

# Continuum limit of baryon-baryon scattering with $SU(3)$ flavor symmetry

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Zoom/Gather@MIT

# H dibaryon

Conjectured  $0^+$ ,  $uuddss$  SU(3) singlet:  $\Lambda\Lambda$  bound state. R. L. Jaffe, 1977

Experiments put upper limit  $B_H < 7$  MeV

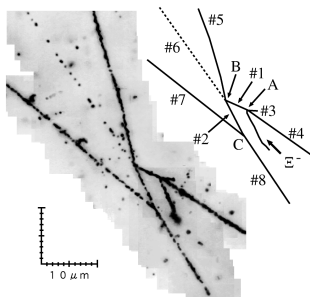
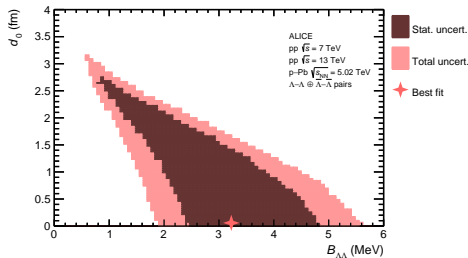


FIG. 2. Photograph and schematic drawing of NAGARA event. See text for detailed explanation.



ALICE, PLB 797, 134822 (2019)

KEK E373, PRL 87, 212502 (2001)

Hyperon-hyperon interactions difficult to study in experiment.  
Important for understanding strangeness in neutron stars.

SU(3) singlet not accessible from strangeness 0 or  $-1$ .

## Previous study

$N_f = 2$  CLS ensembles with quenched strange quark.

*Lattice QCD study of the H dibaryon using hexaquark and two-baryon interpolators*

A. Francis, JRG, P. M. Junnarkar, Ch. Miao, T. D. Rae, H. Wittig, Phys. Rev. D **99**, 074505 (2019)

- ▶ Comparison of interpolating operators

$$\text{Hexaquark: } O_H \sim \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} qqqqqq(x)$$

$$\text{Baryon-baryon: } O_{BB} \sim \sum_{\vec{x}_1, \vec{x}_2} e^{-i\vec{p}_1\cdot\vec{x}_1} e^{-i\vec{p}_2\cdot\vec{x}_2} qqq(x_1) qqq(x_2)$$

→ hexaquark less useful for isolating ground state

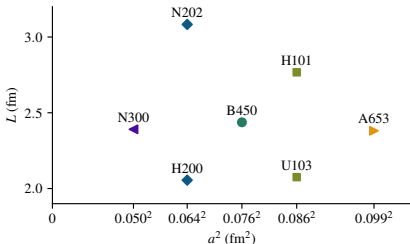
- ▶ Pioneering use of distillation for baryon-baryon operators.

# Calculations with $N_f = 3$

Weakly bound  $H$  dibaryon from  $SU(3)$ -flavor-symmetric QCD

JRG, A. D. Hanlon, P. M. Junnarkar, H. Wittig, arXiv:2103.01054

Ensembles with  $O(a)$  improved Wilson-clover fermions from CLS.



$SU(3)$ -symmetric point with physical  $m_u + m_d + m_s$ .

$m_\pi = m_K = m_\eta \approx 420$  MeV.

Wide range of  $L$  and  $a$ .

Distillation and baryon-baryon interpolators used throughout.

Two octet baryons:  $\mathbf{8} \otimes \mathbf{8} = (\mathbf{1} \oplus \mathbf{8} \oplus \mathbf{27})_S \oplus (\mathbf{8} \oplus \mathbf{10} \oplus \overline{\mathbf{10}})_A$ .

Initially focus on  $\mathbf{1}$ ,  $^1S_0$  for  $H$  dibaryon  $\rightarrow A_1$  or  $A_1^+$  little group irreps.

## Fitting spectrum

In most frames, use  $3 \times 3$  correlator matrix in GEVP.

Perform fits to ratio of diagonalized correlator,

$$R_n(t) \equiv \frac{\bar{C}_{nn}(t)}{C_{\Lambda}^{\vec{p}_1}(t) C_{\Lambda}^{\vec{p}_2}(t)},$$

yields energy difference from noninteracting level.

