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$K\pi$ scattering at physical pion mass using distillation

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THE UNIVERSITY of EDINBURGH

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Outline

Background

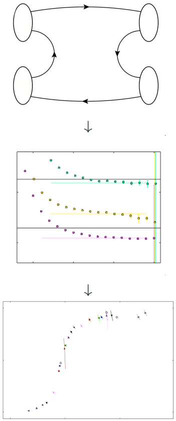
Background

Scattering on the lattice: $K\pi$

- Phenomenological motivations
    - rare decays, e.g. $B \to K^* l^+ l^- (\to K\pi l^+ l^-)$
    - multibody decays, e.g. $B \to K\pi\pi$
- Methodology
    - **correlator data**
    - energy spectrum
    - Lüscher analysis
- What can we get?
    - phase shifts
    - resonance parameters

$\to$ *this project: towards first physical pion mass $K\pi$ scattering*

Workflow illustration



[F. Erben 10.25358/openscience-3858]

Ensemble

- RBC-UKQCD $N_f = 2 + 1$ domain-wall fermion lattice [Blum et al. 10.1103/PhysRevD.93.074505]

volume	$48^3 \times 96$
L	≈ 5.5 fm
$m_\pi L$	≈ 3.8
m_π m_K	≈ 139 MeV ≈ 499 MeV

- Ensemble exploration
  - several datasets with measurements over 9 configurations
  - inversions on 12 time sources per configuration (every 8*th*)
  - low statistics: treat correlators obtained from different time sources as uncorrelated samples (but bin later)

* done using DiRAC Extreme Scaling HPC Service [`https://www.dirac.ac.uk`]

Distillation [M. Peardon et al. 10.1103/PhysRevD.80.054506]

- Projects quark fields onto low-lying covariant 3D-Laplacian subspace

$$\mathcal{S}_{\mathsf{xy}}(t) = \sum_{i=1}^{N_{\mathrm{vec}}} v_{\mathsf{x}}^{(i)}(t) v_{\mathsf{y}}^{(i)}(t)^{\dagger}, \quad \text{eigenvectors } v^{(i)} \text{ of } -\nabla^2 \tag{1}$$

  $\rightarrow$ number inversions $N_{\mathrm{inv}} \propto N_{\mathrm{vec}}$ (*exact distillation*)

- Further: *stochastic distillation* [C. Morningstar et al. 10.1103/PhysRevD.83.114505]
  $\rightarrow$ $N_{\mathrm{inv}} \propto N_{\mathrm{noise}}$: number of Lap-time-spin noises

- Variance reduction: dilution projectors [C. Morningstar et al. 10.1103/PhysRevD.83.114505]
  $\rightarrow$ interlaced dilution: inversions $N_{\mathrm{inv}} \propto N_{\mathrm{noise}} LI$
  * full time-spin dilution

Code

- `Grid`: data parallel C++ lattice library
- `Hadrons`: Grid-based workflow management system for lattice simulations
- Open-source and free software

github.com/paboyle/Grid

github.com/aportelli/Hadrons

Distillation within `Grid` and `Hadrons`

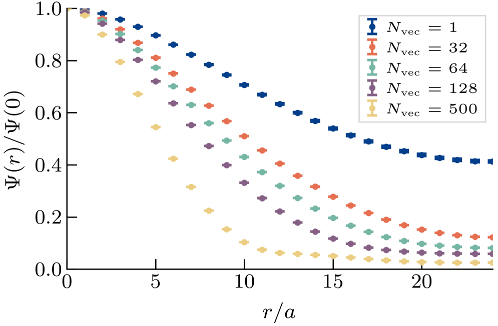
- Started in 2019 by Marshall M. and Erben F. [P. Boyle et al. arxiv:1912.07563]
- Refactorisation: optimised meson fields
- Proper documentation and file specification

Smearing Radius and N_{vec} Dependence

Smearing Radius and N_{vec} Dependence

Spatial distribution [M. Peardon et al. 10.1103/PhysRevD.80.054506]

$$\Psi(r) = \sum_{\hat{r}=1}^{3} \sum_{\mathbf{x},t} \sqrt{\mathrm{tr}\, \mathcal{S}_{\mathbf{x},\mathbf{x}+\mathbf{r}}(t) \mathcal{S}_{\mathbf{x}+\mathbf{r},\mathbf{x}}(t)} \quad (2)$$



