Existence and Non-Existence of Doubly Heavy Tetraquark Bound States

Martin Pflaumer
pflaumer@itp.uni-frankfurt.de

Goethe-Universität Frankfurt am Main
in collaboration with Luka Leskovec, Stefan Meinel, Marc Wagner

The 38th International Symposium on Lattice Field Theory
July 28, 2021

Motivation (1)

Experimental background

- Some experimentally observed states cannot be explained by ordinary quark-antiquark pair
  (e.g. charged Z_b states $Z_b(10610)^+$ and $Z_b(10650)^+$)
- Four-quark structure can describe quantum numbers correctly

Motivation (1)

Experimental background

- Some experimentally observed states cannot be explained by ordinary quark-antiquark pair
  (e.g. charged Z_b states $Z_b(10610)^+$ and $Z_b(10650)^+$)
- Four-quark structure can describe quantum numbers correctly

Theoretical study

- We study theoretically feasible tetraquark system
- Quark content: $\bar{Q}\bar{Q}'qq'$, here: $\bar{b}\bar{b}ud$, $\bar{b}\bar{b}us$, $\bar{b}\bar{c}ud$
- In the limit $m_Q \to \infty$ stable tetraquark was shown

[J. Carlson, L. Heller and J. A. Tjon, Phys. Rev. D **37**, 744 (1988)]
[A. V. Manohar and M. B. Wise, Nucl. Phys. B **399**, 17 (1993)]
[E. J. Eichten and C. Quigg, Phys. Rev. Lett. **119**, no. 20, 202002 (2017)]
[M. Karliner and J. L. Rosner, Phys. Rev. Lett. **119**, no.20, 202001 (2017)]

Motivation (2)

Born-Oppenheimer study of $\bar{b}\bar{b}ud$ tetraquark (with static $\bar{b}$-quarks):

- bound tetraquark with $I(J^P) = 0(1^+)$ and $E_{\text{bind}} \approx -90\,\text{MeV}$
- resonance with $I(J^P) = 0(1^-)$ and $E_{\text{res}} \approx +20\,\text{MeV}$, $\Gamma \approx 100\,\text{MeV}$

[Z. S. Brown and K. Orginos, Phys. Rev. D **86**, 114506 (2012)]
[P. Bicudo *et al.* [ETMC], Phys. Rev. D **87**, no. 11, 114511 (2013)]
[P. Bicudo, K. Cichy, A. Peters, B. Wagenbach, M. Wagner, Phys. Rev. D **92**, no. 1, 014507 (2015)]
[P. Bicudo, J. Scheunert and M. Wagner, Phys. Rev. D **95**, no. 3, 034502 (2017)]
[P. Bicudo, M. Cardoso, A. Peters, M.P. and M. Wagner, Phys. Rev. D **96**, no. 5, 054510 (2017)]

Motivation (2)

Born-Oppenheimer study of $\bar{b}\bar{b}ud$ tetraquark (with static $\bar{b}$-quarks):

- bound tetraquark with $I(J^P) = 0(1^+)$ and $E_{\text{bind}} \approx -90\,\text{MeV}$
- resonance with $I(J^P) = 0(1^-)$ and $E_{\text{res}} \approx +20\,\text{MeV}$, $\Gamma \approx 100\,\text{MeV}$

[Z. S. Brown and K. Orginos, Phys. Rev. D **86**, 114506 (2012)]
[P. Bicudo *et al.* [ETMC], Phys. Rev. D **87**, no. 11, 114511 (2013)]
[P. Bicudo, K. Cichy, A. Peters, B. Wagenbach, M. Wagner, Phys. Rev. D **92**, no. 1, 014507 (2015)]
[P. Bicudo, J. Scheunert and M. Wagner, Phys. Rev. D **95**, no. 3, 034502 (2017)]
[P. Bicudo, M. Cardoso, A. Peters, M.P. and M. Wagner, Phys. Rev. D **96**, no. 5, 054510 (2017)]