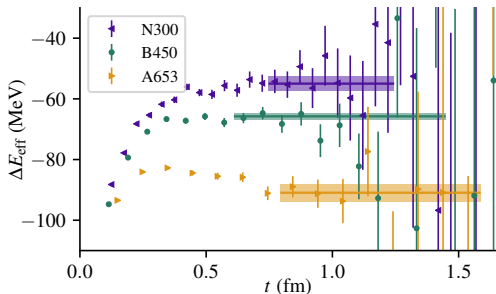
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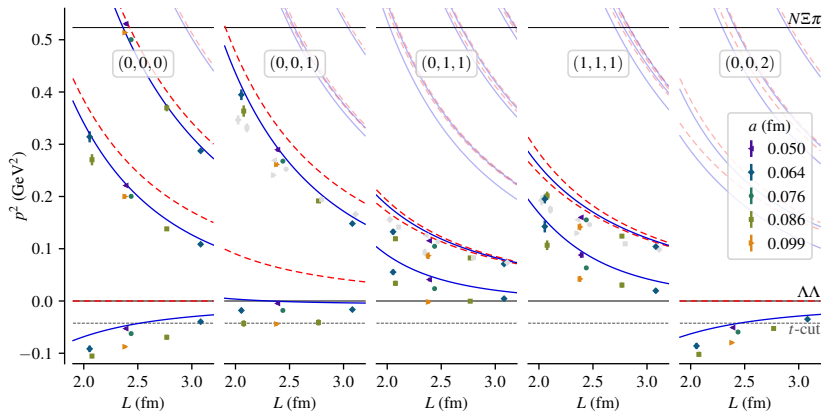
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Effective energy difference:  
ground state in  $(0, 0, 1)$   
(closest to pole location).

Three ensembles with  
similar  $L$ , different  $a$ .

# Spectrum summary

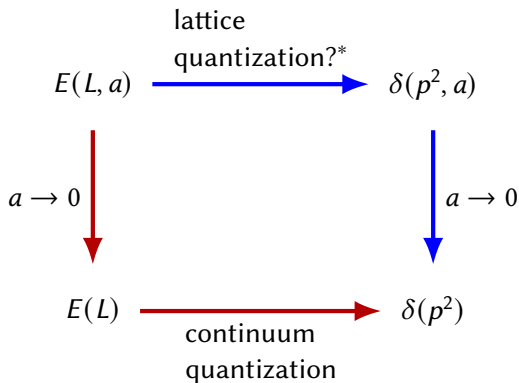


Points: lattice energy levels.

Red dashed curves: noninteracting levels.

Blue curves: interacting levels in continuum.

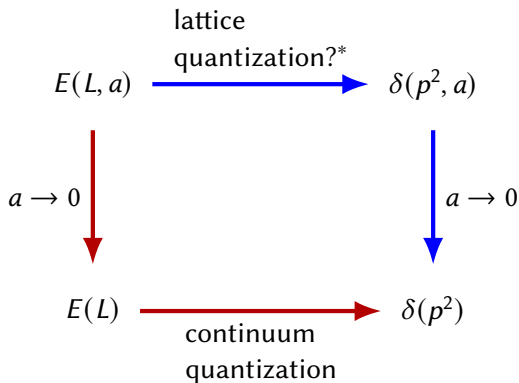
# Two paths to continuum phase shift



\*finite-volume quantization at nonzero lattice spacing was studied for a contact interaction in [C. Körber, E. Berkowitz, T. Luu, 1912.04425](#)



## Two paths to continuum phase shift



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Our strategy: apply continuum quantization conditions to  $E(L, a)$ , i.e. follow **blue path** neglecting artifacts in quantization condition.

