Optimizing Distillation for charmonium and glueballs

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The Distillation framework

The Basics:

Smear the quark field via projection $\psi \to VV^\dagger\psi$ onto the N_v eigenvectors with smallest eigenvalues of the 3D covariant Laplacian ∇^2.
[M. Peardon et al. Phys. Rev. D80 (2009) arXiv:0905.2160]

The building blocks are:

- Elementals: $\Phi[t] = V[t]^\dagger \Gamma V[t]$ (Γ from $\bar{q}\Gamma q$)
- Perambulators: $\tau[t_1, t_2] = V[t_1]^\dagger D^{-1} V[t_2]$

The limitations:

× N_v scales with volume for fixed smearing cut-off $\lambda \leq \lambda_{max}$.

× Several inversions ($4 \times N_v \times N_t$) per gauge configuration.

A first improvement:

Stochastic distillation: Alleviates the cost of inversions but sacrifices the exact calculation of $\tau[t_1, t_2]$.
[C. Morningstar et al. Phys. Rev. D83 (2011) arXiv:1104.3870]

Another alternative:

Exploit quark distillation profiles.

- Quark distillation profile: $J_{ab}[t] = \delta_{ab} f(\lambda_a[t])$
- Modified distilled quarks: $\psi \to VJV^\dagger\psi$
- Modified elementals: $\Phi[t] = J[t]^\dagger V[t]^\dagger \Gamma V[t] J[t]$

Find the best way to use the eigenvectors in the elementals.

The steps:

Build correlation matrix $C_{ab}(t) = \langle O_a(t)\bar{O}_b(0)\rangle$:

- Profile J_a in sink and J_b in source.
- $a, b = 1, \dots, N_B$

Prune $C(t)$ into $\bar{C}(t)$ via its N_s most significant singular vectors u_i at a time t_0:

- $\bar{C}_{ij}(t) = u_i^\dagger C(t) u_j$
- ✓ Improves stability and keeps useful operators.

[J. Balog et al. Phys. Rev. D60 (1999) arXiv:hep-lat/9903036]
[F. Niedermayer et al. Nucl. Phys. B597 (2001) arXiv:hep-lat/0007007]

Solve generalized eigenvalue problem:

- $\bar{C}(t) w_k(t, t_0) = \rho_k(t, t_0) \bar{C}(t_0) w_k(t, t_0)$

[C. Michael & I. Teasdale, Nucl. Phys. B215 (1983)]
[M. Lüscher & U. Wolff, Nucl. Phys. B339 (1990)]
[B. Blossier et al. JHEP 04 (2009) arXiv:0902.1265]

The outcome:

- ✓ From principal correlators $\rho_k(t, t_0)$ we get effective masses.
- ✓ From vectors $w_k(t, t_0)$ and u_i we get optimal meson profiles for energy states.

Pruned basis meson profiles:

- $f_i^{(p)}(\lambda_a, \lambda_b) = \sum_{k=1}^{N_B} u_i^{(k)} f_k(\lambda_a)^* f_k(\lambda_b)$
- $u_i^{(k)}$: k-th entry of i-th singular vector.

Optimal meson profiles:

