An improvement of glueball mass calculations using gradient flow

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Lattice 2021, July 26-30

Contents of this talk

- Removing ultraviolet noise from the gauge fields is necessary for glueball spectroscopy in lattice QCD.

→ smearing of gauge fields

- The Yang-Mills gradient flow is an alternative approach

in this talk,

We study an application of the gradient flow technique to the construction of the extended glueball operators

Gradient flow

Gradient flow[1] has some characteristic properties

[1] M. Lüscher, JHEP. 1008, 071 (2010).

- **We focus on its smoothing effect on the gauge fields**

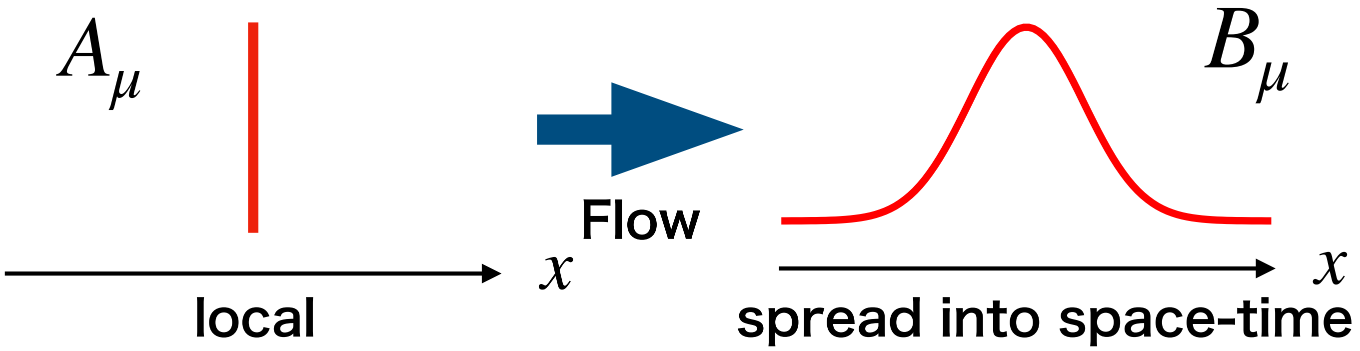
Flow equation

$$\partial_t B_\mu(t,x) = D_\nu G_{\nu\mu}(t,x),\ B_\mu(t=0,x) = A_\mu(x)$$

$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + [B_\mu, B_\nu],\ D_\mu = \partial_\mu + [B_\mu, \cdot\ ]$$

t : diffusion time

The solution B_μ takes a gaussian form



gradient flow can remove ultraviolet noise from the gauge fields

The effects from gradient flow

Flow in the temporal direction causes some problems

$t \to$ Large ➡ B_μ becomes dominant in the time correlation

Gauss

ex) $f(\tau) = e^{-\tau^2}$ (Gauss form)

Effective mass $-\log \dfrac{f(\tau+1)}{f(\tau)} \to -\dfrac{d}{d\tau} \log f(\tau) = 2\tau$ **linear** $\neq$ plateau

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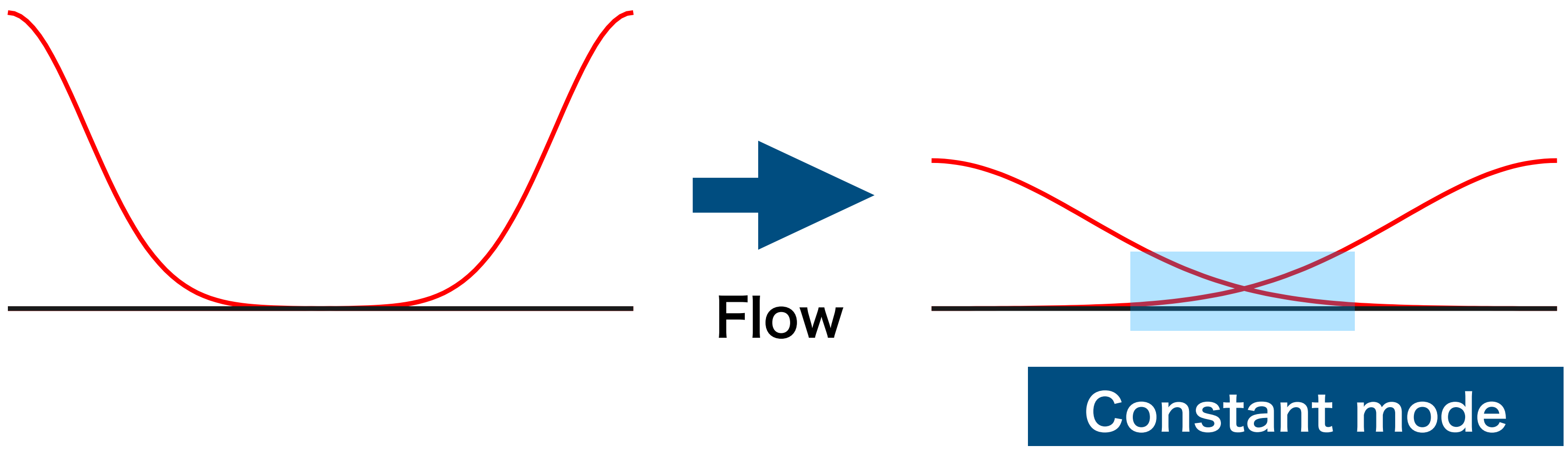
2pt. func.

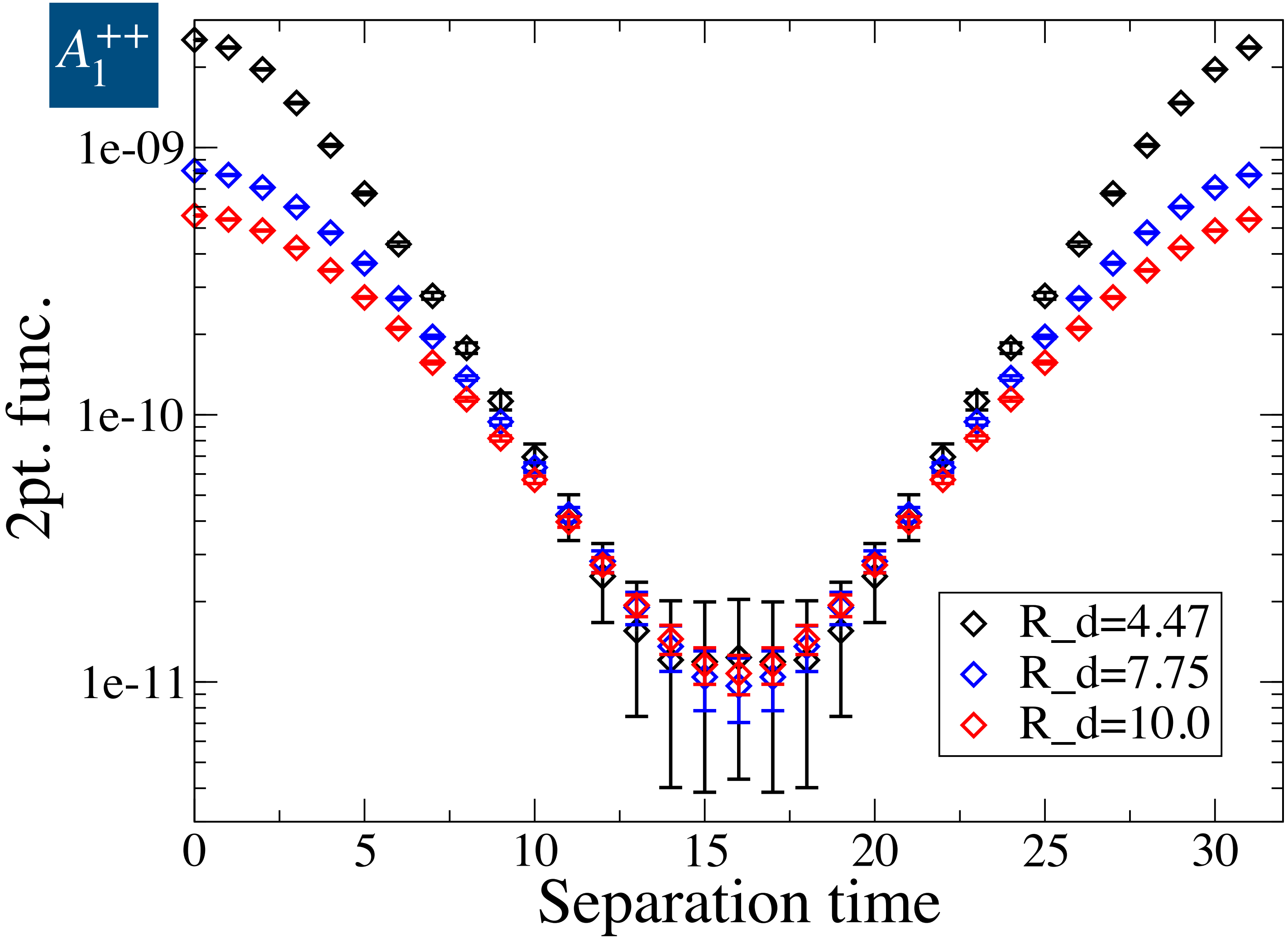


These properties are inconvenient for spectroscopy

2pt. func. with gradient flow

Diffusion radius $R_d = \sqrt{8t}$, t **: flow time**

$\beta(=6/g^2)$	$L^3 \times T$	Meas.	a[fm]
6.4	$32^3 \times 32$	3000	0.0513(3)



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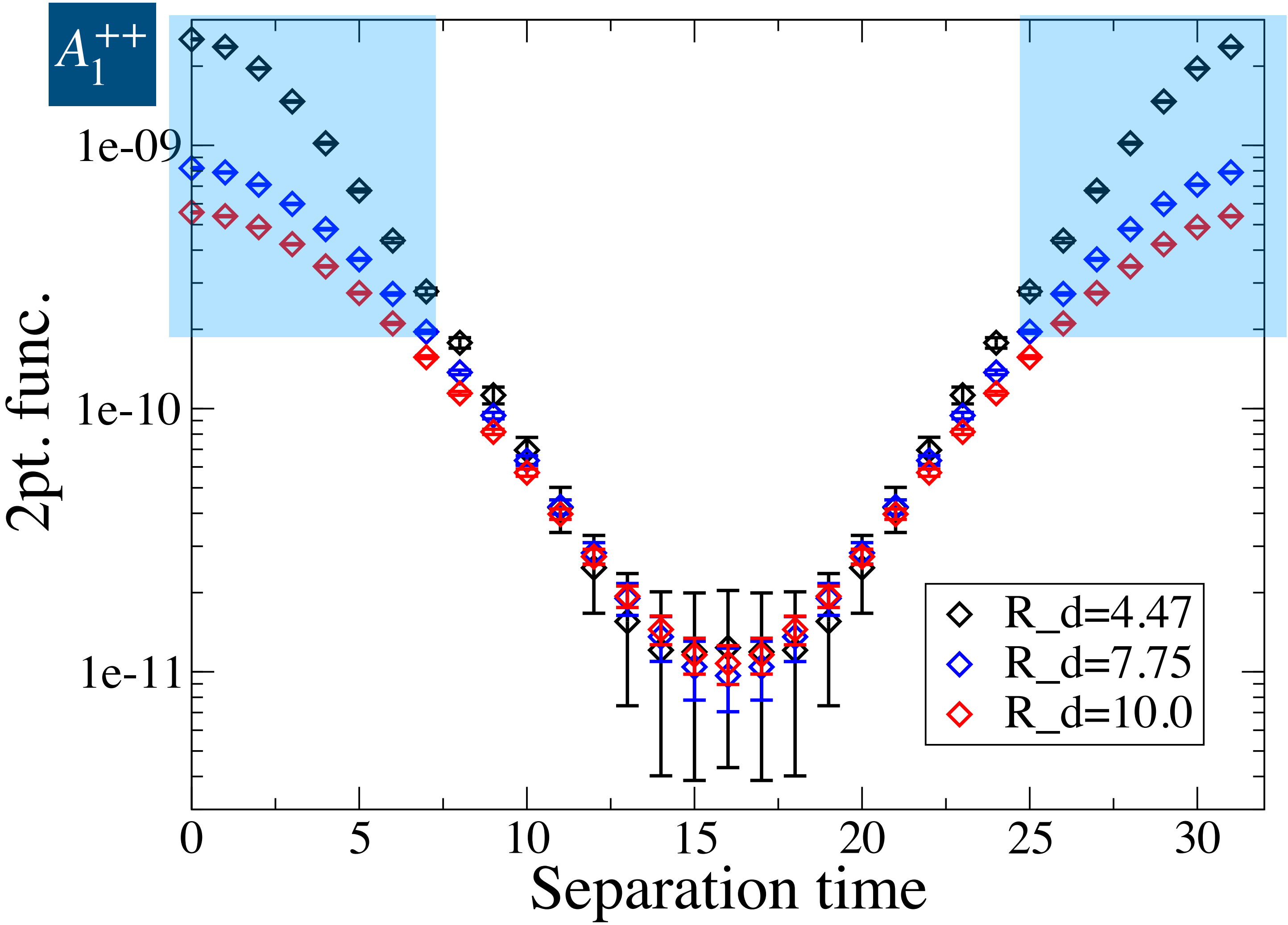

