Nuclear force with LapH smearing

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with
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(for HAL QCD Collaboration)

Introduction: Nuclear Force from LQCD

- Nuclear force (NN int.) is the most important target for HAL QCD
  - application to atomic nuclei and neutron stars

- Yet there are several issues to be addressed
  - unphysical m_π
    ... Is deuteron bound at heavy m_π?
  - bad S/N ratio
  - spin-orbit force (parity-odd)

Yamazaki *et al.*, PRD**92** (2015) 014501
Ishii *et al.*, PLB**712** (2012) 437
Iritani *et al.*, JHEP**10** (2016) 101

Murano *et al.*, PLB**735** (2014) 19

- Elastic-state saturation is required to apply the HAL QCD method

$$R(\boldsymbol{r},t) \equiv \left\langle 0 \middle| T\, N(\boldsymbol{x},t)N(\boldsymbol{y},t)\bar{\mathcal{J}}(0) \middle| 0 \right\rangle \times e^{2m_N t}$$
$$\simeq \sum_{n=0}^{n_{\rm th}} A_n \psi(\boldsymbol{r}, W_n) e^{-\Delta W_n t}$$

- Improve the source operator s.t. $A_n = \langle n \,|\, \bar{J}(0) \,|\, 0 \rangle$ is small for $n > n_{th}$
  → LapH method (this talk) / one-end trick (c.f. Y.Akahoshi)

Free LapH Method

- Free Laplacian operator

Peardon *et al.*, PRD**80** (2009) 054506

$$\Delta(\boldsymbol{x},\boldsymbol{y}) = \sum_{k=1}^{3}\left\{\delta^3(\boldsymbol{y},\boldsymbol{x}+\hat{\boldsymbol{k}}) + \delta^3(\boldsymbol{y},\boldsymbol{x}-\hat{\boldsymbol{k}}) - 2\delta^3(\boldsymbol{y},\boldsymbol{x})\right\}$$

$$= \begin{pmatrix}\boldsymbol{v}_1 & \boldsymbol{v}_2 & \cdots & \boldsymbol{v}_N\end{pmatrix}\begin{pmatrix}\lambda_1 & & & \mathbf{0}\\ & \lambda_2 & & \\ & & \ddots & \\ \mathbf{0} & & & \lambda_N\end{pmatrix}\begin{pmatrix}\boldsymbol{v}_1^\dagger\\ \boldsymbol{v}_2^\dagger\\ \vdots\\ \boldsymbol{v}_N^\dagger\end{pmatrix}$$

take N_l low-lying states (plain waves)

- Laplacian Heaviside (LapH) smearing operator

$$\mathcal{S}(\boldsymbol{x},\boldsymbol{y}) = \sum_{n=1}^{N_l}\omega_l\, v_l(\boldsymbol{x})v_l^*(\boldsymbol{y})$$

- remove $\lambda_n > \Lambda$, as they are not significant for low-energy scattering

- ω_l : weight factor for mode l → check later

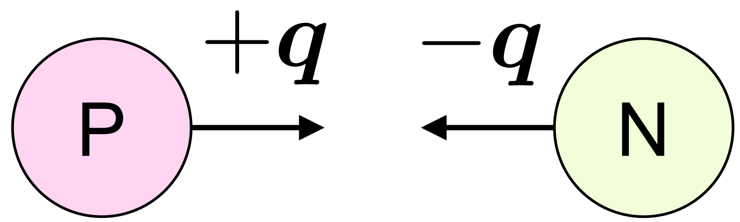
- Compt. cost is reduced from covariant LapH: $\mathcal{O}(N_l^4)$ → free LapH: $\mathcal{O}(N_l^3)$