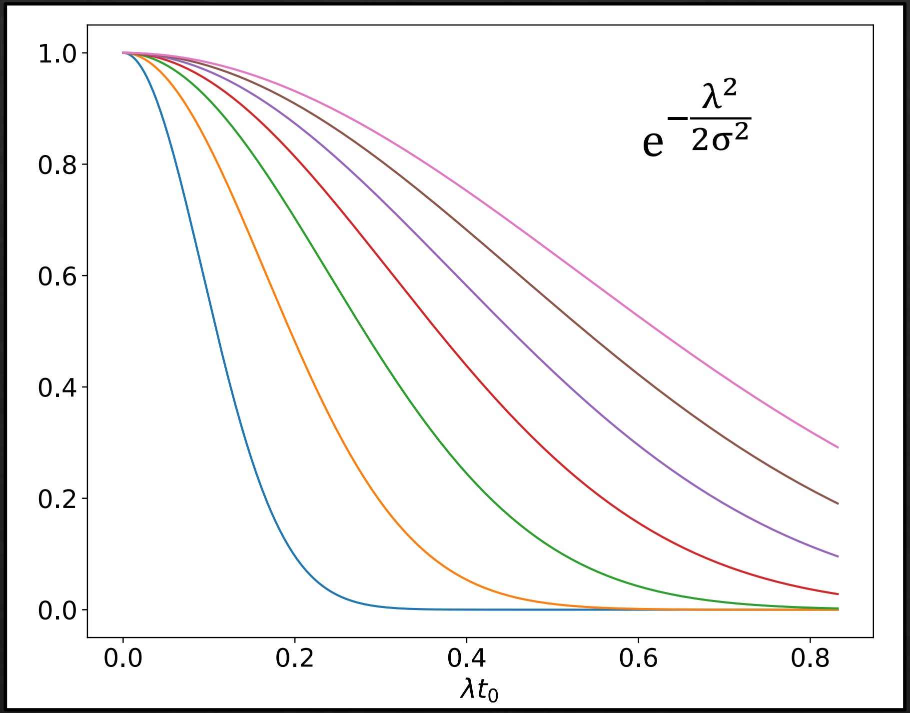
- $\tilde{f}_e(\lambda_a, \lambda_b) = \sum_{i=1}^{N_s} w_e^{(i)} f_i^{(p)}(\lambda_a, \lambda_b)$
- $w_e^{(i)}$: i-th entry of e-th eigenvector.

Weighted meson elemental:

- ✓ $\tilde{\Phi}_e[t]_{ij}^{\alpha\beta} = \tilde{f}_e(\lambda_i[t], \lambda_j[t]) v_i^\dagger[t] \Gamma[t]^{\alpha\beta} v_j[t]$

In this work

- 48×24^3 ensemble with 2 dynamical quarks.
- Masses at half the physical charm quark mass.
- Lattice spacing $a = 0.0658$ fm.
- $N_v = 200$ eigenvectors.



$\sigma t_0 = \{0.092, 0.165, 0.238, 0.311, 0.384, 0.457, 0.529\}$

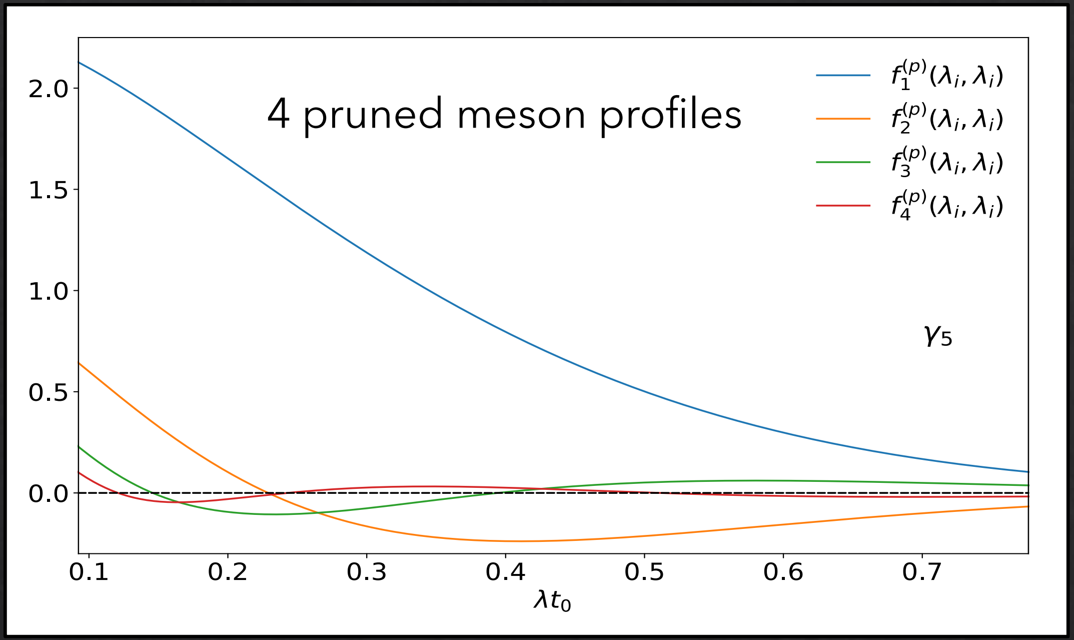
✓ Cover range of 200 eigenvalues.