in small T

Gaussian shape

2pt. func. with gradient flow

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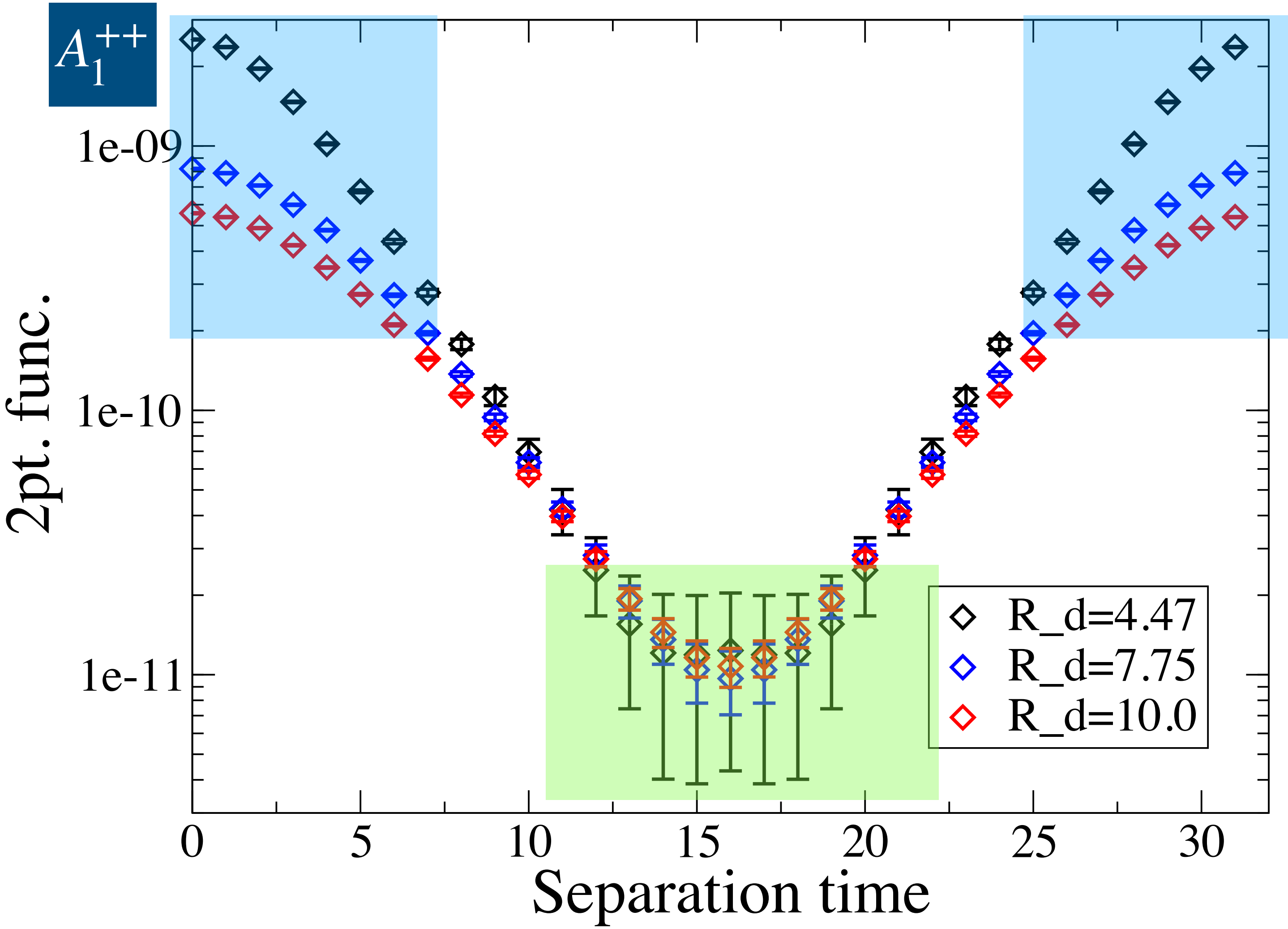

in small T
Gaussian shape

in large T
constant mode

2-point function takes gaussian shape

Effective mass with Gradient flow

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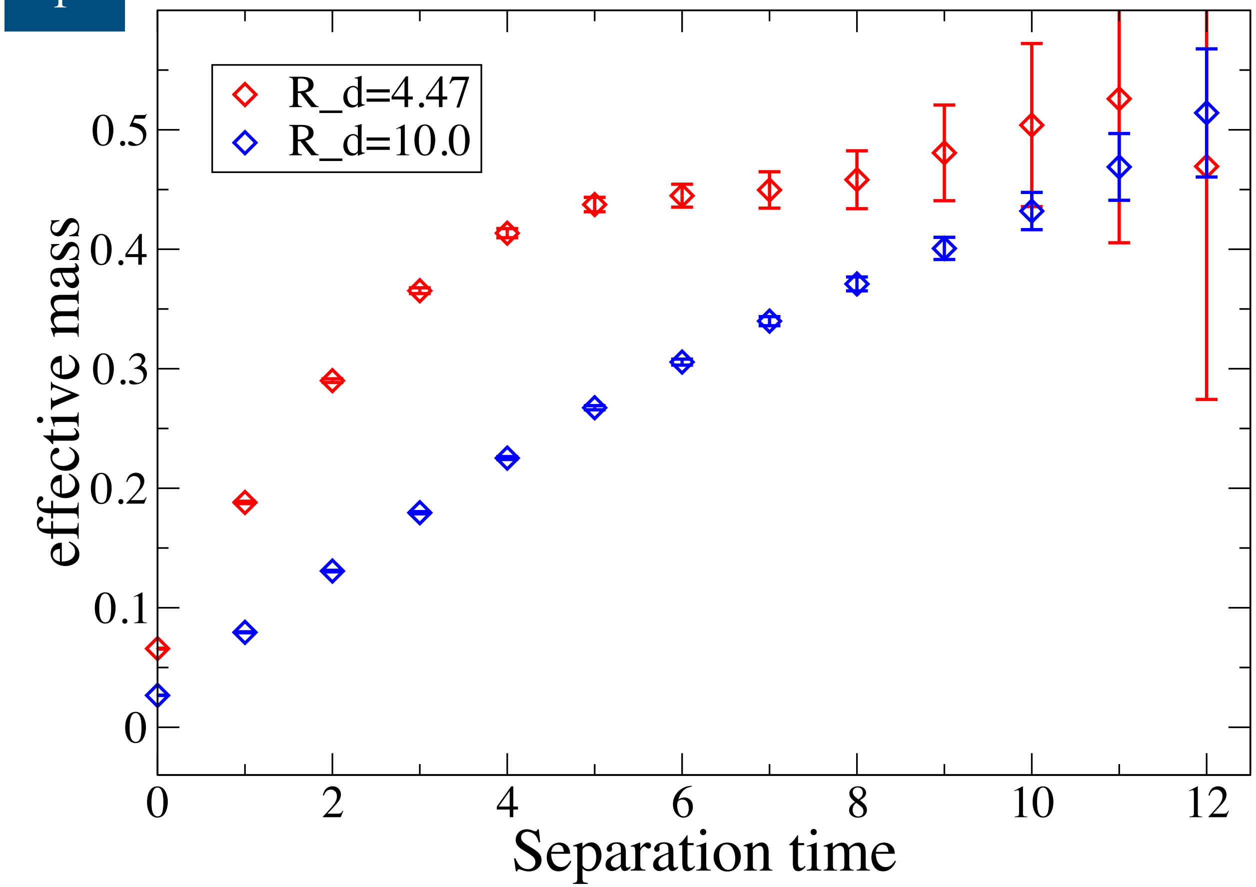
A_1^{++}



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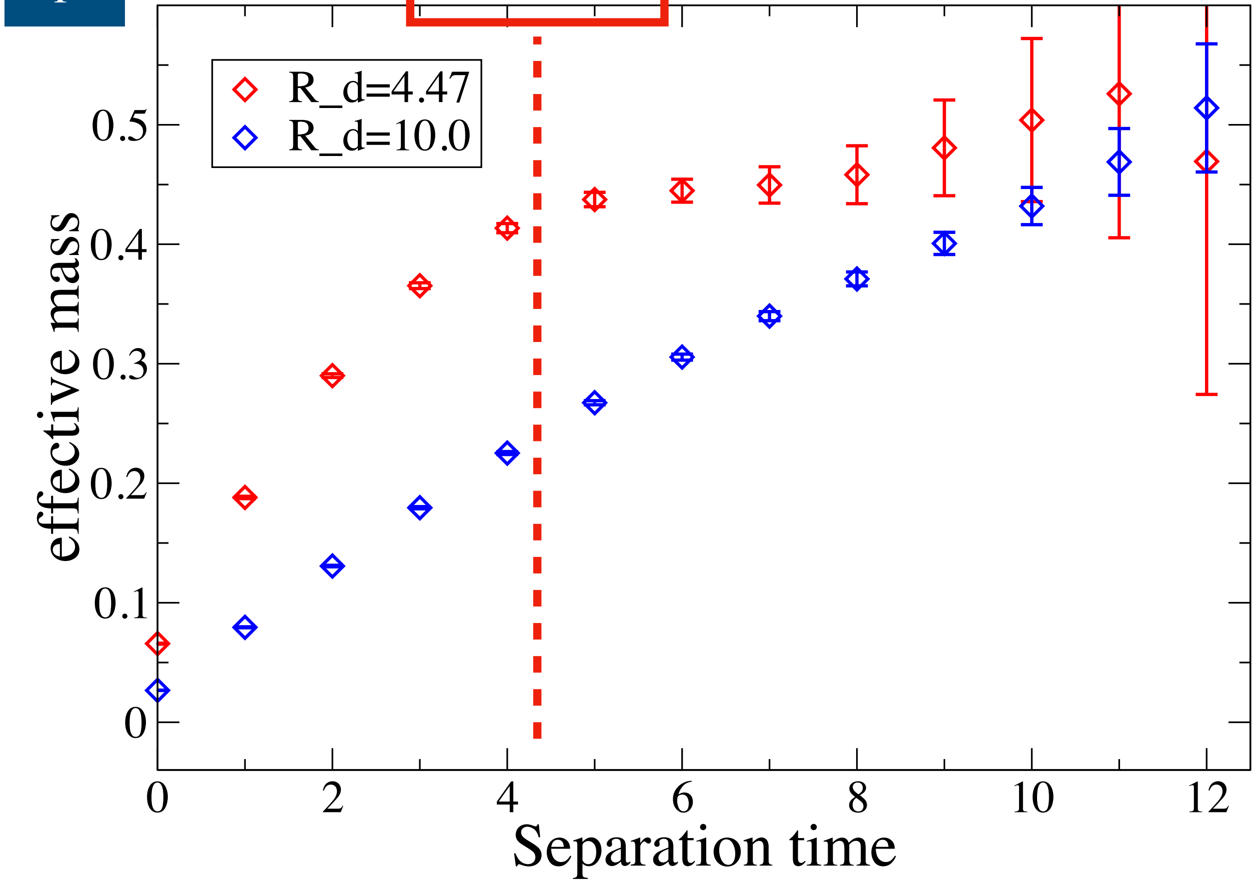
$R_d = 4.47$