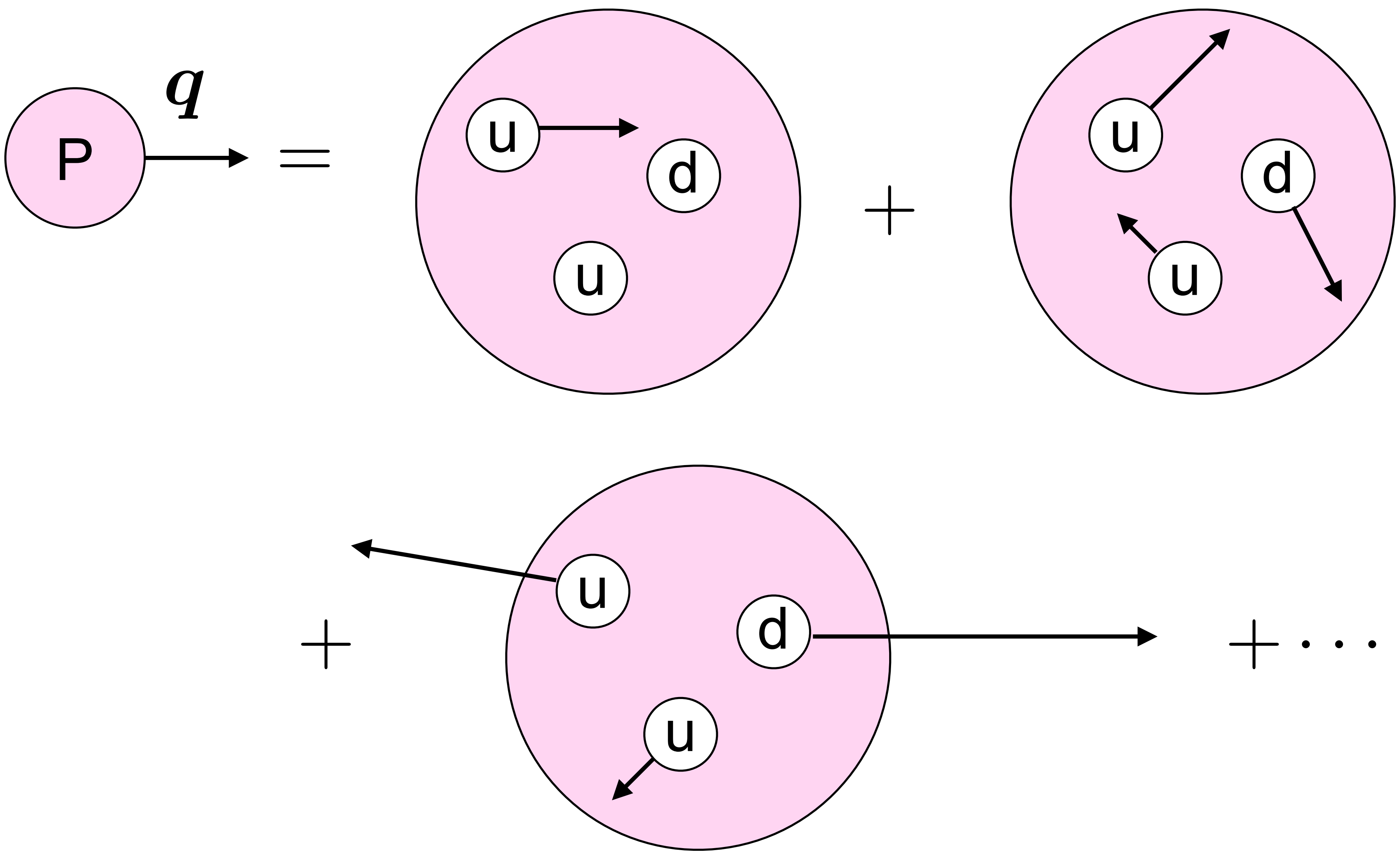
- $N_l = 1$ corresponds to wall-source

Free LapH Method

- Schematic picture of the LapH smeared 2N source operator

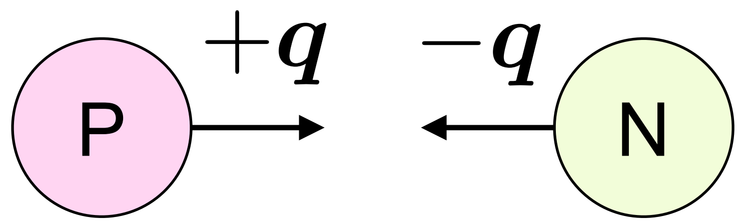
$\bar{J}(t) =$

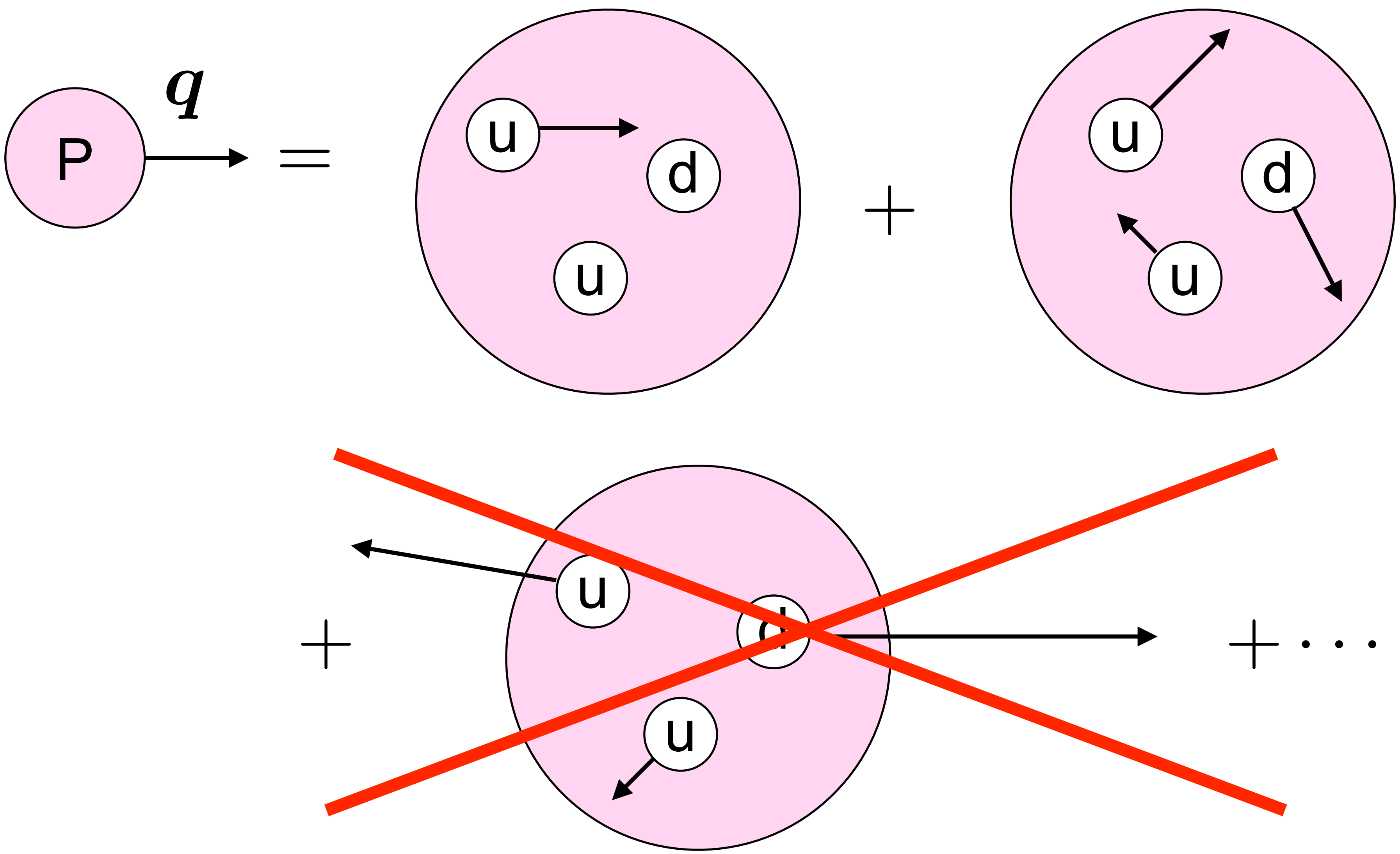




Free LapH Method