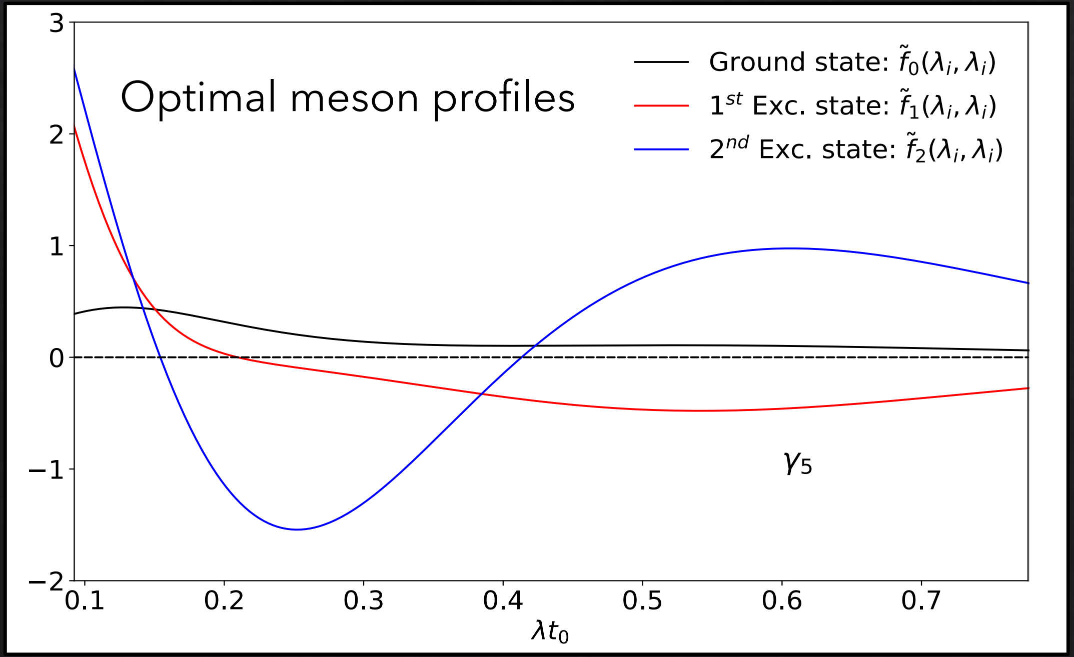
✓ Impose decay at large λ.

✓ Numerically stable compared to other basis functions such as λ^k.

Pruning and GEVP at $t_0 = 3$

γ_5 (A_1^{-+}) iso-vector meson distillation profiles ($\lambda_i = \lambda_j$)