short flow case



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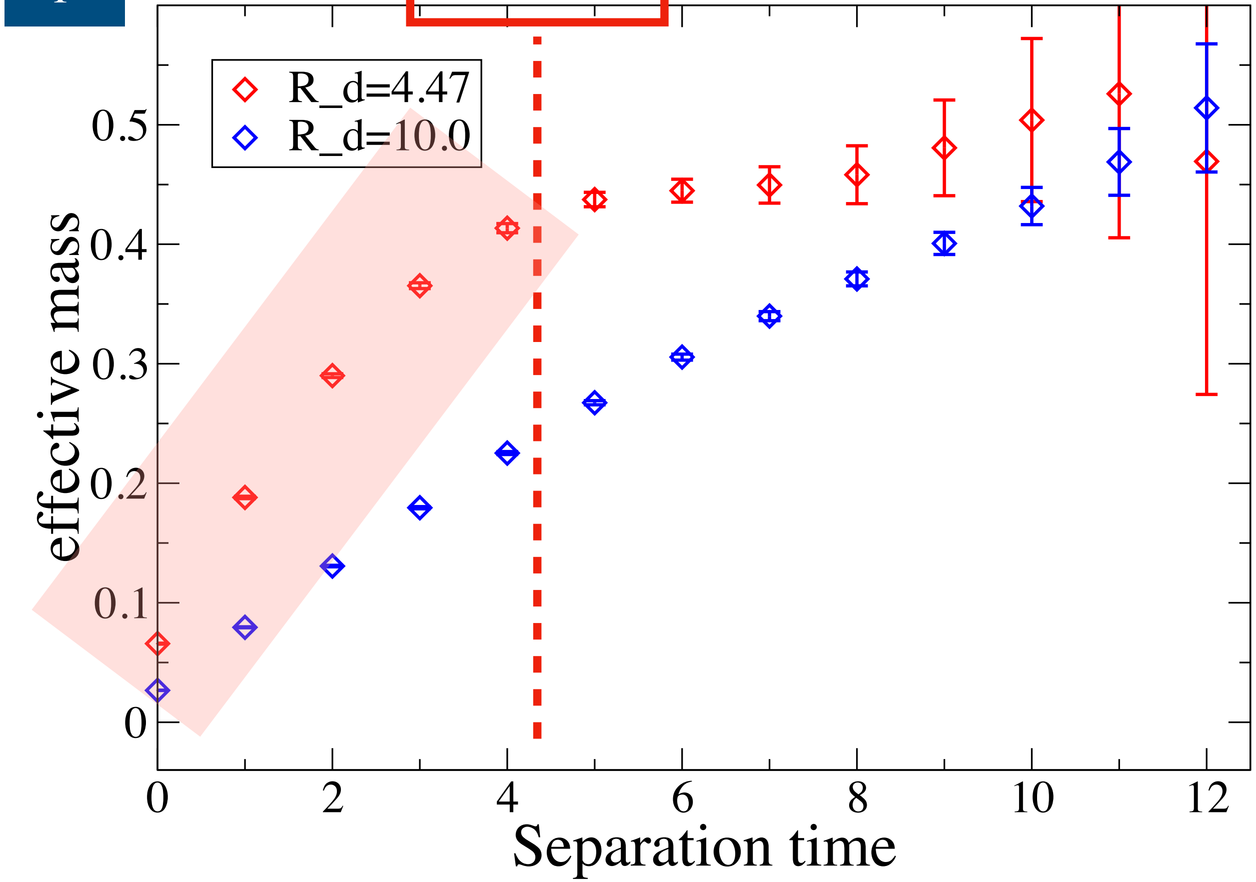

short flow case

inside R_d **:**

linear behavior from gaussian

Effective mass with Gradient flow

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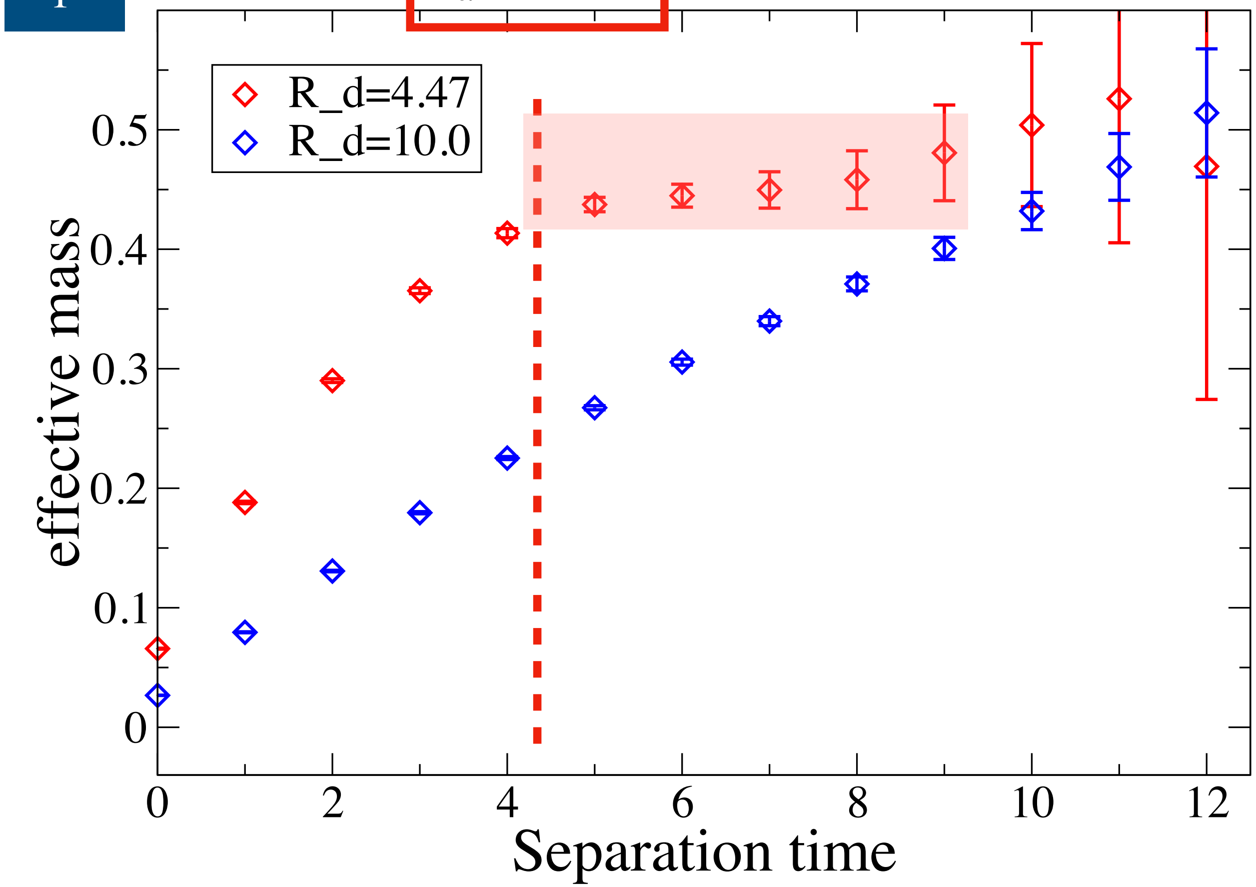

short flow case

inside R_d :

linear behavior from gaussian

outside R_d :

plateau

Effective mass with Gradient flow

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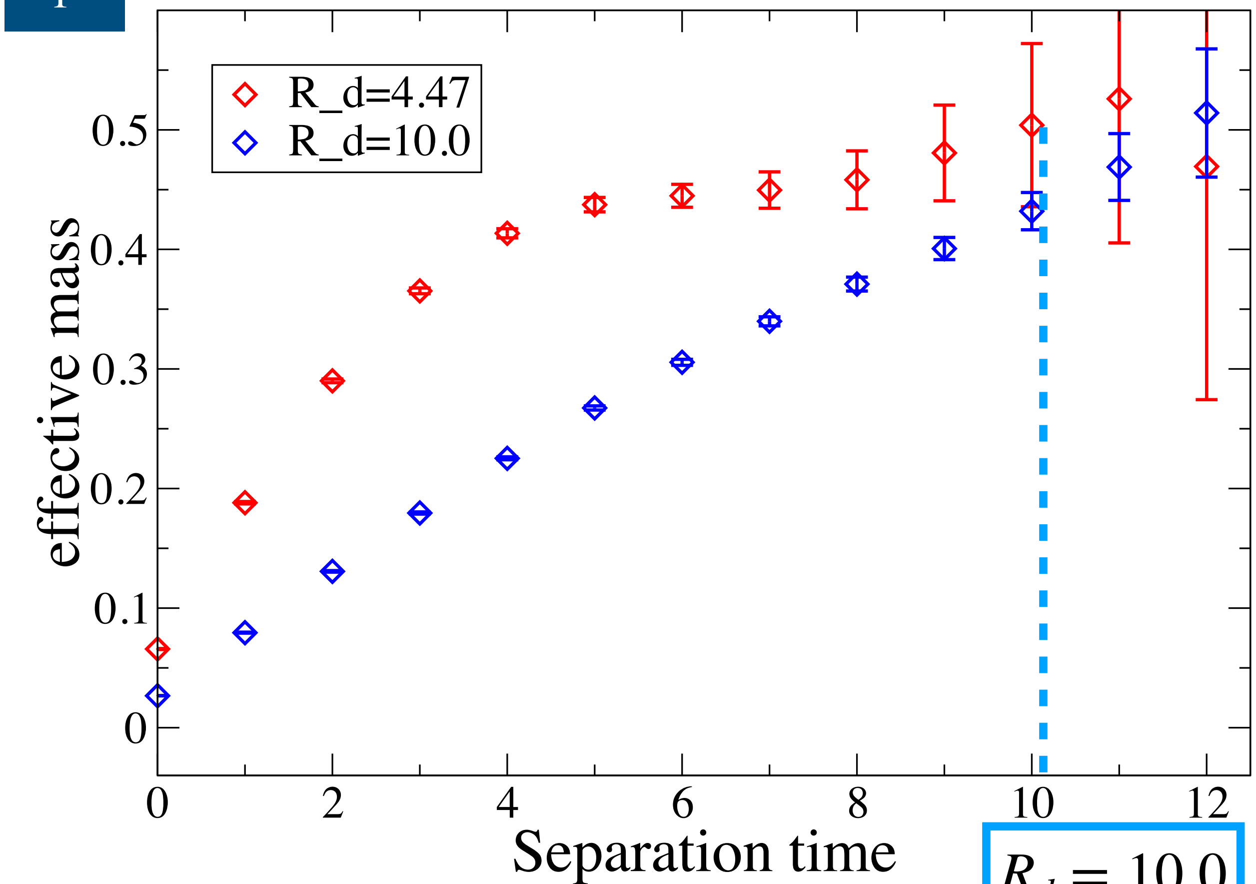