# Combined phase shift fits

S-wave quantization condition:

$$p \cot \delta(p) = \frac{2}{\sqrt{\pi L y}} Z_{00}^{\vec{p}L/(2\pi)} \left( 1, \left( \frac{pL}{2\pi} \right)^2 \right)$$

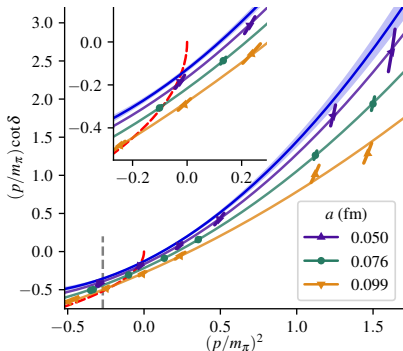
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Fit ansatz:

$$p \cot \delta(p) = \sum_{i=0}^{N-1} c_i p^{2i}, \quad c_i = c_{i0} + c_{i1} a^2.$$



# Combined phase shift fits

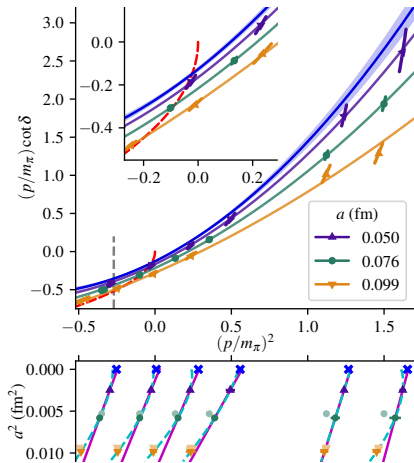
S-wave quantization condition:

$$p \cot \delta(p) = \frac{2}{\sqrt{\pi L \gamma}} Z_{00}^{\vec{p}L/(2\pi)} \left( 1, \left( \frac{pL}{2\pi} \right)^2 \right)$$

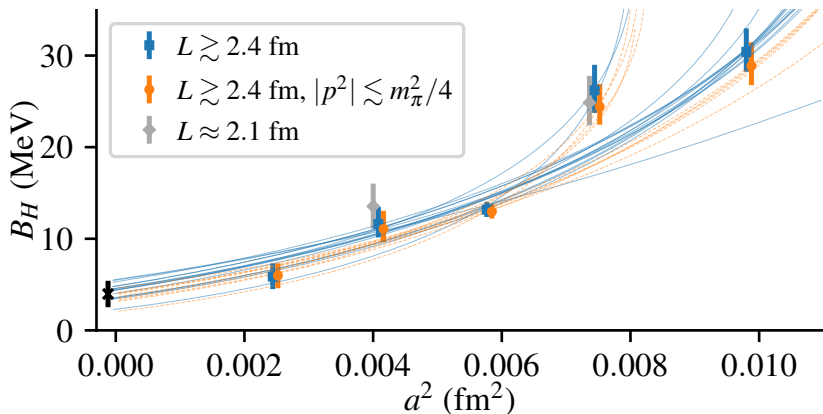
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*Cross check:* extrapolate energies at fixed volume.

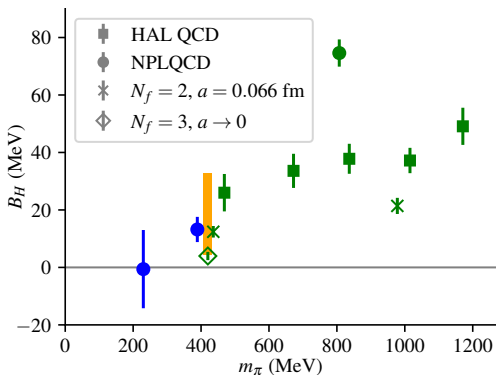


## Binding energy versus $a$



Fits to spectrum of different subsets of ensembles.  
Strong dependence on lattice spacing.