This work: Search for doubly-heavy tetraquark bound states in $\bar{b}\bar{b}ud$, $\bar{b}\bar{b}us$, $\bar{b}\bar{c}ud$ with full lattice QCD using **Non-Relativistic QCD**

[A. Francis, R. J. Hudspith, R. Lewis and K. Maltman, Phys. Rev. Lett. **118**, no. 14, 142001 (2017)]
[P. Junnarkar, N. Mathur and M. Padmanath, Phys. Rev. D **99**, no. 3, 034507 (2019)]
[A. Francis, R. J. Hudspith, R. Lewis and K. Maltman, Phys. Rev. D **99**, no. 5, 054505 (2019)]
[L. Leskovec, S. Meinel, M.P. and M. Wagner, Phys. Rev. D **100**, no.1, 014503 (2019)]
[R. J. Hudspith, B. Colquhoun, A. Francis, R. Lewis and K. Maltman, Phys. Rev. D **102**, 114506 (2020)]
[M.P., L. Leskovec, S. Meinel and M. Wagner, arXiv:2009.10538 [hep-lat]].

Interpolating Operators

- **Local operators**:
  - Four quarks at the same space-time position
  - Jointly projected to zero momentum
  - Several local mesonic and diquark-antidiquark operators

Interpolating Operators

- **Local operators**:
  - Four quarks at the same space-time position
  - Jointly projected to zero momentum
  - Several local mesonic and diquark-antidiquark operators
- **Nonlocal operators**:
  - Two mesons separated in space-time position
  - Separately projected to zero momentum
  - Several mesonic scattering operators

Interpolating Operators

- **Local operators**:
  - Four quarks at the same space-time position
  - Jointly projected to zero momentum
  - Several local mesonic and diquark-antidiquark operators
- **Nonlocal operators**:
  - Two mesons separated in space-time position
  - Separately projected to zero momentum
  - Several mesonic scattering operators
- Expectation:
  - Local operators: good overlap to ground state (stable four-quark)
  - Nonlocal operators: sizable overlap to first excited state (2 meson state)

$\Rightarrow$ Isolate ground state from higher excitations, especially first excited state

Lattice Setup

- Use gauge link configuration generated by RBC and UKQCD collaboration
  [Y. Aoki *et al.* [RBC and UKQCD Collaborations], Phys. Rev. D **83**, 074508 (2011)]
  [T. Blum *et al.* [RBC and UKQCD Collaborations],Phys. Rev. D **93**, no. 7, 074505 (2016)]

- $2+1$ flavours domain-wall fermions and Iwasaki gauge action
- Five different ensembles which differ in

lattice spacing	a	$\approx 0.083\,\text{fm} \ldots 0.114\,\text{fm}$,
lattice size	L	$\approx 2.65\,\text{fm} \ldots 5.48\,\text{fm}$,
pion mass	m_π	$\approx 139\,\text{MeV} \ldots 431\,\text{MeV}$

- Here: only results for one ensemble presented
- explored also dependence on L, m_π via scattering analysis and chiral extrapolation
- Smeared point-to-all propagators for the up and down quarks

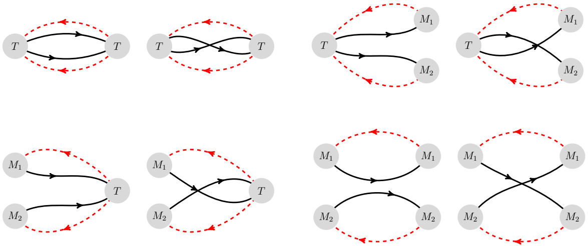
Energy Spectrum for the $\bar{Q}\bar{Q}'qq'$ system