long flow case

$R_d = 10.0$

Effective mass with Gradient flow

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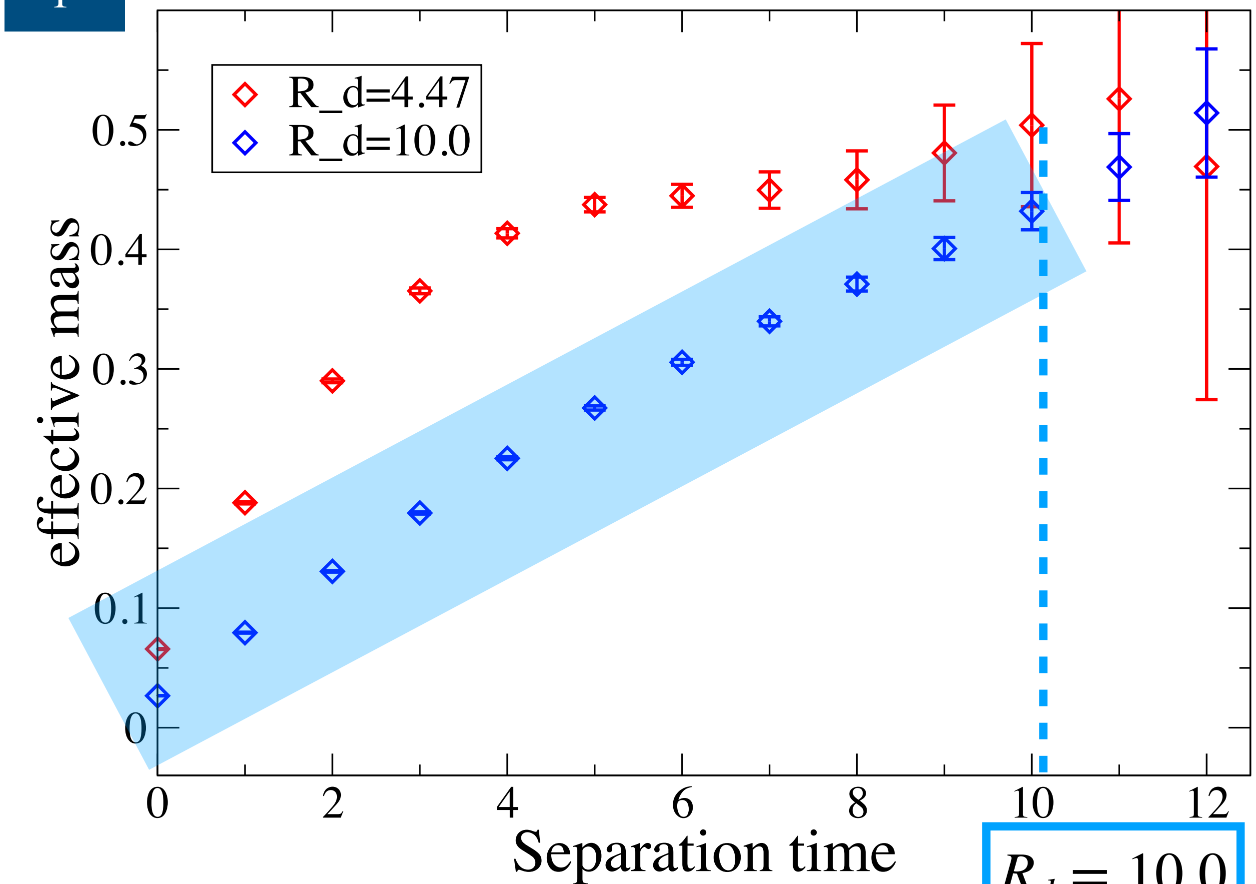

$R_d = 10.0$

long flow case

Linear dependence fully dominated

The plateau no longer appears

Effective mass with Gradient flow

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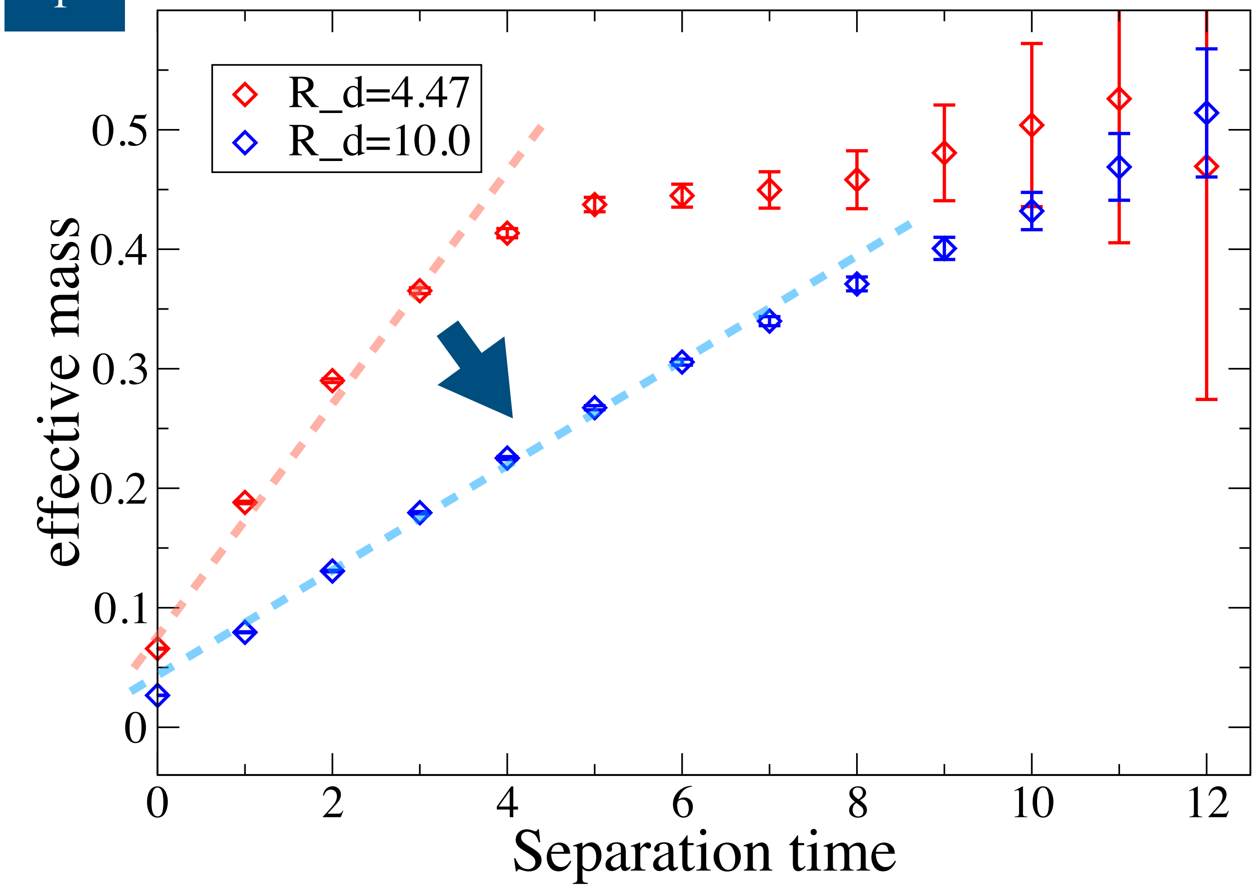

slope of linear part

Effective mass with Gradient flow

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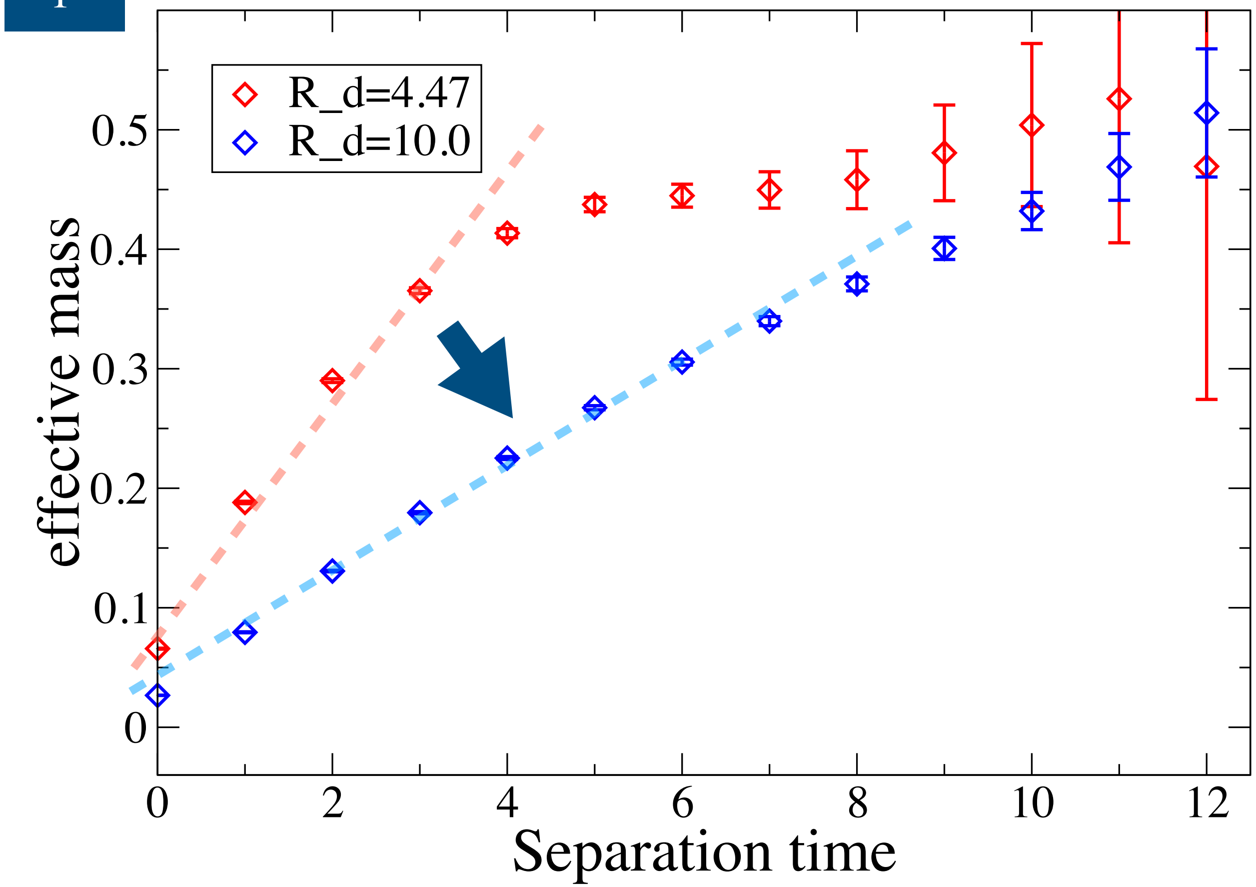