- Smearing profile for several values of N_{vec}
- Larger N_{vec} approaches point source ✓

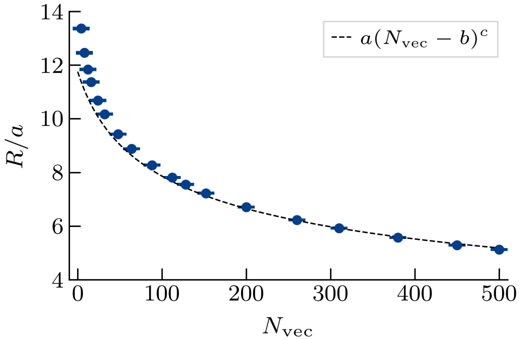
* stout-smearing parameters $\rho = 0.2, n = 3$

Smearing Radius

- Define R:

$$\frac{\int_0^R \Psi(r)dr}{2\int_0^{aL/2} \Psi(r)dr} = 0.341 \qquad (3)$$

Study dependence on $N_{\rm vec}$ ⟶



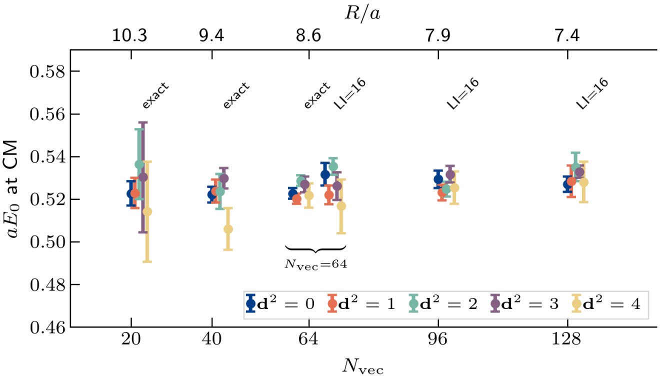
- Curve flattening: simple power-law fit with exponent $c \approx -0.3$
- Given such flattening and overall cost $\propto N_{\rm vec}$, reasonable to explore $N_{\rm vec} \sim 100$
- Schemes

	$LI = 4$	$LI = 8$	$LI = 16$	$LI = 32$	exact	exact	exact
$N_{\rm vec}$	64	64	64, 96, 128	64	20	40	64
$N_{\rm inv}$	384	768	1536	3072	960	1920	3072

* stochastic distillation with 2 noise vectors

Vector-to-vector correlators ($\bar{s}\gamma_i l$)

- Fit to $Z_0 \left(e^{-E_0 t} + e^{-(N_t - t)E_0}\right)$ for now
- Use exact distillation fit ranges as reference
- Boost E_0 from moving frames (A1 irrep) to center-of-momentum frame



$\rightarrow$ E_0 roughly consistent for $N_{\text{vec}} \gtrsim 60$

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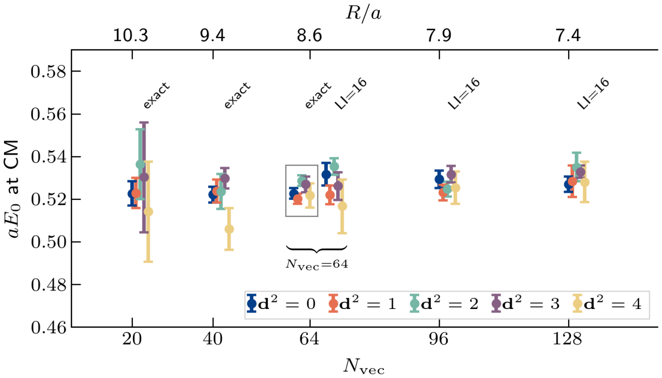
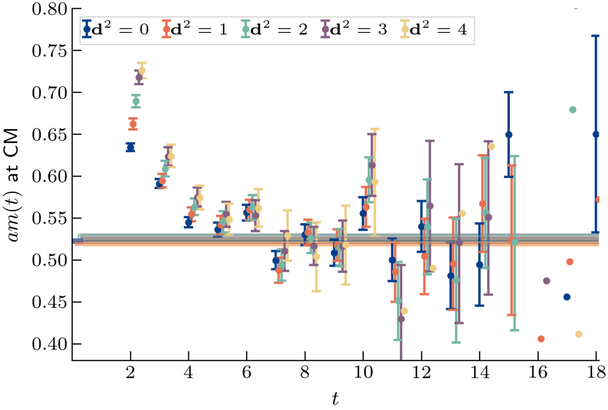