- Schematic picture of the LapH smeared 2N source operator

$\bar{J}(t) =$ 



Simulation Setup

- 2+1$_f$ confs by PACS-CS Collaboration — Aoki *et al.*, PRD**79** (2009) 034503
  - $L^3 \times T = 32^3 \times 64$
  - $La \simeq 2.9\,\mathrm{fm}$
  - $(\kappa_{ud}, \kappa_s) = (0.13700, 0.13640)$
  - $m_\pi = 701\,\mathrm{MeV}$
  - $m_N = 1581\,\mathrm{MeV}$
- Statistics: 399 confs × 2(forward/backward propagations)
- Relative momentum between source nucleons: $|\vec{q}\,| = 0$
- LapH modes
  - $N_l = 1 \Leftrightarrow |\vec{p}\,|^2 = 0$
  - $N_l = 7 \Leftrightarrow |\vec{p}\,|^2 \leq 1 \cdot (2\pi/L)^2$
  - $N_l = 19 \Leftrightarrow |\vec{p}\,|^2 \leq 2 \cdot (2\pi/L)^2$

Weight factor

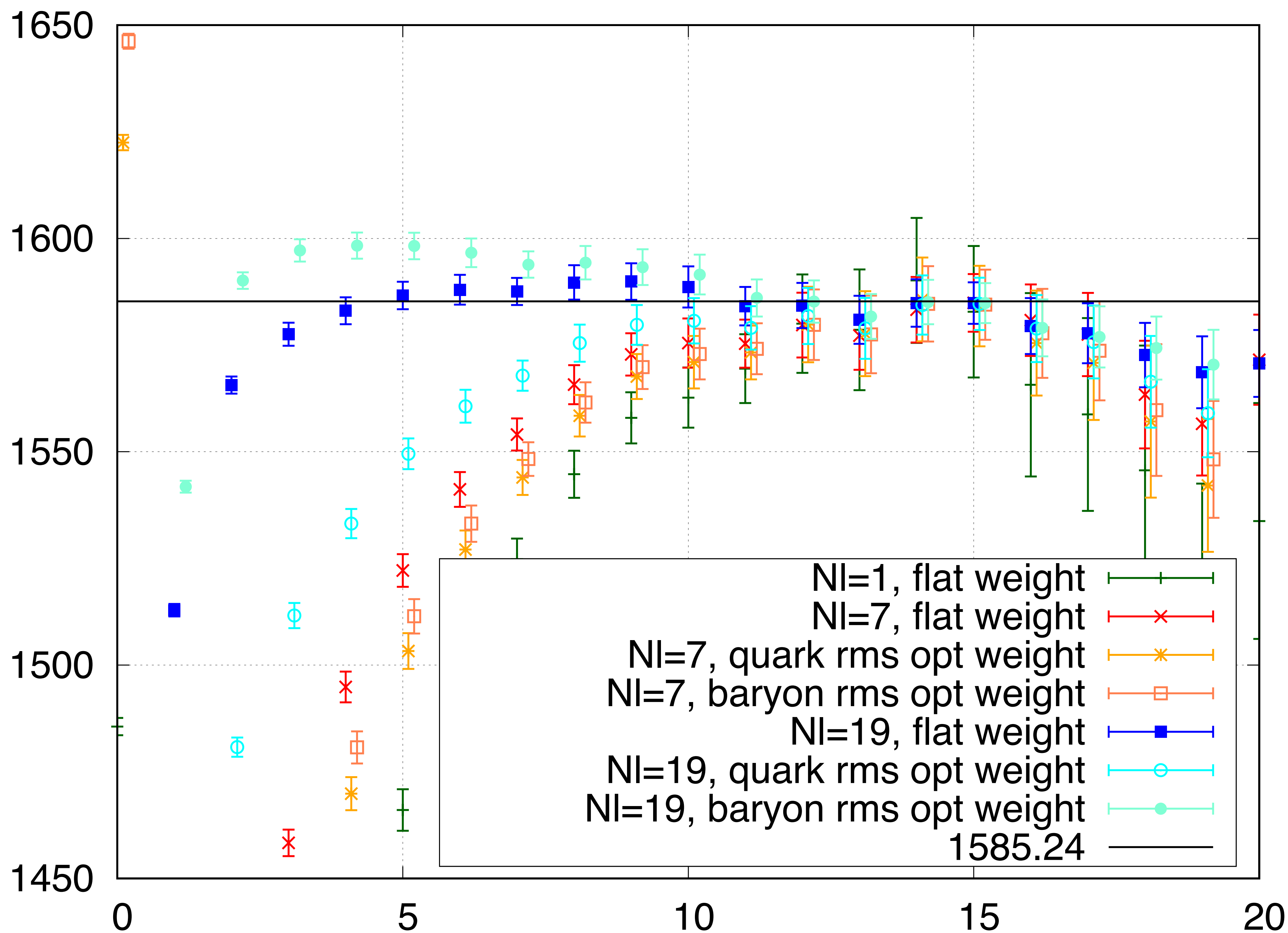
$$\mathcal{S}(\boldsymbol{x},\boldsymbol{y}) = \sum_{n=1}^{N_l} \omega_l \, v_l(\boldsymbol{x}) v_l^*(\boldsymbol{y})$$