slope of linear part

R_d→large

slope→small

Effective mass with Gradient flow

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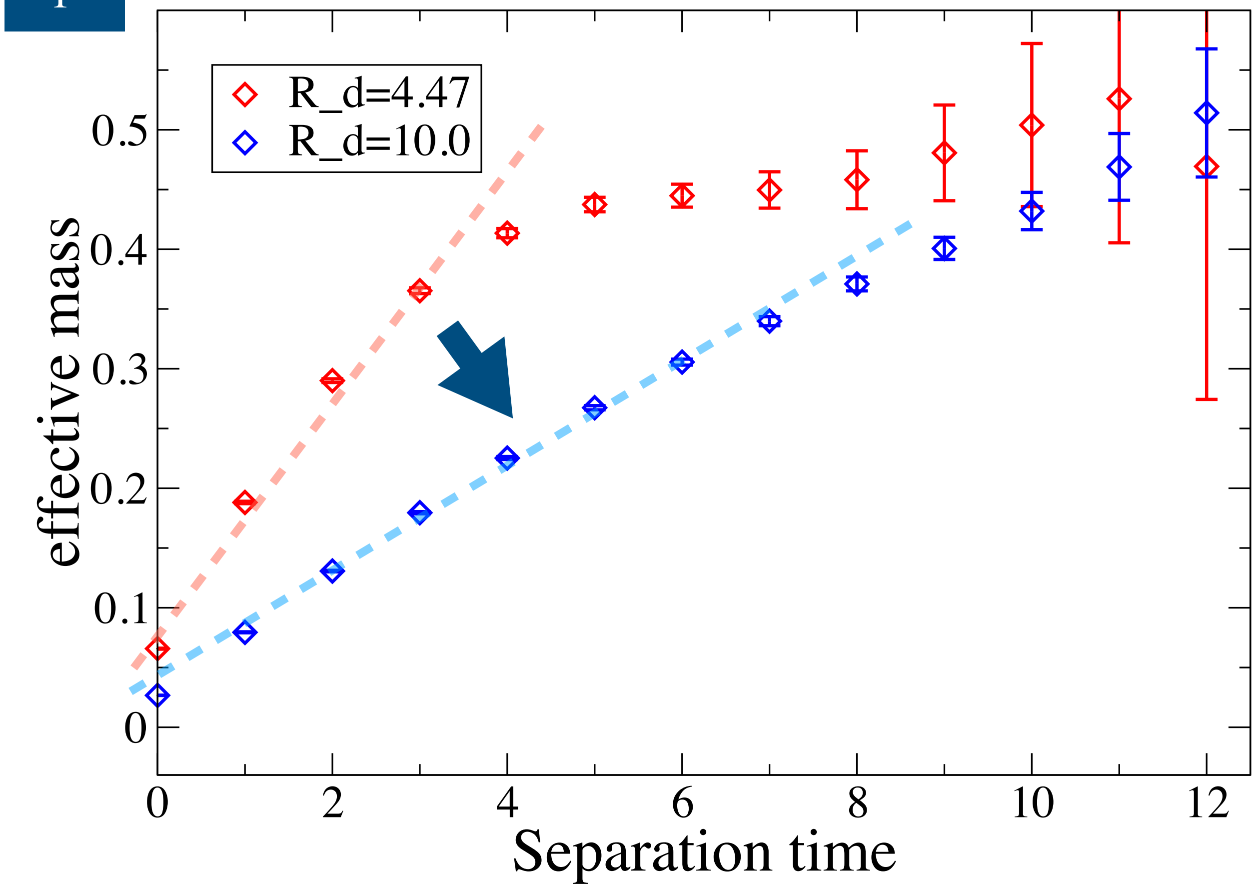

slope of linear part

R_d→large

slope→small

as expected from the gaussian shape of 2pt. func.

Effective mass with Gradient flow

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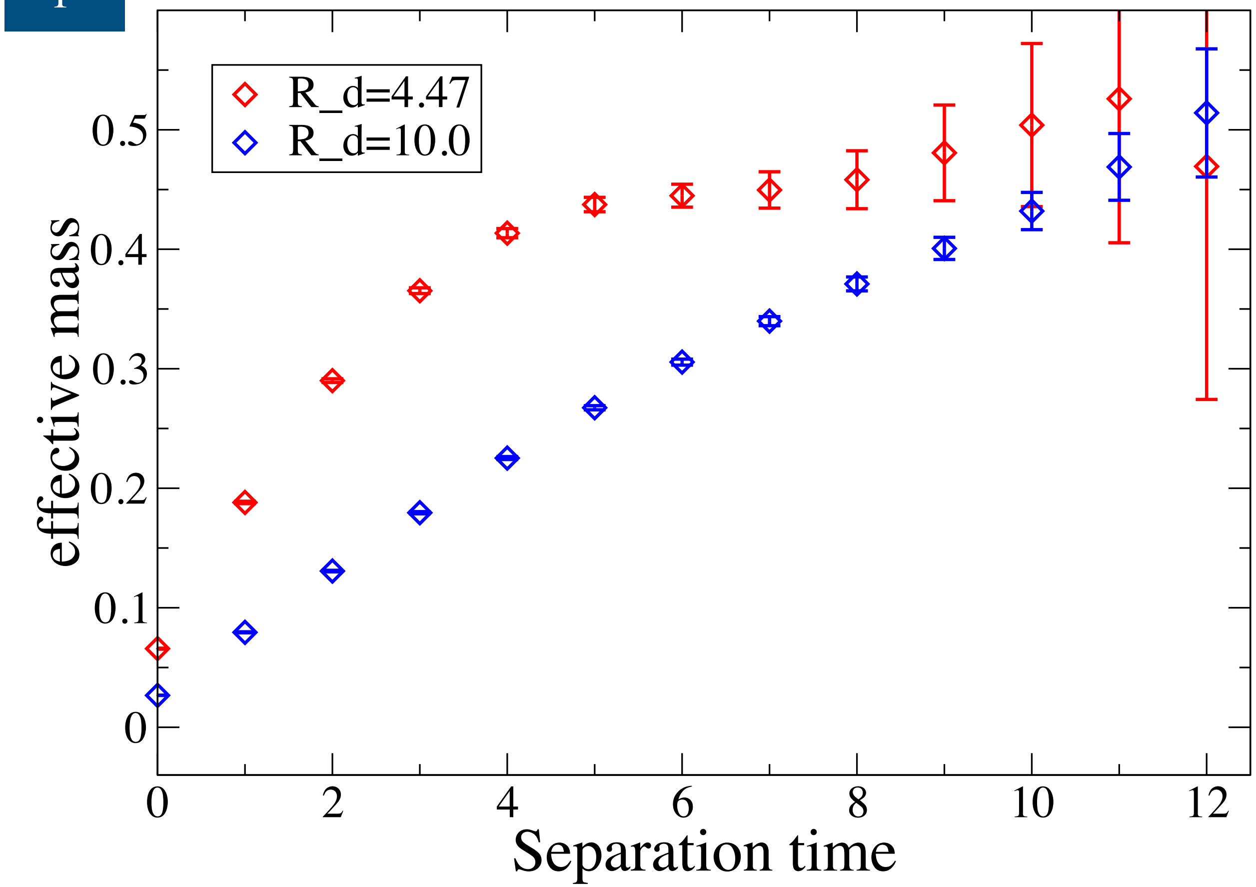

The longer flow is needed to improve a signal of 2pt. func.

However,

after the longer flow, the gaussian behavior becomes dominant in the temporal correlation

Spatial flow

Gradient flow on the lattice

$$\frac{\partial}{\partial t} V_\mu(x,t) V_\mu(x,t)^{-1} = -\, g_0^2 \partial_{x,\mu} S_W[V(t)],$$

$$V_\mu(x, t=0) = U_\mu(x),\ S_W[V(t)] = \beta \sum_{x,\mu\neq\nu} \left\{ 1 - \frac{1}{N} \mathrm{Re\ tr} V_{\mu\nu}(x,t) \right\}$$

using only the spatial gradient

"Spatial flow"

$$\frac{\partial}{\partial t} V_i(x,t) V_i^{-1}(x,t) = -g_0^2 \partial_{x,i} S_{\mathrm{splaq}}[V(t)],$$

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i, j: spatial direction

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i, j: spatial direction

"Action" with only the spatial links

2pt. func. with Spatial flow

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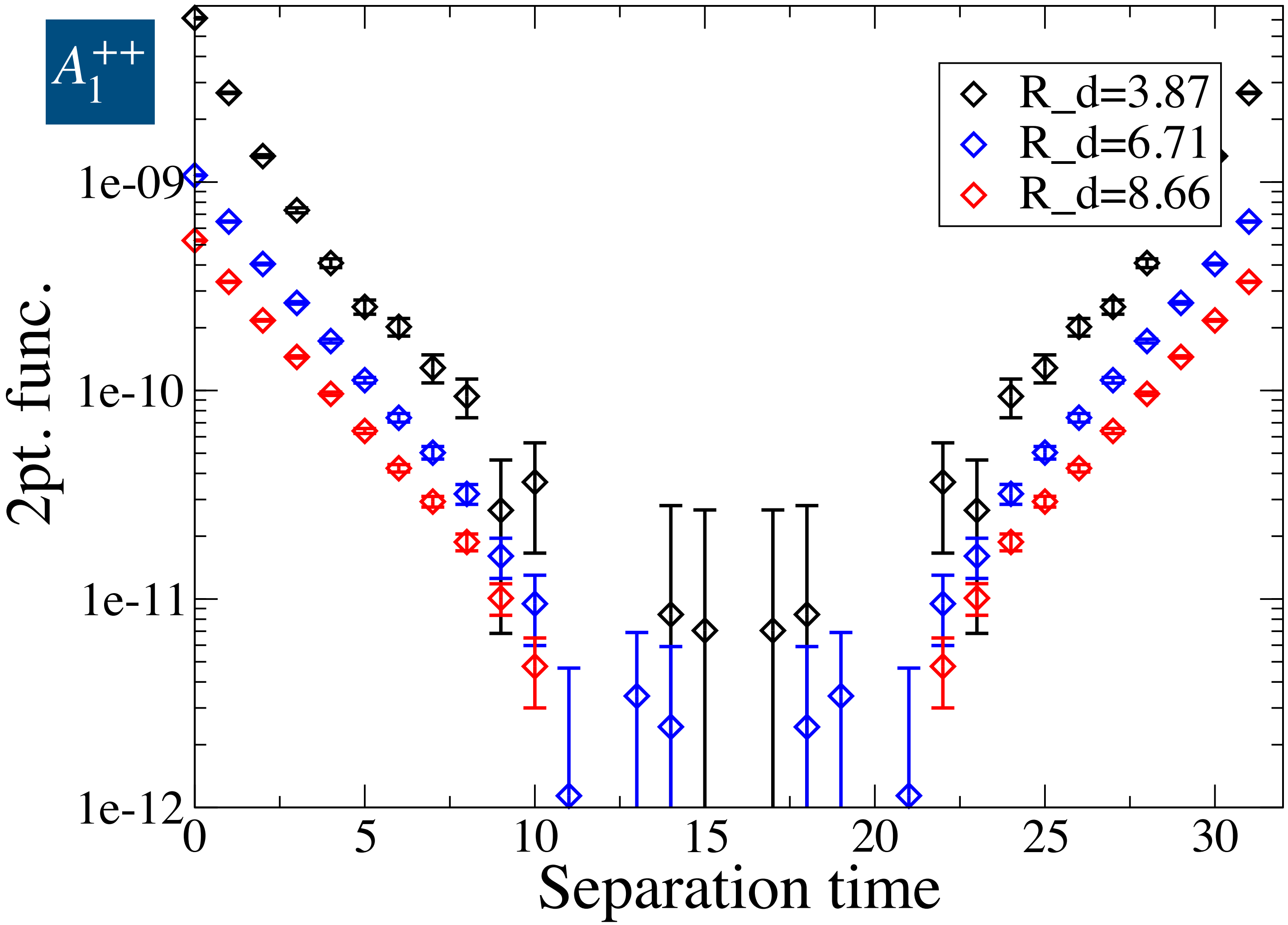

