][\bar{c}C\gamma_i \bar{b}]$$

$$[uC\gamma_0\gamma_5 d][\bar{c}C\gamma_i\gamma_0 \bar{b}]$$

Meson-Meson type

$$[uC\gamma_i \bar{b}][dC\gamma_5 \bar{c}]$$

$$[uC\gamma_5 \bar{b}][dC\gamma_i \bar{c}]$$

$$[u\bar{b}][dC\gamma_i\gamma_5 \bar{c}]$$

$$[u\bar{c}][dC\gamma_i\gamma_5 \bar{b}]$$

$$\varepsilon_{ijk}[uC\gamma_i \bar{b}][dC\gamma_k \bar{c}]$$

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$$[u\bar{b}][dC\gamma_i\gamma_5 \bar{c}]$$

$$[u\bar{c}][dC\gamma_i\gamma_5 \bar{b}]$$

$$\varepsilon_{ijk}[uC\gamma_i \bar{b}][dC\gamma_k \bar{c}]$$

Lattice and action details

- Lattices used (to be used) for this study :

HISQ 2+1+1 flavours gauge configurations from MILC

$24^3 \times 64$, $a = 0.12$ fm, $m_l/m_s = 1/5$, $m_\pi L = 4.54$, $m_\pi = 305$ MeV

$32^3 \times 96$, $a = 0.089$ fm, $m_l/m_s = 1/5$, $m_\pi L = 4.5$, $m_\pi = 312$ MeV

$48^3 \times 144$, $a = 0.058$ fm, $m_l/m_s = 1/5$, $m_\pi L = 4.51$, $m_\pi = 319$ MeV

PHYSICAL REVIEW D 87, 054505 (2013) (MILC)

- **Bottom quark:** NRQCD with improved coefficients: $\mathcal{O}(\alpha_s^2 a^2 v^4)$
u(d), s, c: **Overlap**