- Due to point-to-all propagators, only non-symmetric correlation matrix available (no scattering operator at source)
- Apply multi-exponential matrix fitting: employable also for non-symmetric matrices

$$C_{jk}(t) \approx \sum_{n=0}^{N-1} Z_j^n Z_k^n \mathrm{e}^{-E_n t},$$

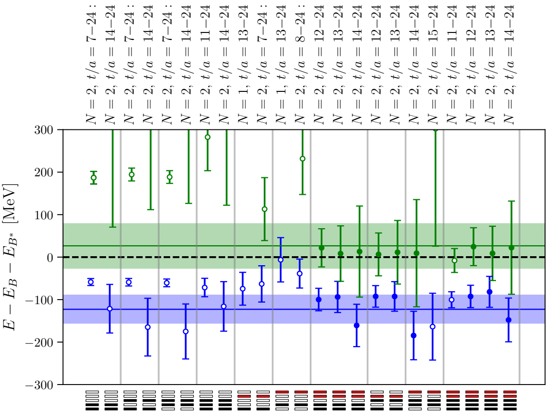
E_n : n-th energy eigenvalue

$Z_j^n = \langle \Omega | \mathcal{O}_j | n \rangle$: overlap factor



Schematic representation of Wick contractions for different correlation matrix elements

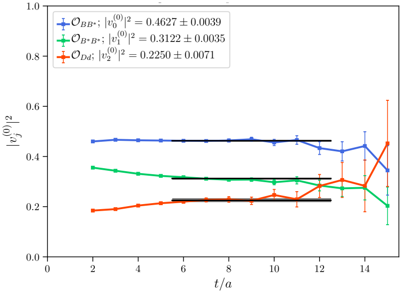
Fit Results for $\bar{b}\bar{b}ud$

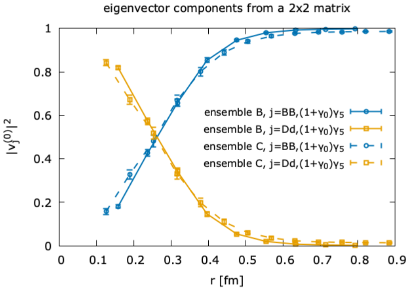


Results for the lowest two $\bar{b}\bar{b}ud$ energy levels relative to the BB^* threshold. Black box: local operator included. Red box: scattering operator included.

- Found evidence for bound state with $E_{\text{binding}} = -128$ MeV
- First excited state corresponds to threshold

Eigenvector Components for the Ground State for $\bar{b}\bar{b}ud$





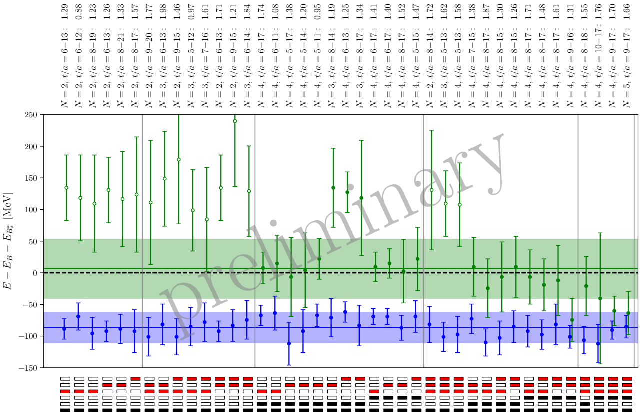
left: Normalized eigenvector components for the 3×3 matrix using only local operators.
right: Normalized eigenvector components for a 2×2 matrix in the Born-Oppenheimer approximation in dependence of the spacial separation r (plot from 2101.00723 $\rightarrow$ talk by M.Wagner).