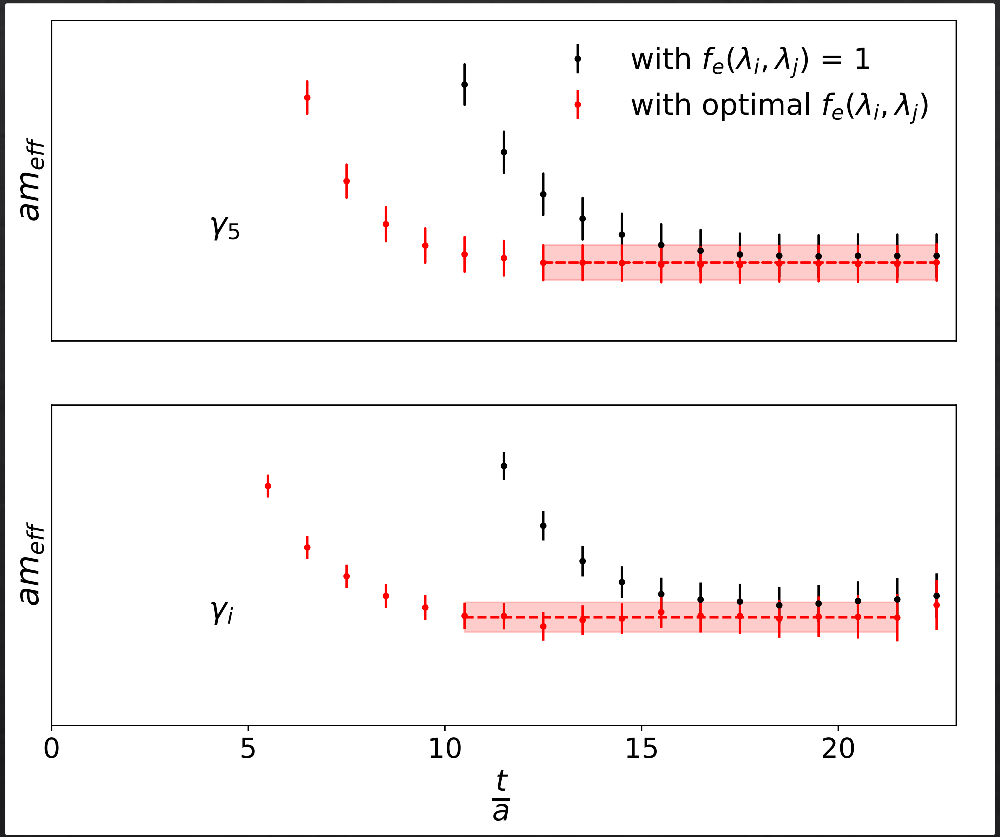


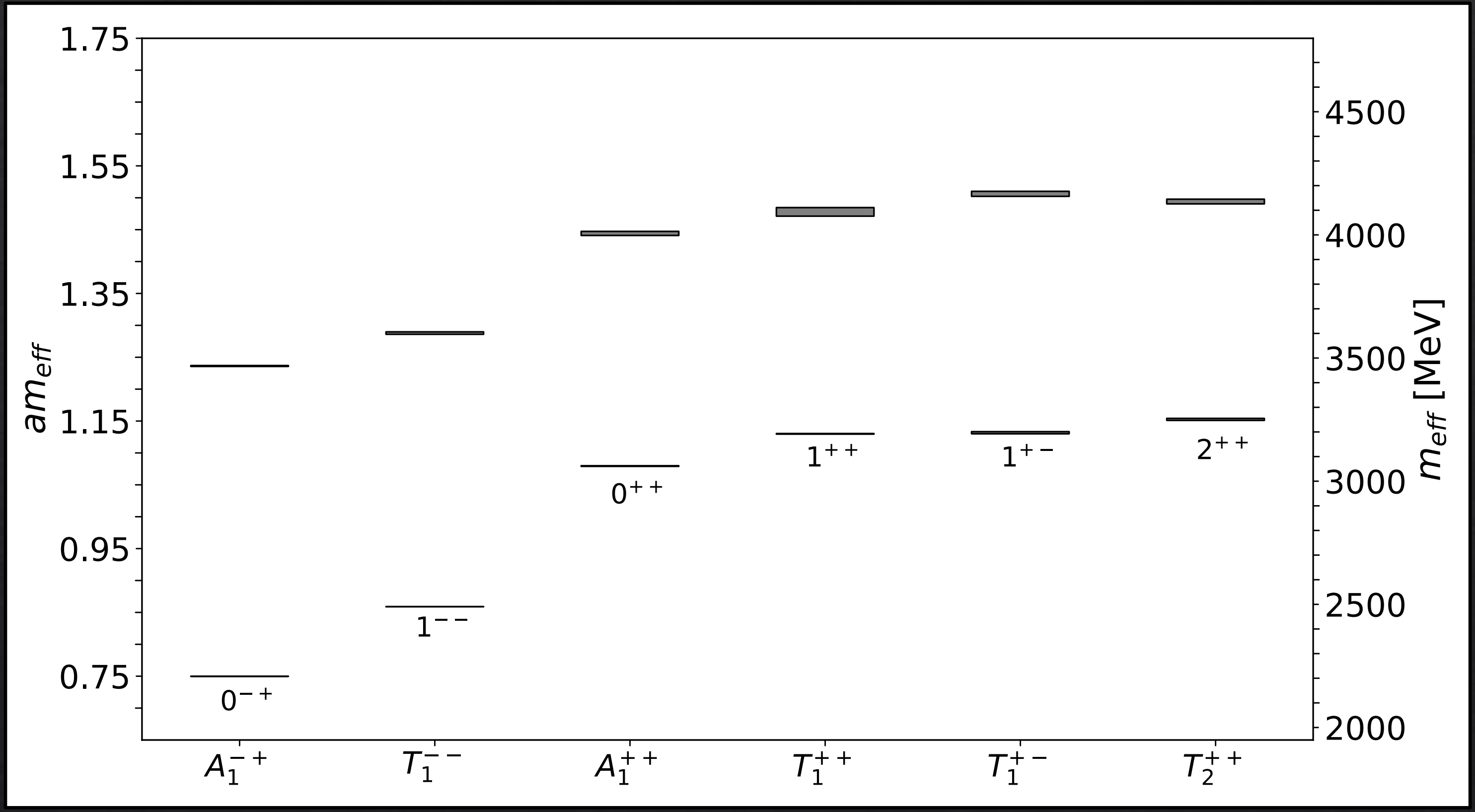
The iso-vector meson spectrum

4080 configurations for local operators (A_1, T_1).
1500 configurations for derivative operator (T_2).

Derivative operator taken from [*J. J. Dudek et al. Phys. Rev. D77 (2008) arXiv:0707.4162*]

Earlier mass plateaus in ground state are achieved.
(Y axis omitted for display simplicity.)