→ E_0 roughly consistent for $N_{\rm vec} \gtrsim 60$

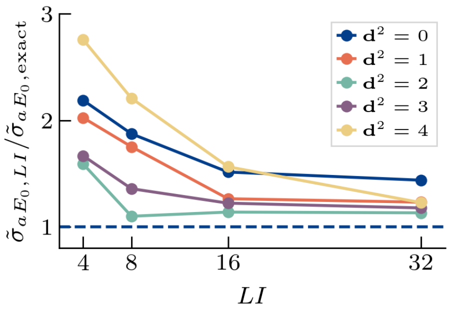
- **Effective mass** at $N_{\text{vec}} = 64$ (exact distillation)
  - Fit result: E_0 bands



- Caveats:
  - low-statistics and correlations
  - excited states (future work)

Exact and Stochastic Distillation

Exact and Stochastic Distillation at $N_{\rm vec} = 64$



Cost comparison: fit data

- Cost-normalised standard deviation $\tilde{\sigma} \equiv \sigma\sqrt{N_{\rm inv}}$
- Ratio to exact distillation (only MC noise at dashed line)

- $N_{\rm noise} = 2$ here ; would need at least 4 in the full analysis

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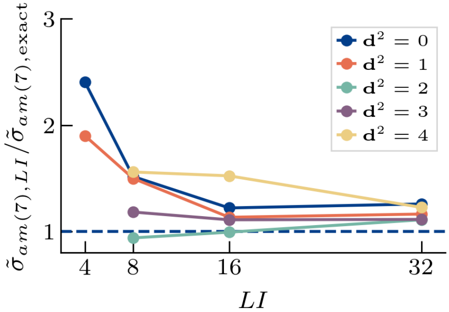
Exact and Stochastic Distillation at $N_{\text{vec}} = 64$



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Conclusions and Outlook

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- `Grid/Hadrons` distillation code for large simulations (open source)
- N_{vec} **dependence of smearing radius and single-particle correlators**
  - smearing radius flattening
  - $N_{\mathrm{vec}} \approx 60$ looks consistent to resolve momenta up to $\mathbf{d}^2 = 4$ at correlator level

    $\rightarrow$ *needs further study*
- **Comparison between several distillation schemes at** $N_{\mathrm{vec}} = 64$
  - cost comparison between exact and stochastic distillation ($N_{\mathrm{noise}} = 2$)
  - exact has better cost-benefit at correlator level, besides being simpler to handle
- **Next**
  - multi-hadron correlators (`Hadrons` contractor)
  - higher statistics, GEVP and Lüscher analysis

Thanks for the attention. Questions or comments?

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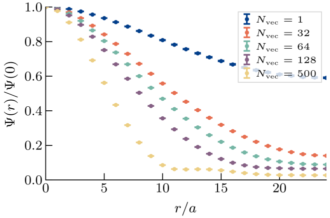
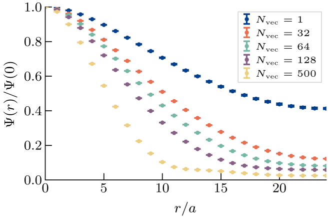
Backup

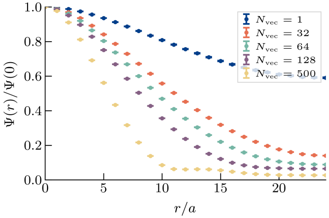
Smearing spatial distribution for $n_{\text{stout}} = 0, 3, 12$ ($\rho = 0.2$)



$n_{\text{stout}} = 0$



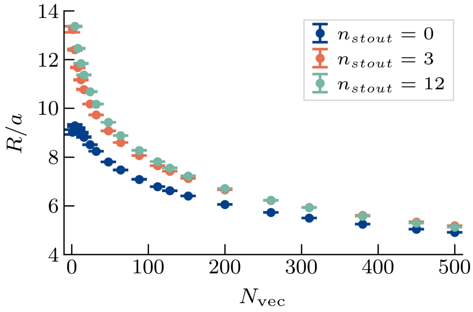
$n_{\text{stout}} = 3$



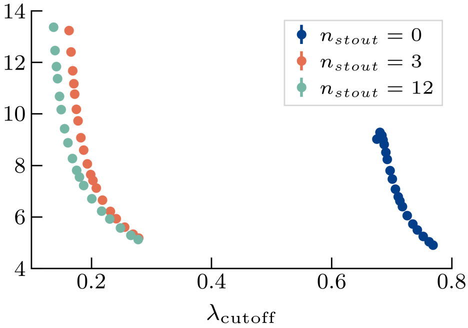
$n_{\text{stout}} = 12$

Backup

Smearing radius for $n_{\mathrm{stout}} = 0, 3, 12$ ($\rho = 0.2$)