Percentages of the ground state

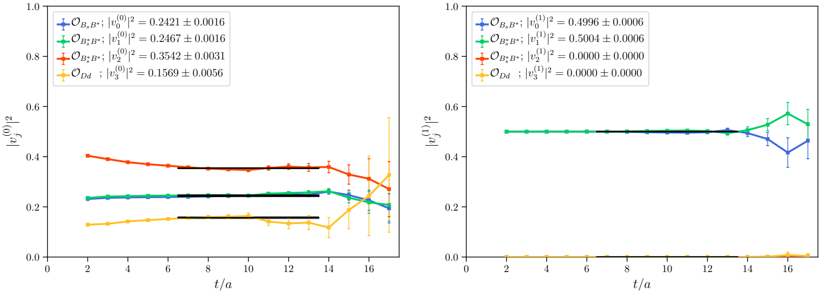
- this study: $\sim 77\%$ BB and $\sim 23\%$ Dd
- 2101.00723: $\sim 60\%$ BB and $\sim 40\%$ Dd

  $\Rightarrow$ Tendency to mesonic dominated state.

Preliminary Results for $\bar{b}\bar{b}us$



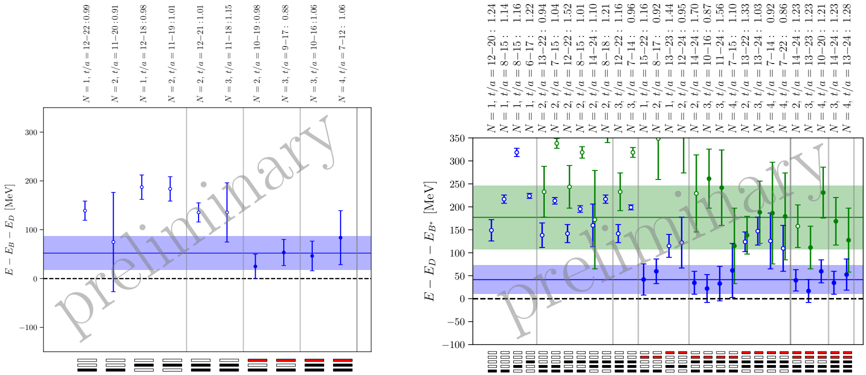
- Found evidence for bound state with $E_{\text{binding}} \approx -80$ MeV
- First excited state corresponds to threshold



Normalized eigenvector components for the 4×4 matrix using only local operators. **left:** Ground state. **right:** First excited state.

- Ground state is again dominated by mesonic structures
  ($\sim 85\%$ BB and $\sim 15\%$ Dd)
- First excited state is linear combination of B_sB^* and B_s^*B
  $\Rightarrow$ "Isospin 1" state decouples

Preliminary Results for $\bar{b}\bar{c}ud$



left: $\bar{b}\bar{c}ud$, $J = 0$. **right:** $\bar{b}\bar{c}ud$, $J = 1$.

- No evidence for bound states in $\bar{b}\bar{c}ud$ systems
- Lowest energy level corresponds to threshold

Summary

- Study bound states in doubly heavy tetraquarks
- Consider *local* and *nonlocal* interpolating operators

Summary

- Study bound states in doubly heavy tetraquarks
- Consider *local* and *nonlocal* interpolating operators
- Predict a bound state in the $\bar{b}\bar{b}ud$ channel with in $I(J^P) = 0(1^+)$ with $E_{\text{binding}} = (-128 \pm 24 \pm 10)\,\text{MeV}$
- Evidence for bound state in $\bar{b}\bar{b}us$, $I(J^P) = \frac{1}{2}(1^+)$ sector with $E_{\text{binding}} \approx -80\,\text{MeV}$
- No evidence for bound tetraquark in $\bar{b}\bar{c}ud$, both $0(1^+)$ and $0(0^+)$

Summary

- Study bound states in doubly heavy tetraquarks
- Consider *local* and *nonlocal* interpolating operators
- Predict a bound state in the $\bar{b}\bar{b}ud$ channel with in $I(J^P) = 0(1^+)$ with $E_{\text{binding}} = (-128 \pm 24 \pm 10)\,\text{MeV}$
- Evidence for bound state in $\bar{b}\bar{b}us$, $I(J^P) = \frac{1}{2}(1^+)$ sector with $E_{\text{binding}} \approx -80\,\text{MeV}$
- No evidence for bound tetraquark in $\bar{b}\bar{c}ud$, both $0(1^+)$ and $0(0^+)$

Outlook

- More detailed analysis of threshold states in $\bar{b}\bar{c}ud$ $\rightarrow$ candidates for resonances?