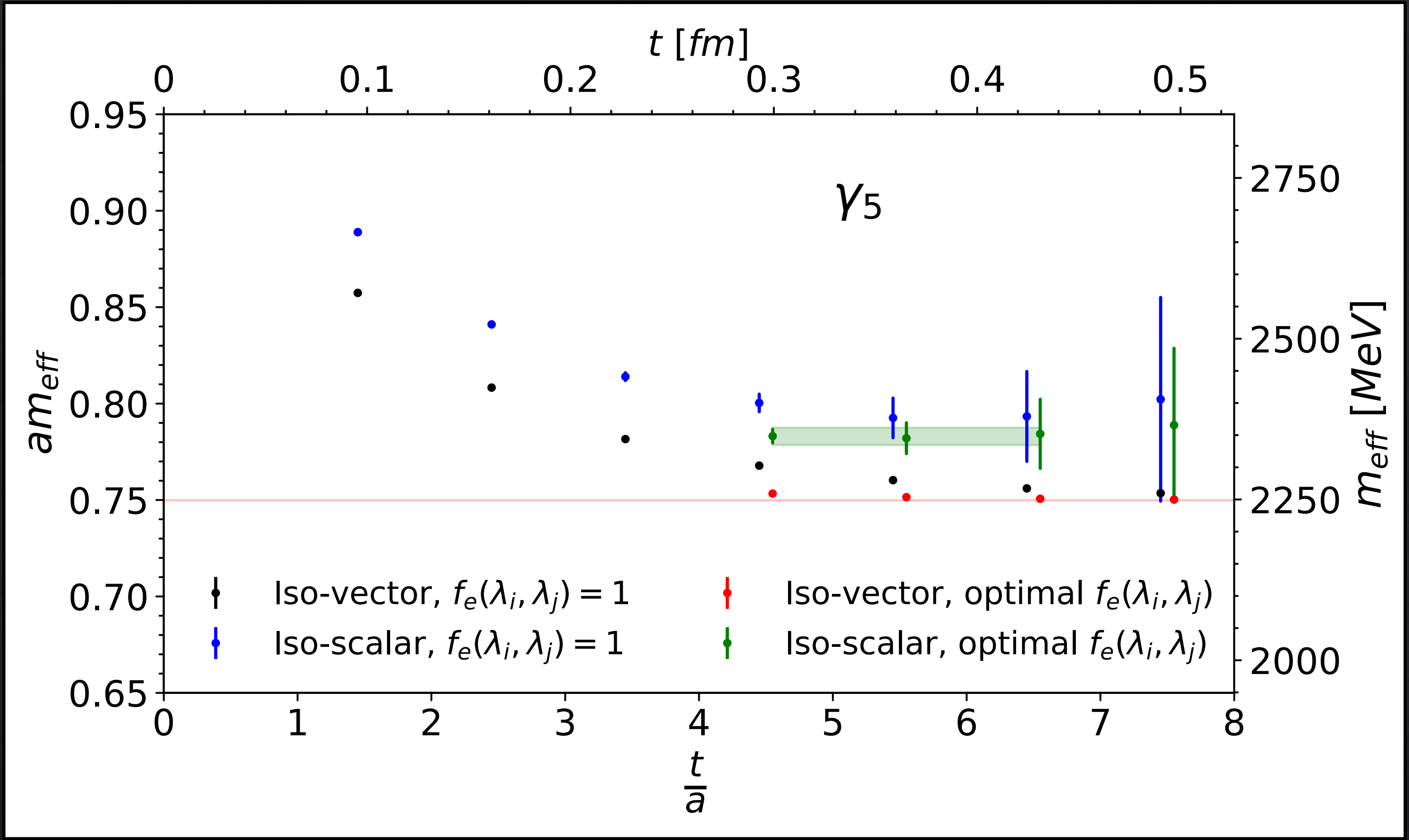




- ✓ Decreased contamination from excited states in ground state compared with standard distillation.

- ✓ Access to excited states already with just one Γ .
- ✓ Including multiple Γ's in same channel could improve signal and avoid possibly missing a state.

The iso-scalar and iso-vector $\bar{c}\gamma_5 c$



Distillation allows us to directly calculate iso-scalar correlations which involve disconnected correlators (Costly via other techniques).

- Difference gives 99 ± 15 MeV.

✓ Non-negligible difference in this setup.

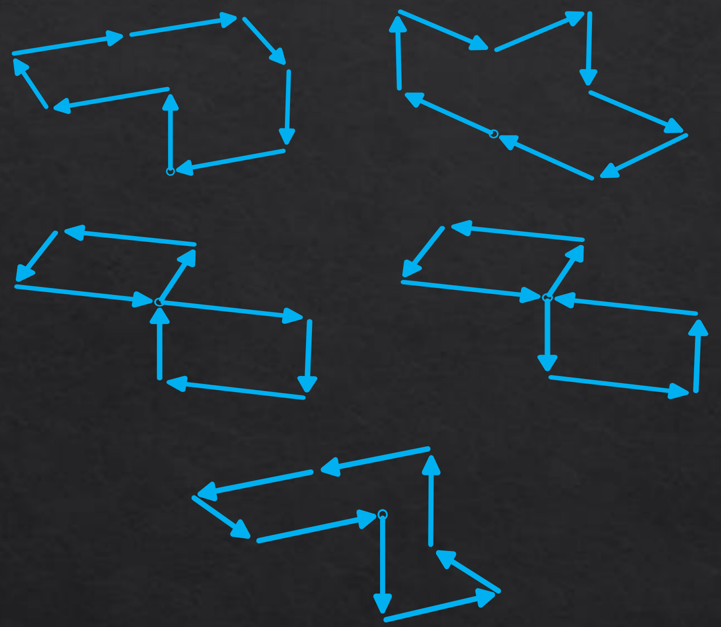
Indirect measurement gives 7.3 ± 1.2 MeV in $N_f = 2 + 1 + 1$ QCD + QED.
[D. Hatton et al. Phys. Rev. D102 (2020) arXiv:2005.01845]