# Binding energy: comparison with literature

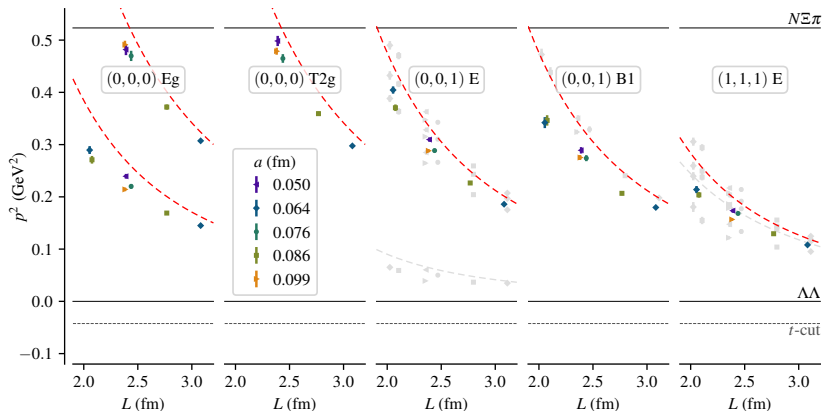


green:  $m_s = m_{ud}$   
blue:  $m_s = m_s^{\text{phys}} > m_{ud}$

## $H$ dibaryon: current status

- ▶ Analysis being revised in response to referee comments.  
→ more conservative errors on spectrum.
- ▶ Additional ensemble providing sixth lattice spacing  
 $a = 0.039$  fm.
- ▶ Main conclusions unchanged:
  - ▶ large discretization effects
  - ▶ small continuum binding energy

# PRELIMINARY: singlet spectrum in nontrivial irreps

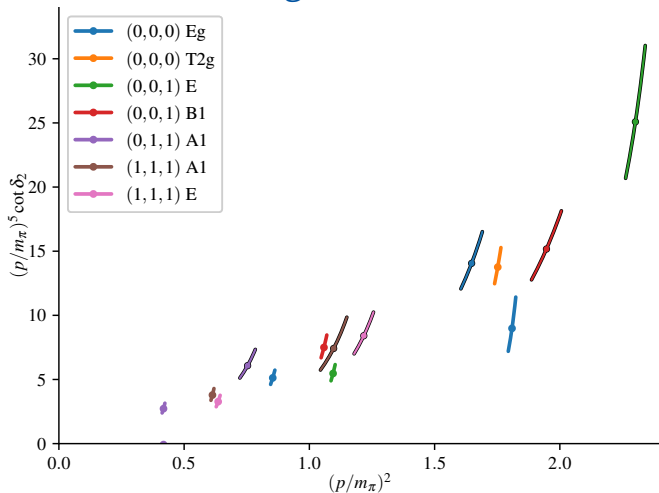


Focus on spin zero for  $^1D_2$ .

Many spin-one levels (gray) discarded: relevant for  $^3P_1$ ,  $^3P_2$ – $^3F_2$ , etc.  
Omitted irreps have only spin-one levels.



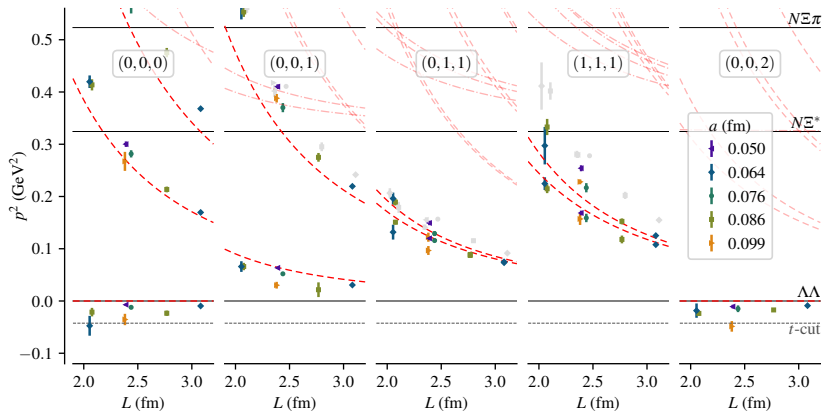
# PRELIMINARY: singlet $^1D_2$



Ensembles H200 and N202:  $a = 0.064$  fm,  $L = 2.1$  and  $3.1$  fm.

Finite-volume matrix in quantization condition computed using TwoHADRONsINBox,  
C. Morningstar *et al.*, Nucl. Phys. B **964**, 477–507 (2017)

# PRELIMINARY: 27-plet spectrum in trivial irreps



27 includes  $NN I = 1$ .

$\delta_0$  passes through zero. Fit rational functions to  $p \cot \delta_0$ ?

Dineutron is probably a virtual bound state.

## Conclusions / outlook

- ▶ Distillation works well for the baryon-baryon spectrum.
- ▶  $H$  dibaryon is weakly bound at  $SU(3)$  point.
- ▶ Discretization effects can be very large!

Looking forward:

- ▶ Much more physics to be extracted from symmetric-point data.
- ▶ Next talk goes away from  $SU(3)$  point.