Summary

- Study bound states in doubly heavy tetraquarks
- Consider *local* and *nonlocal* interpolating operators
- Predict a bound state in the $\bar{b}\bar{b}ud$ channel with in $I(J^P) = 0(1^+)$ with $E_{\text{binding}} = (-128 \pm 24 \pm 10)\,\text{MeV}$
- Evidence for bound state in $\bar{b}\bar{b}us$, $I(J^P) = \frac{1}{2}(1^+)$ sector with $E_{\text{binding}} \approx -80\,\text{MeV}$
- No evidence for bound tetraquark in $\bar{b}\bar{c}ud$, both $0(1^+)$ and $0(0^+)$

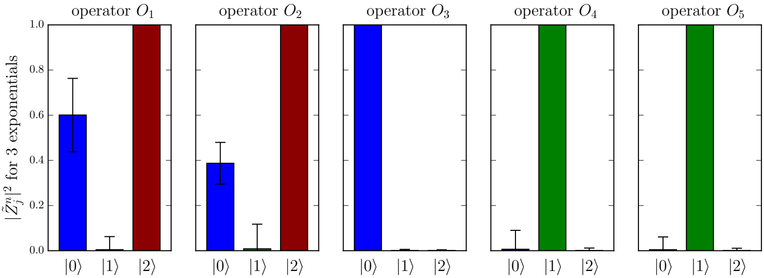
Outlook

- More detailed analysis of threshold states in $\bar{b}\bar{c}ud$ $\rightarrow$ candidates for resonances?

Thank You for Your Attention!

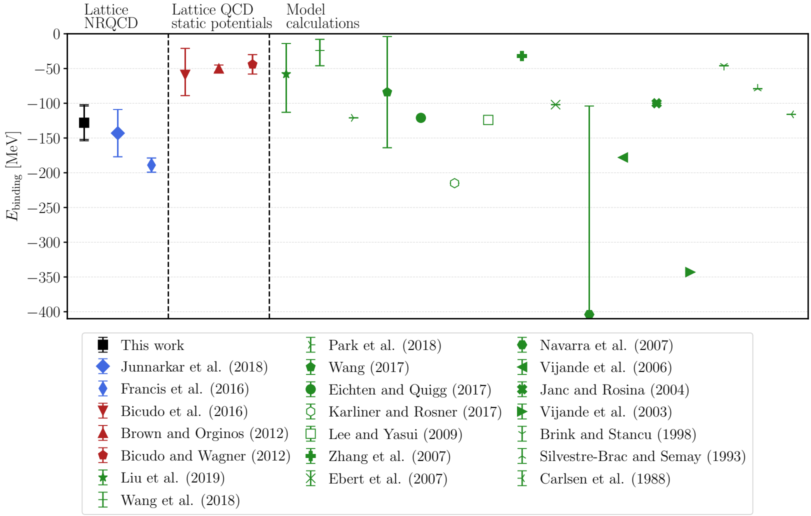
For fixed j: Z_j^n indicates relative importance of energy eigenstates $|n\rangle$

$$\mathcal{O}_j^\dagger|\Omega\rangle = \sum_{n=0}^{\infty} |n\rangle\langle n|\mathcal{O}_j^\dagger|\Omega\rangle = \sum_{n=0}^{\infty} Z_j^n|n\rangle.$$



The normalized overlap factors $|\tilde{Z}_j^n|^2 = \frac{|Z_j^n|^2}{\max_m(|Z_j^m|^2)}$ as determined on ensemble C005.