weight factor can be tuned

We use 3 types

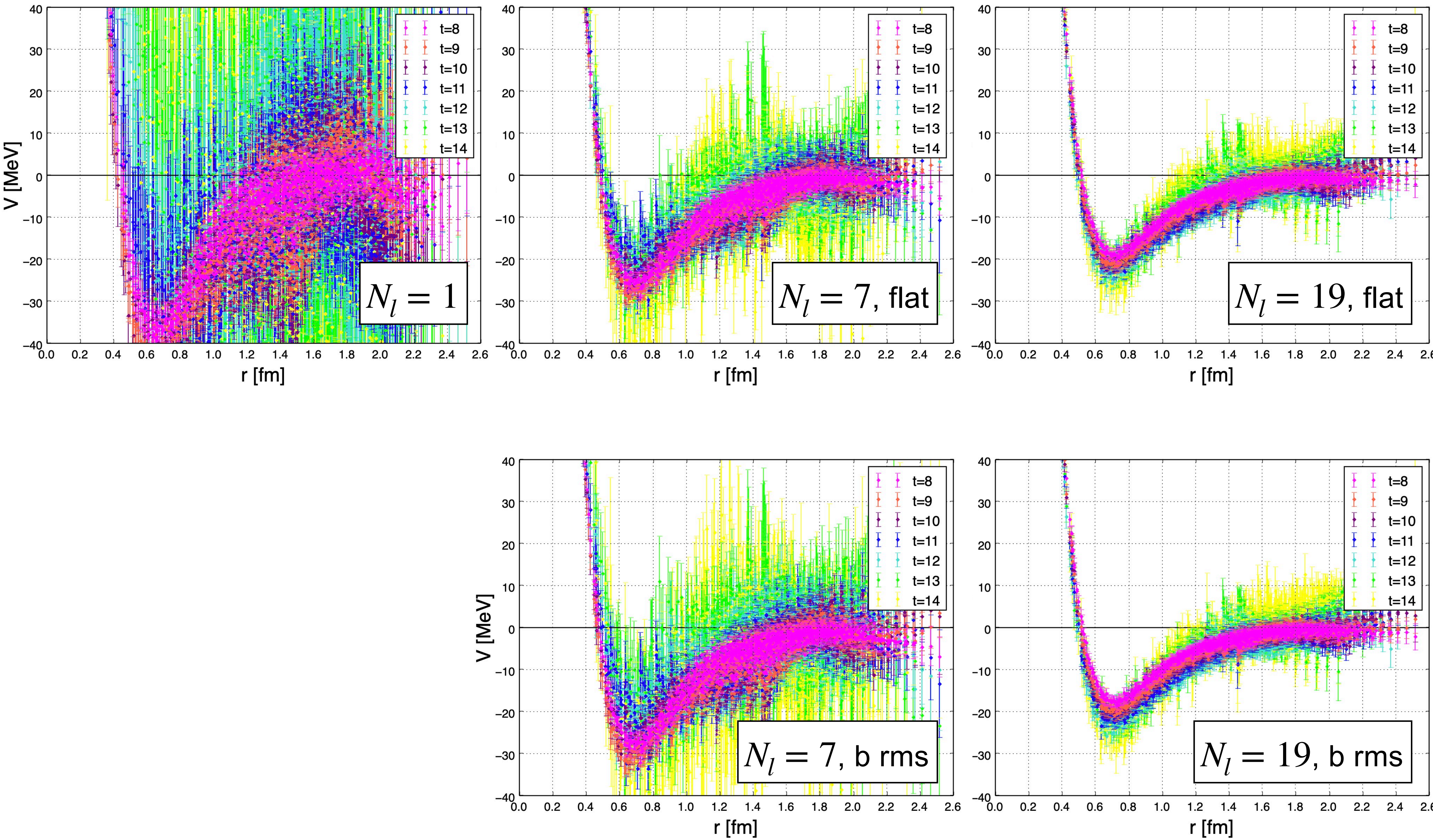
1. flat : $\omega_l = 1$ for all l
2. baryon rms optimized
   ... minimize baryon-level rms of the smearing function
   $$\sqrt{\langle r^2 \rangle^b} = \sqrt{\sum_{\boldsymbol{r}} \boldsymbol{r}^2 |\mathcal{S}(\boldsymbol{r},0)|^6}$$
3. quark rms optimized
   ... minimize $\sqrt{\langle r^2 \rangle^q} = \sqrt{\sum_{\boldsymbol{r}} \boldsymbol{r}^2 |\mathcal{S}(\boldsymbol{r},0)|^2}$