Optimal $f_e(\lambda_i, \lambda_j)$ from iso-vector is used for the iso-scalar because the iso-vector data is much cleaner to analyze.

Glueball calculations

Wide variety of Wilson loop operators for multiple irreducible representations:

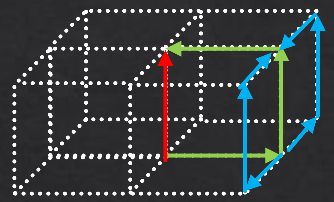
5 loop shapes



- ✓ Single and double winding of each shape.
- ✓ Same shapes with double length too.

[B. Berg & A. Billoire, Nucl. Phys. B221 (1983)]
[C. J. Morningstar & M. Peardon, Phys. Rev. D60 (1999) arXiv:9901004]

3D HYP smearing



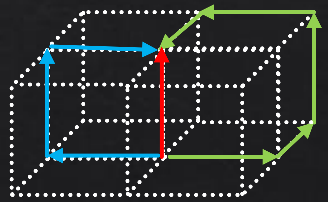
Original link staple
Decorated link staple
Final link

[A. Hasenfratz & F. Knechtli, Phys. Rev. D64 (2001) arXiv:0103029]

Improved APE smearing

$= + \frac{\alpha_1}{4}$ (3-link staple sum)

$+ \frac{\alpha_2}{16}$ (5-link wiggly staple sum)