Comparison of Different Results for $\bar{b}\bar{b}ud$



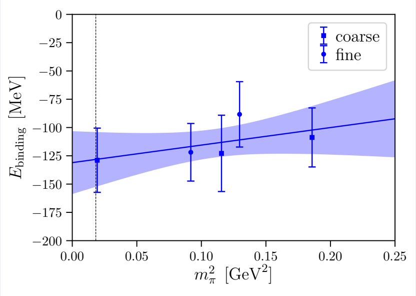
Comparison of $\bar{b}\bar{b}ud$ tetraquark binding energies with $I(J^P) = 0(1^+)$ (black: this work; blue: lattice NRQCD; red: lattice QCD computations of static $\bar{b}\bar{b}$ potentials and solving the Schrödinger equation; green: effective field theories and potential models).

Scattering Analysis and Chiral Extrapolation for $\bar{b}\bar{b}ud$

Scattering Analysis

- Relate *finite volume* energy spectrum E_n to *infinite volume* scattering amplitude
- Use Lüscher's formula to determine phase shift and *infinite volume* binding energy
- Confirmation that ground state is stable tetraquark.

Chiral Extrapolation



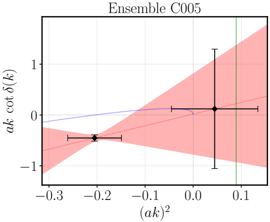
Fit of the pion-mass dependence of E_{binding}. The vertical dashed line indicates the physical pion mass.

$$E_{\text{binding}}(m_{\pi,\text{phys}}) = (-128 \pm 24 \pm 10)\,\text{MeV}$$
$$m_{\text{tetraquark}}(m_{\pi,\text{phys}}) = (10476 \pm 24 \pm 10)\,\text{MeV}$$

Scattering Analysis

- Relate *finite volume* energy spectrum E_n to *infinite volume* scattering amplitude for 2 energy levels in T_1^+ irrep
- Use Lüscher's formula and scattering momenta k_n^2 to determine phase shift
- Apply effective-range-expansion (ERE)

$$k \cot \delta_0(k) = \frac{1}{a_0} + \frac{1}{2} r_0 k^2 + \mathcal{O}(k^4).$$



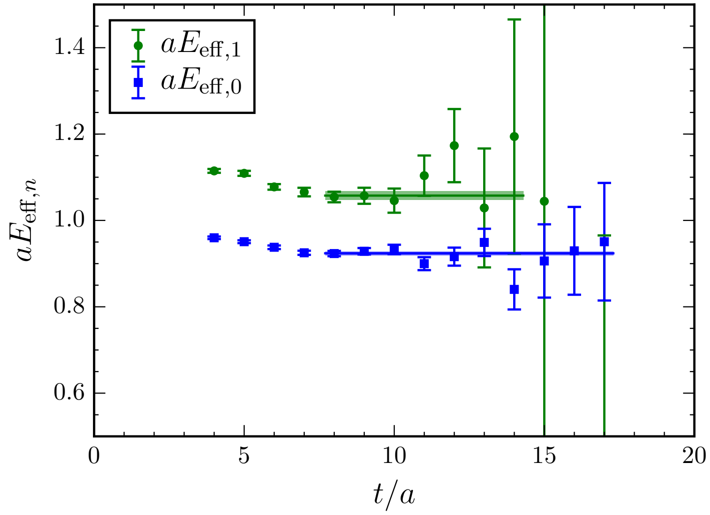
Plot of the effective-range-expansion for C005. Blue curve: $ak \cot(\delta(k)) + |ak|$. Vertical green line: Inelastic $B^* B^*$ threshold

- Search bound state pole of scattering amplitude below threshold at

$$\cot \delta_0(k_{\mathrm{BS}}) = i, \quad \text{so:} \quad -|k_{\mathrm{BS}}| = \frac{1}{a_0} - \frac{1}{2} r_0 |k_{\mathrm{BS}}|^2$$

- Results essentially identical to the finite-volume energy levels
- Confirmation that ground state is stable tetraquark.

GEP-results



Effective masses $aE_{\text{eff},n}$ for $n = 0, 1$ as a function of t/a for a 3×3 correlation matrix.