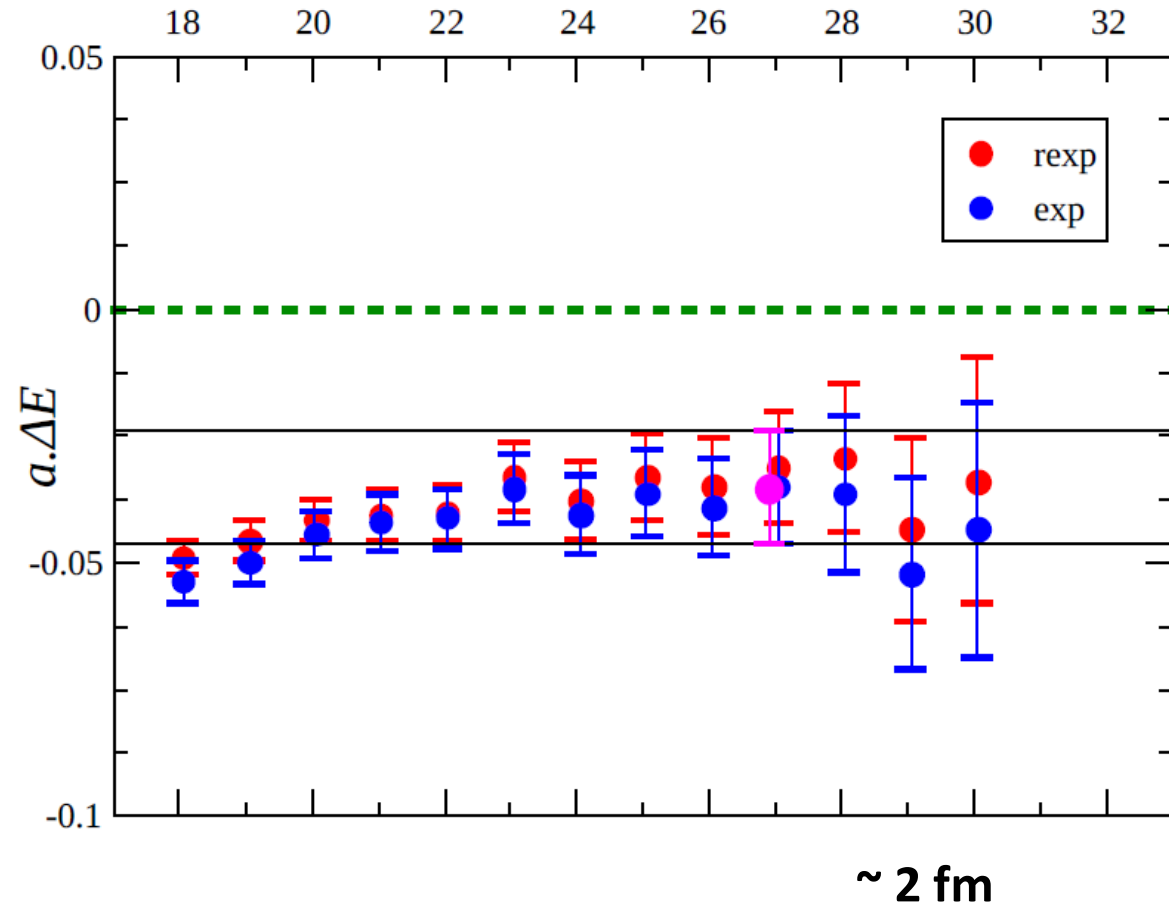
$$\bar{b}\bar{c} q_1 q_2 0(1^+)$$

□ Along with HQET-inspired operators we also use the **diquark-antidiquark type operators with the light diquark having a spin 1** keeping in mind that the charm quark is much lighter in mass compared to the bottom quark.

□ **Preliminary Findings:**

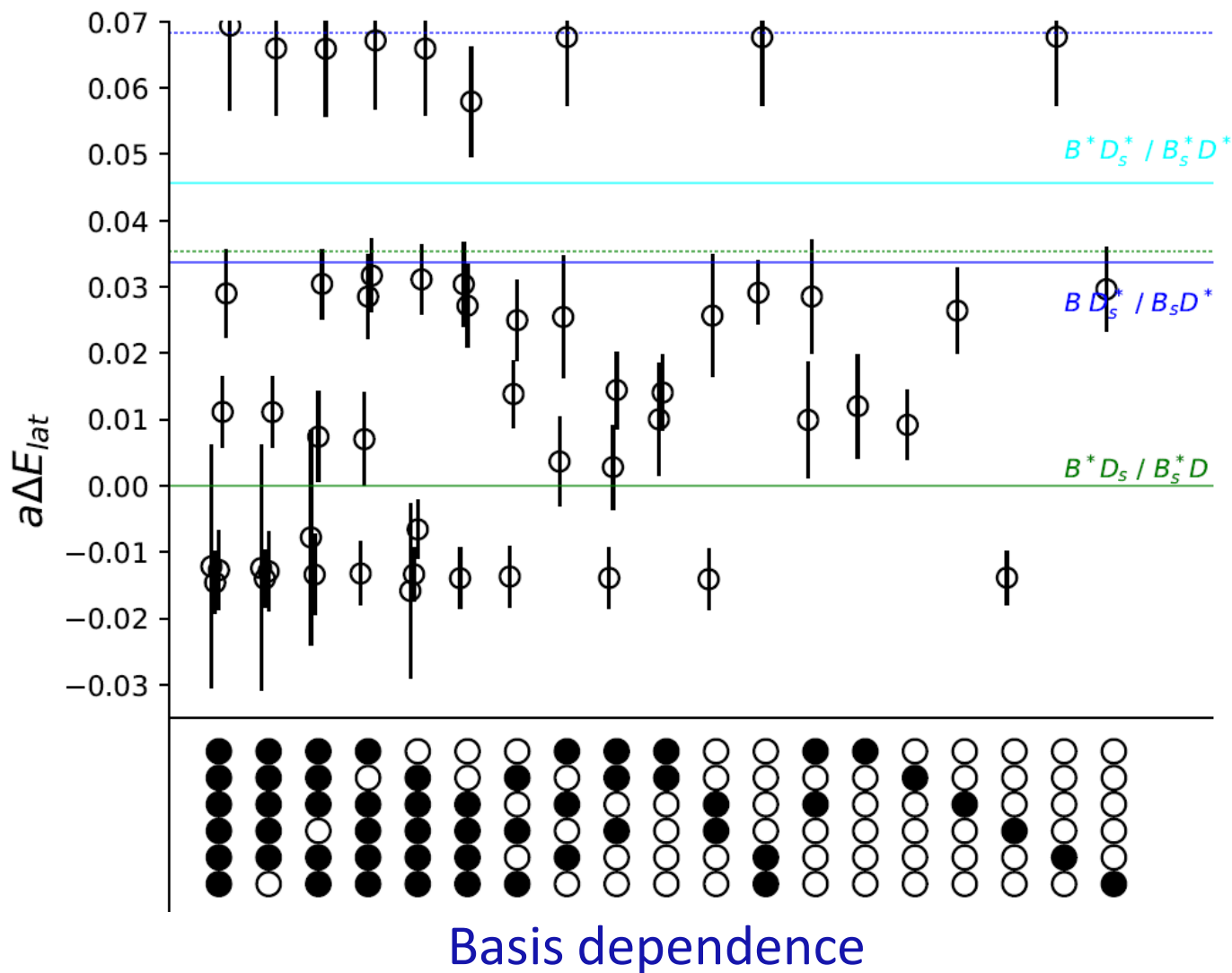
- **Multiple energy levels around elastic threshold.**
- **At least one energy level below two-meson thresholds.**