in small T

in large T

2pt. func. with Spatial flow

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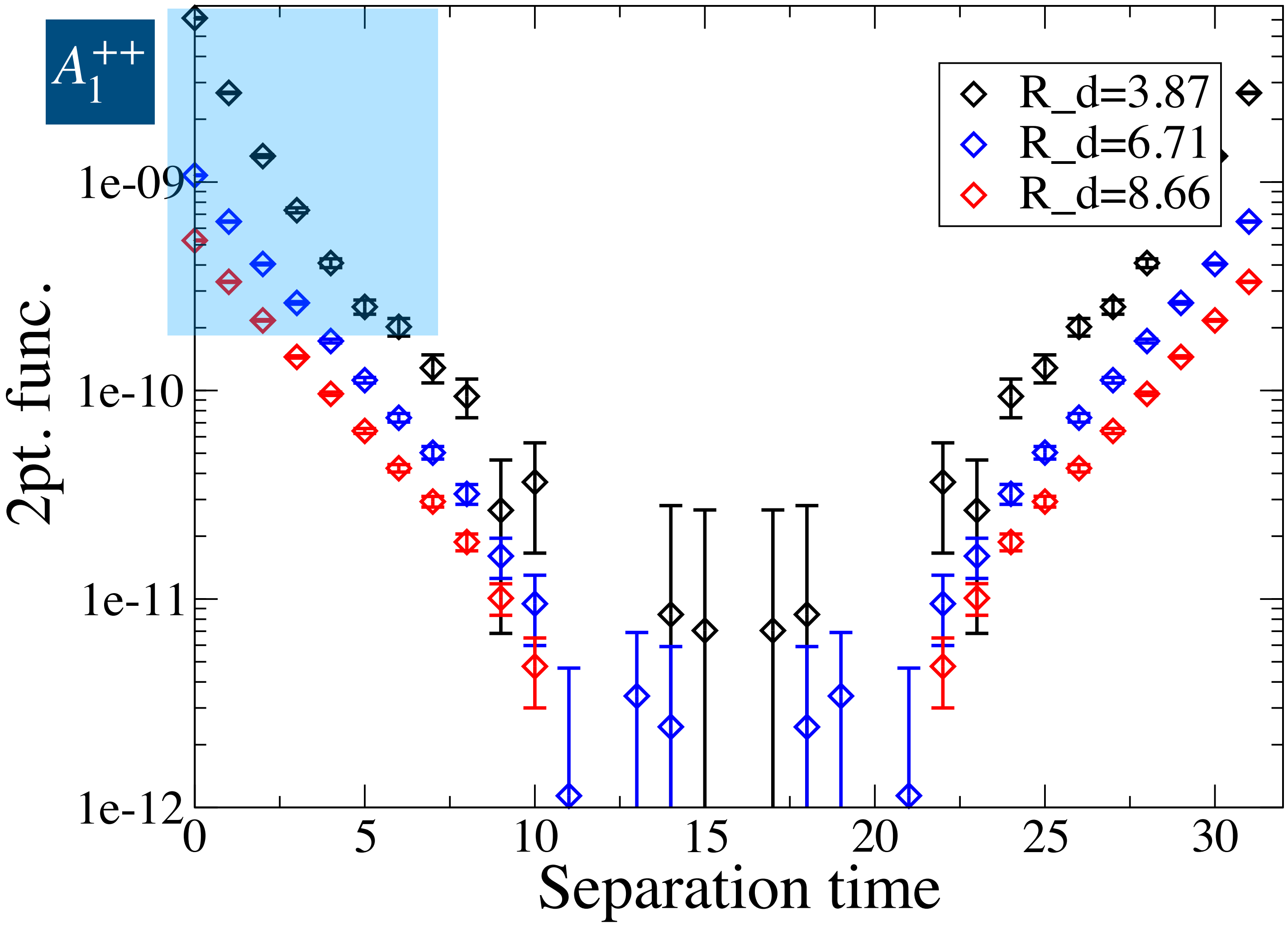

in small T

exponential behavior

in large T

2pt. func. with Spatial flow

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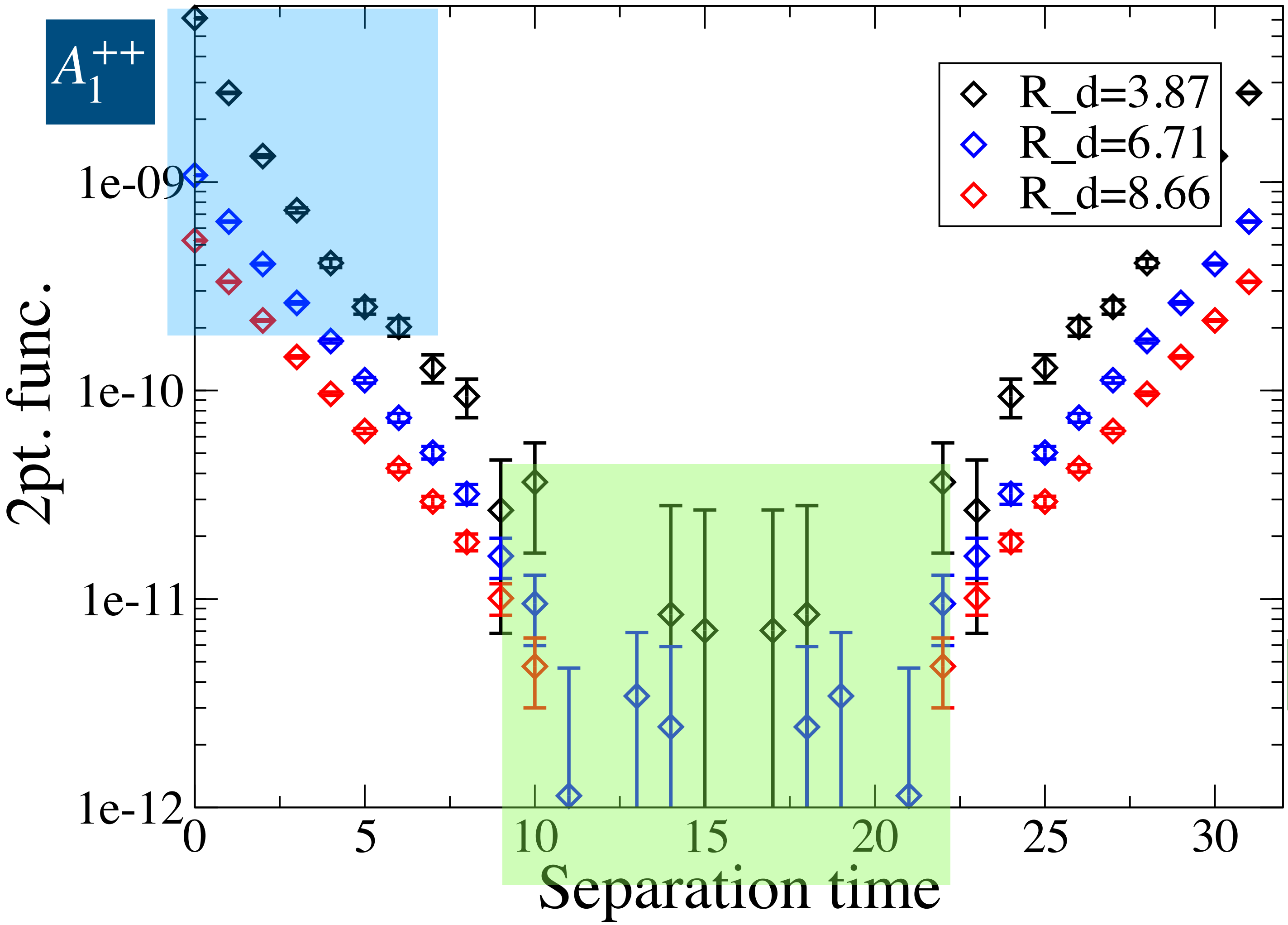

in small T
exponential behavior

in large T
no constant modes

Effective mass with spatial flow

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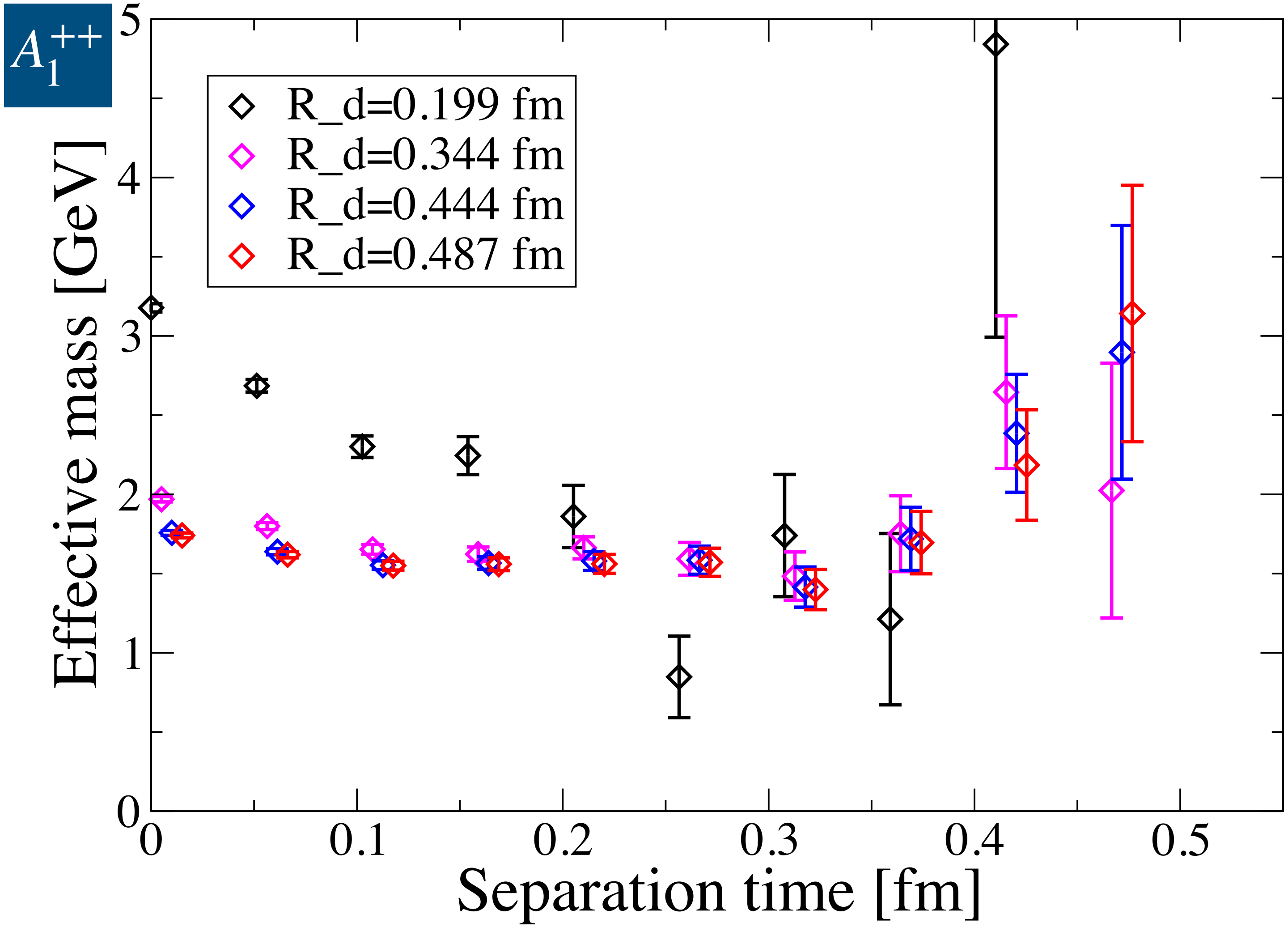