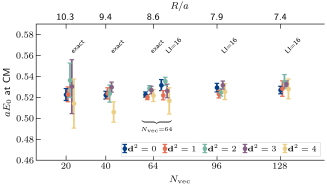


radius vs N_{vec}

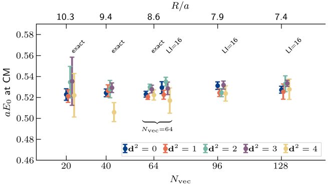


radius vs $\lambda_{\mathrm{cutoff}}$

Varying bin size (E_0 vs N_{vec})



bin size=1



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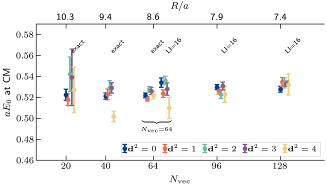
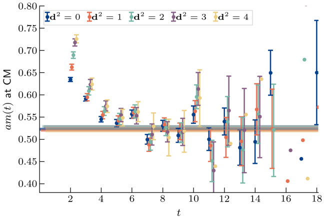


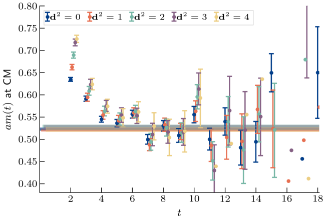
bin size=4

Backup

Varying bin size (m_{eff} vs t)



bin size=1



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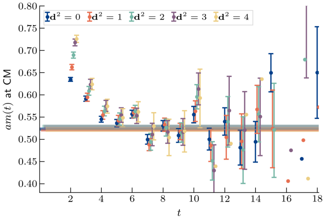
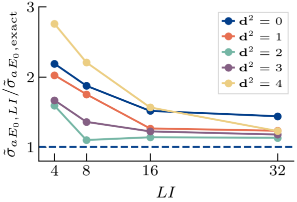


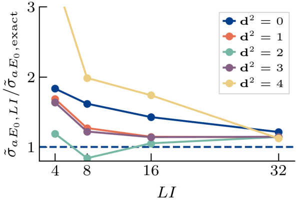
bin size=4

Backup

Varying bin size ($\frac{\sigma_{E_0,LI}\sqrt{N_{\mathbf{inv},LI}}}{\sigma_{E_0,exact}\sqrt{N_{\mathbf{inv},exact}}}$ vs LI)



bin size=1



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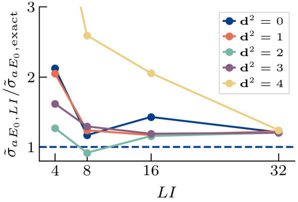
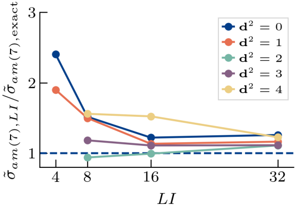


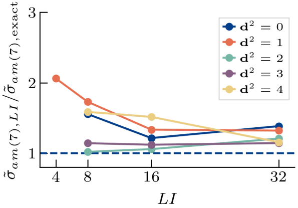
bin size=4

Fluctuates a bit but it is not changing conclusions

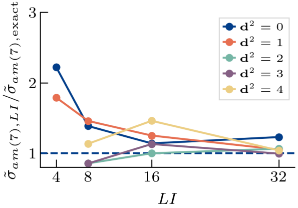
Varying bin size $\left(\frac{\sigma_{m_{\mathrm{eff}}(t=7),LI}\sqrt{N_{\mathrm{inv},LI}}}{\sigma_{m_{\mathrm{eff}}(t=7),exact}\sqrt{N_{\mathrm{inv},exact}}}\right)$ vs LI)



bin size=1



bin size=2



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