$\bar{b}\bar{c} q_1 q_2 0(1^+)$



Preliminary

$\bar{b}\bar{c} q_1 q_2 0(1^+)$



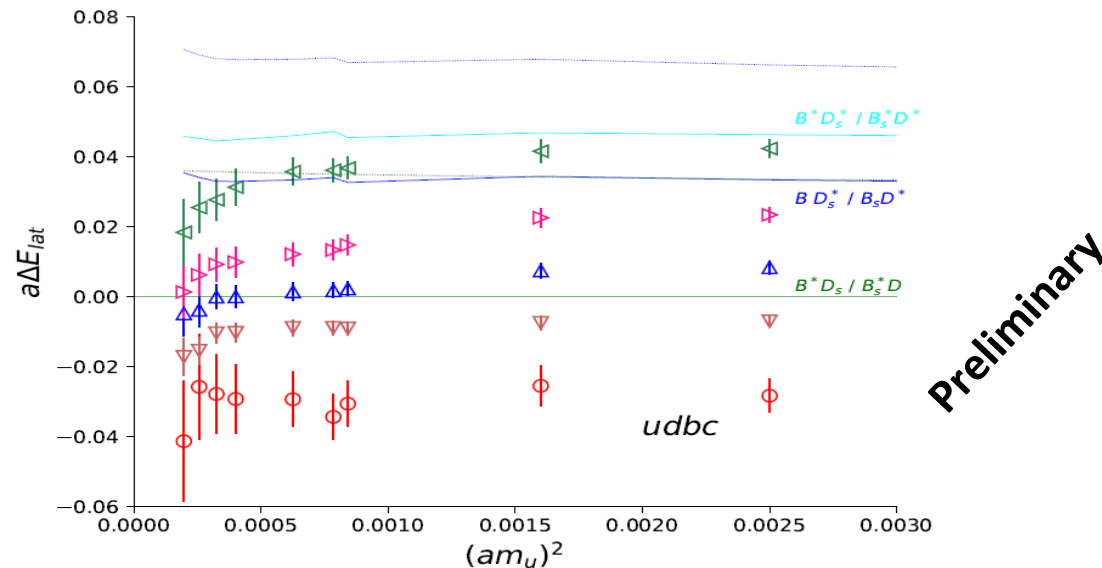
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□ Preliminary Findings:

- Multiple energy levels around elastic threshold.
- **At least one energy level below two-meson thresholds.**



More work is in progress to cross-check this finding with more statistics and another volume

What's next?

- A level below threshold appears with more number of interpolating fields, particularly with the inclusion of **non-HQET types**. Perhaps need even more number of interpolating fields?
- GEVP issue? Checking that.
- Problem with wall source-point sink? Should not be an issue as signals are obtained more than 1.8 fm away
 - Will study this with a point source.
- Finite volume effect?
 - Will study this with another volume

Conclusions and Outlooks

- ✚ **There is a resurgence of interest in the study of bound states with heavy quark(s).**
- ✚ **Exotic states with heavy quarks have been observed having four and five quark configurations.**
- ✚ **Lattice QCD calculations are playing a crucial role in studying these states.**
- ✚ **Multiple lattice QCD studies suggest the existence of a deeply bound doubly-bottom four-quark state. More works with multiple volumes are needed to firmly confirm this.**
- ✚ **Non-HQET interpolating operators are important for diquarks involving charm quarks.**
- ✚ **Preliminary findings indicate the existence of an energy level below the respective lowest threshold for the bottom-charm fourquarks. This needs to be further investigated with multiple sources and multiple volumes. Stay tuned!**