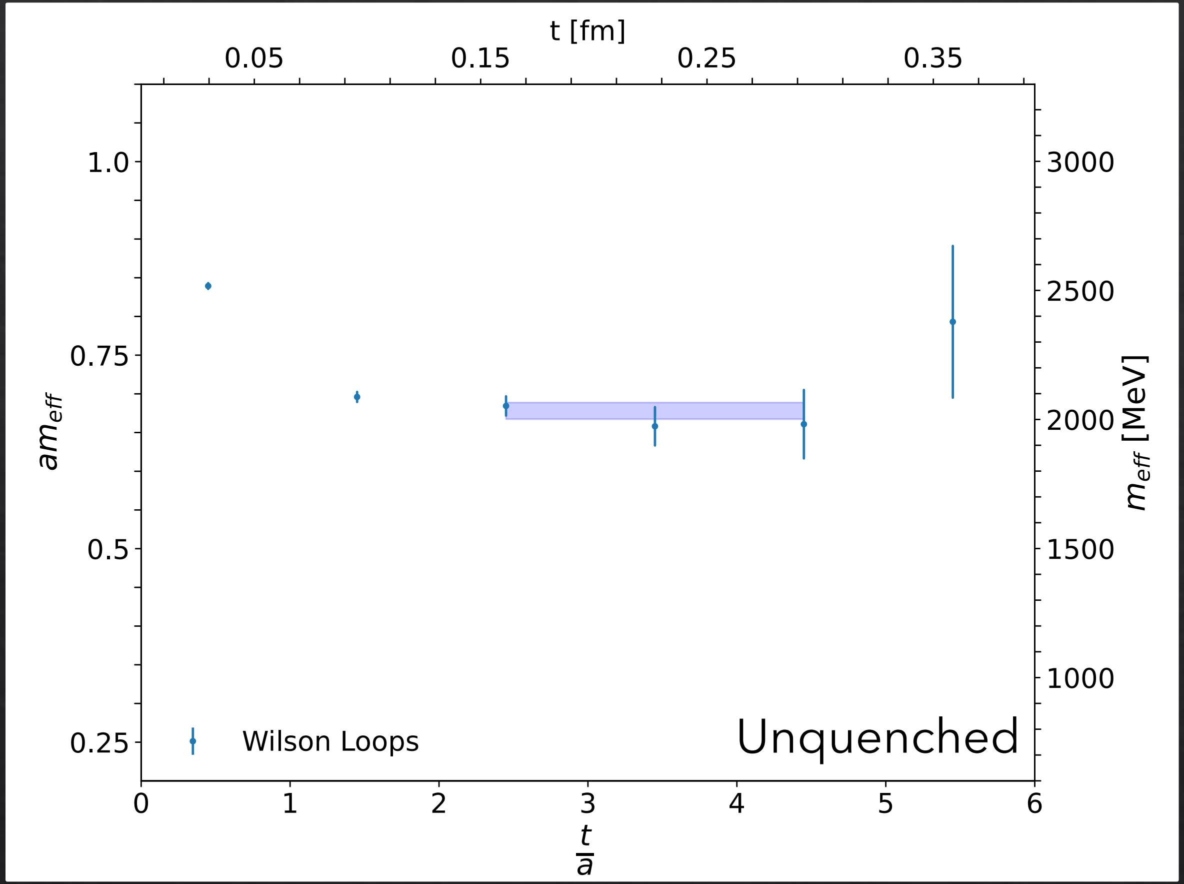
[B. Lucini et al. JHEP 06 (2004) arXiv:hep-lat/0404008]