The length of flow becomes sufficient around $R_d \simeq 0.45$ fm

“Spatial flow” does not cause the problems in spectroscopy

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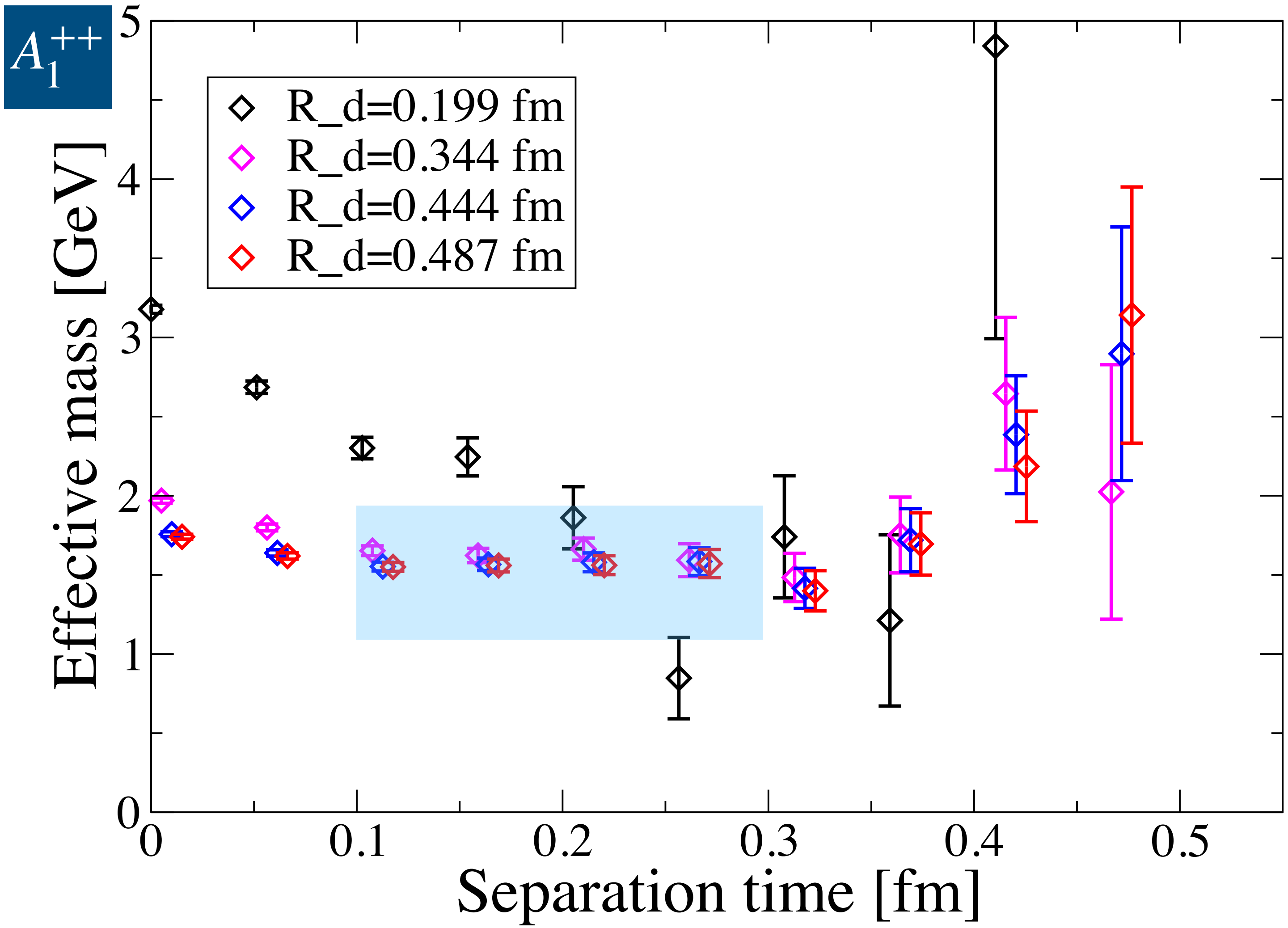

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plateau

“Spatial flow” does not cause the problems in spectroscopy

Stout smearing

$\beta(=6/g^2)$	$L^3 \times T$	Meas.	a[fm]
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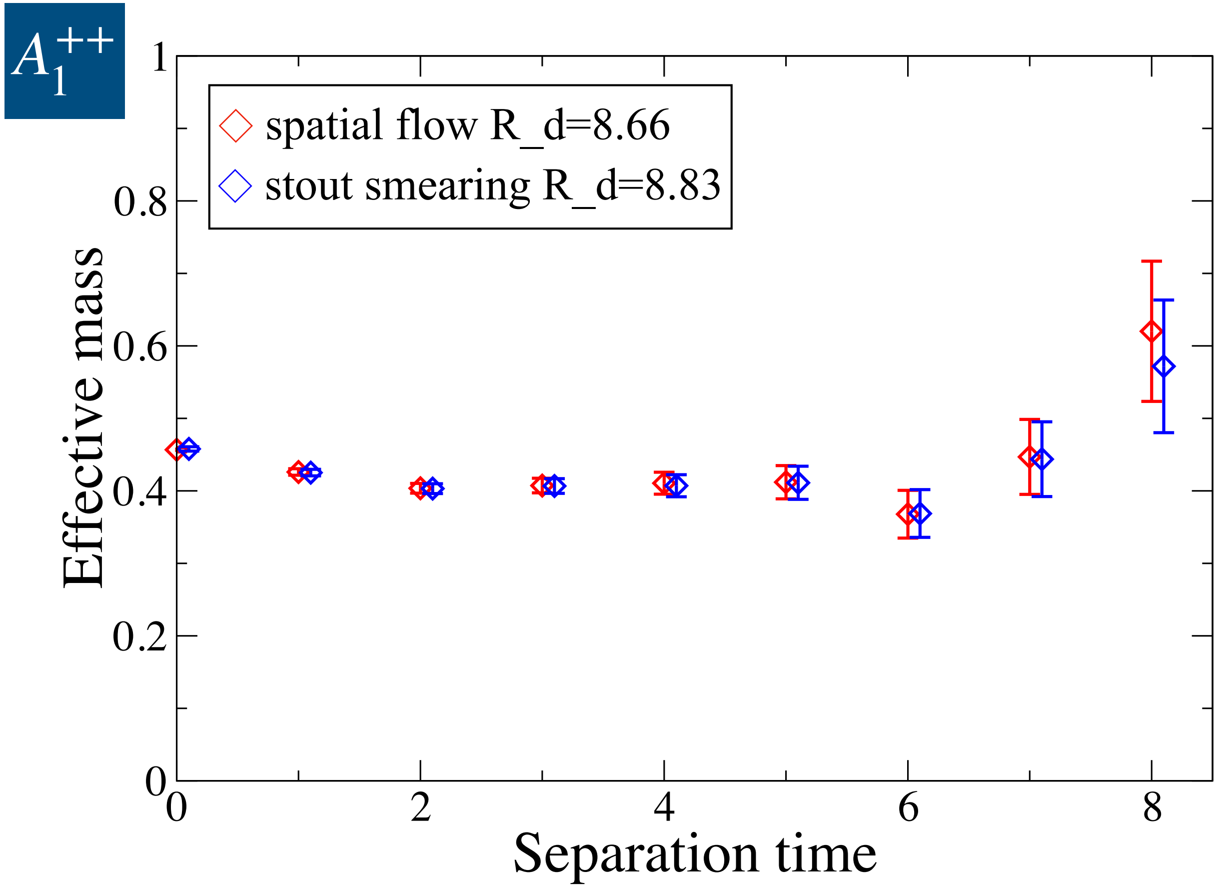
We also apply the "stout smearing" [1] to glueball calculations

[1] C. Morningstar and M. Peardon, Phys. Rev. D 69, 054501 (2004)



usual cases — steps~10 << glueball — steps~100 (very large steps)

Almost identical

in fact,

infinitesimal step stout smearing = gradient flow

M. Lüscher, JHEP. 1008, 071 (2010).

The results of glueball calculations are not sensitive to whether the smearing is continuous or discrete

Calculation set up

We calculate the glueball mass on these 3 sets of configurations.

$\beta(=6/g^2)$	$L^3 \times T$	Meas.	a[fm]	$L \times a$[fm]	diffusion rad. [fm]
6.20	$24^3 \times 24$	4000×4*	0.06775	1.6260	0.4545
6.40	$32^3 \times 32$	3000	0.0513	1.6416	0.4443
6.71	$48^3 \times 48$	600×4*	0.0345	1.6560	0.4818

*We also performed multiple measurements on the rotated configurations. ($t \leftrightarrow x$, $t \leftrightarrow y$, $t \leftrightarrow z$)

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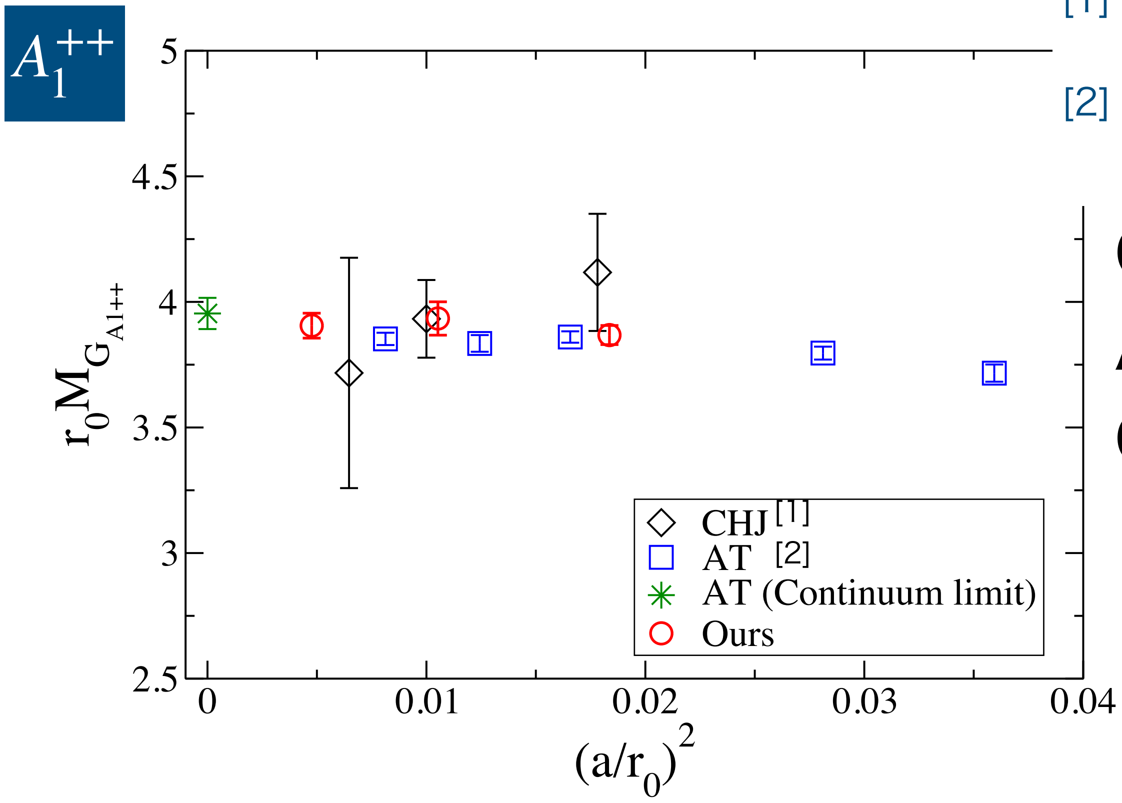
- **We chose the length of spatial flow as** $R_d \simeq 0.45$ fm

Comparison



[1] A. Chowdhury, et al., JHEP02 (2016) 134.