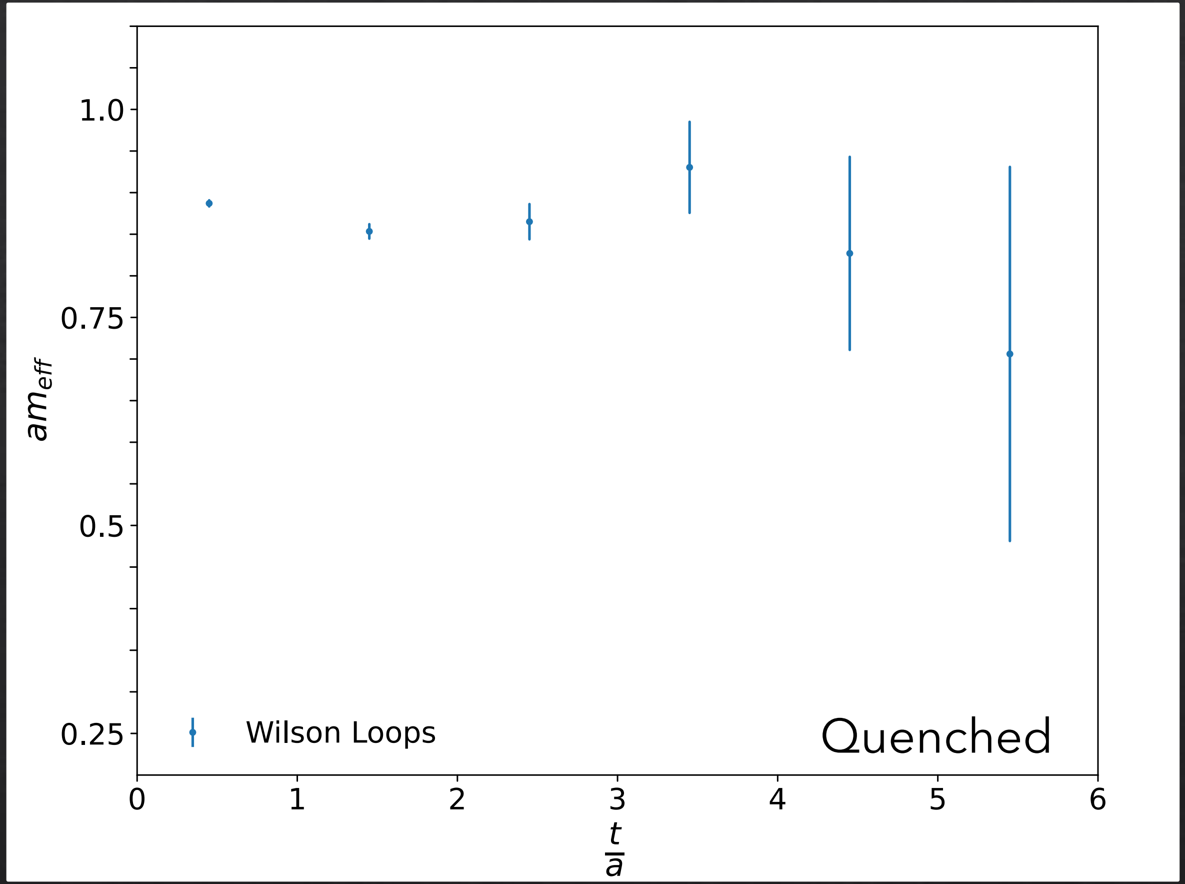
Glueball operators ($C = 1$)

$$G_i^{(R,P=\pm)}(t) = \frac{1}{V}\sum_x \sum_{g=0}^{23} \rho_g^R \, Re\left(Tr\left(W_{i,g}(t,x) \pm W'_{i,g}(t,x)\right)\right)$$

- i: Multi-index for the loop shape, winding, length and smearing scheme.
- $W_{i,g}(t,x)$: Wilson loop at (t,x) operator from parameter set i after applying cubic transformation g to its shape.
- $W'_{i,g}(t,x)$: Parity twin of $W_{i,g}(t,x)$.
- ρ_g^R: Projection coefficients for irrep R.

The 0^{++} Glueball





- Tuning of smearing parameters / selection of operators done in low statistics.
- Pruning + GEVP to find effective masses.
- Measurements in 10200 cfgs.

Same measurements (7100 cfgs.) in quenched ensemble of equal lattice size and almost equal t_0.

Excited states are more suppressed than in unquenched case.

Conclusions

- ✓ Distillation can be optimized by exploiting quark and meson distillation profiles.
- ✓ This method was tested to compute the spectrum of $N_f = 2$ QCD with mass $\frac{m_{charm}}{2}$.
- ✓ Iso-scalar operator for γ_5 is accessible and there is a non-negligible difference with the iso-vector.
- ✓ Ground state of the 0^{++} glueball is accessible but more contaminated by excited states than in the quenched theory for this setup.

Thank you for your attention