[2] A. Athenodorou and M. Teper, JHEP11 (2020) 172.

CHJ: gradient flow

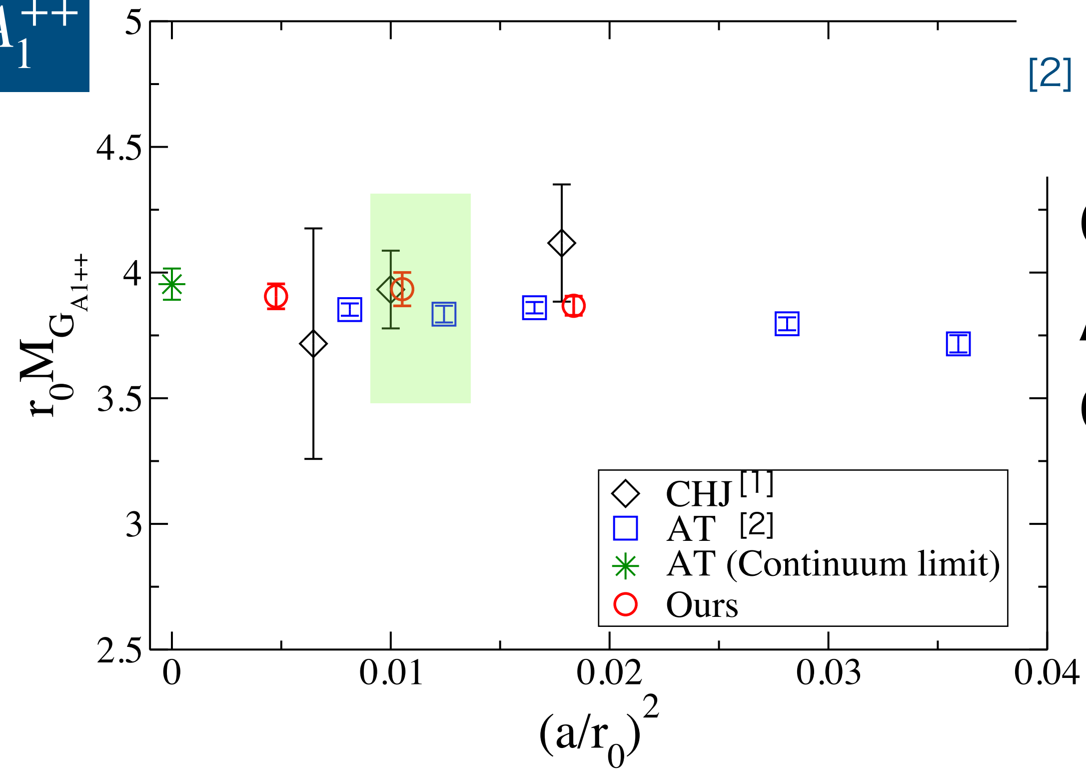
AT: conventional smearing

Ours: spatial flow

Consistent with previous studies

Comparison

A_1^{++}



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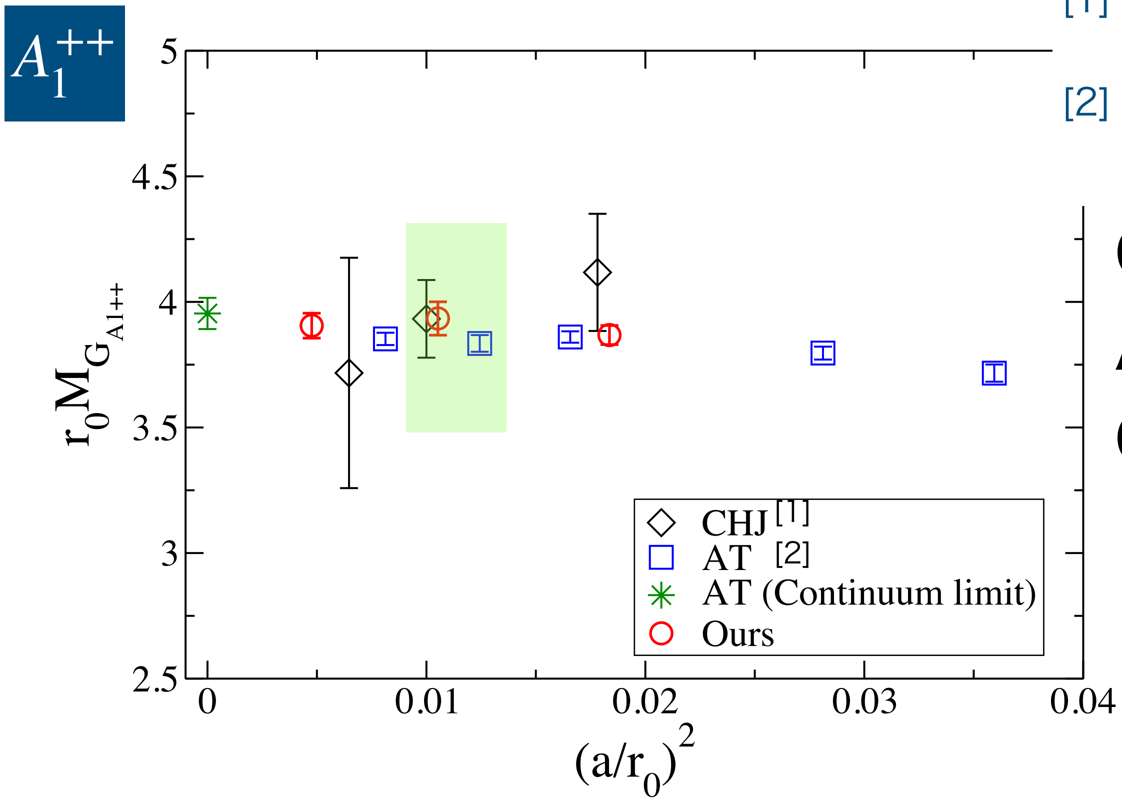
	$\beta(=6/g^2)$	Meas.	$\frac{\mathrm{ERR}}{\mathrm{AVE}} \times \sqrt{\mathrm{MEAS}}$
CHJ [1]	6.42	1958	1.739
AT [2]	6.338	80000	2.447
Ours	6.40	3000	0.925

Comparison



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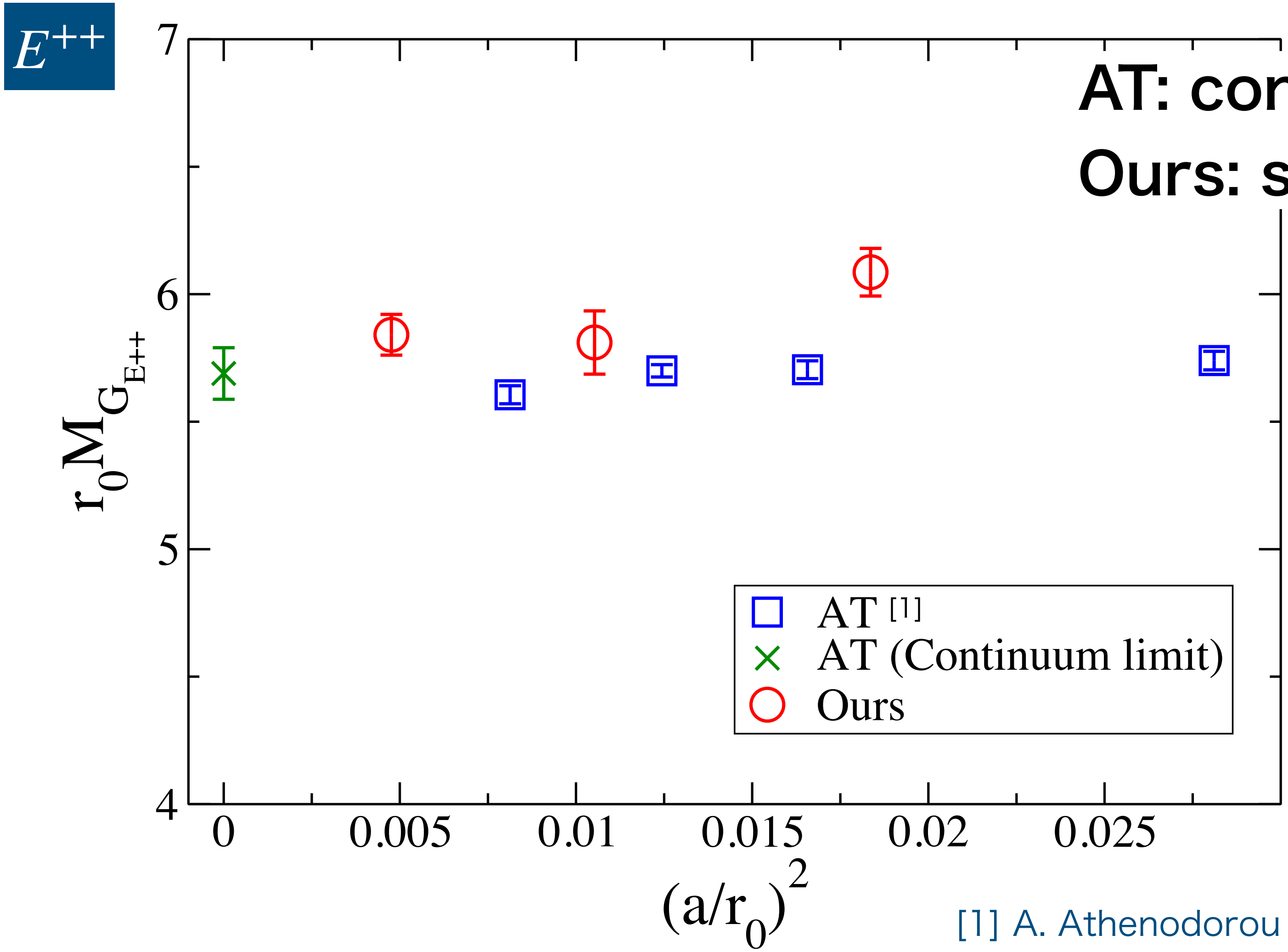
vs gradient flow: 2.6x effective

vs conventional smearing: 1.9x effective

Comparison

tensor glueball operator $(2^{++} = E^{++} \oplus T_2^{++})$



AT: conventional smearing

Ours: spatial flow

[1] A. Athenodorou and M. Teper, JHEP11 (2020) 172.

Summary

- The Yang-Mills gradient flow method is an alternative approach instead of smearing of the gauge fields
- A naive application of the original gradient flow method has some problems in glueball mass calculations with the longer flow time.
- “Spatial flow”, which is defined by the spatial gradient of the Wilson plaquette action, has no such problems.
- Spatial flow is more effective than original gradient flow or conventional smearing, while the stout smearing with a very large number of steps becomes almost identical with the spatial flow in the glueball spectroscopy