Results: Nucleon Effective Mass

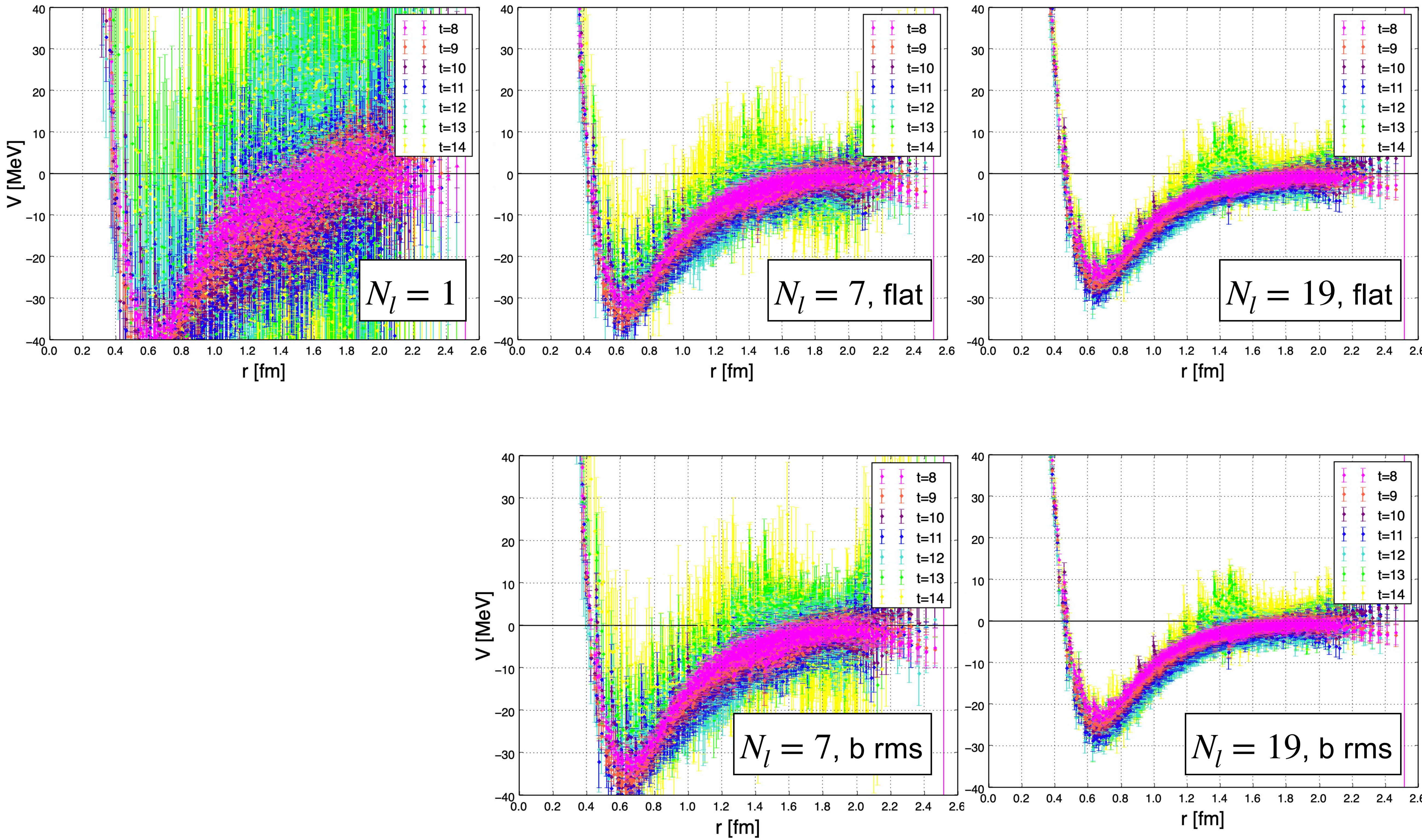


"flat" or "baryon rms opt" will be good

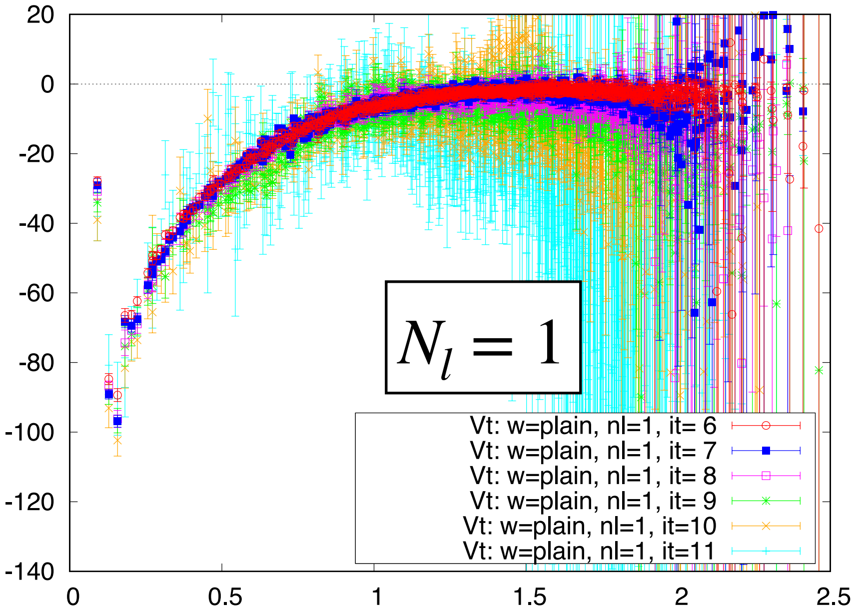
Results: NN Potential ($^1S_0$, I=1)

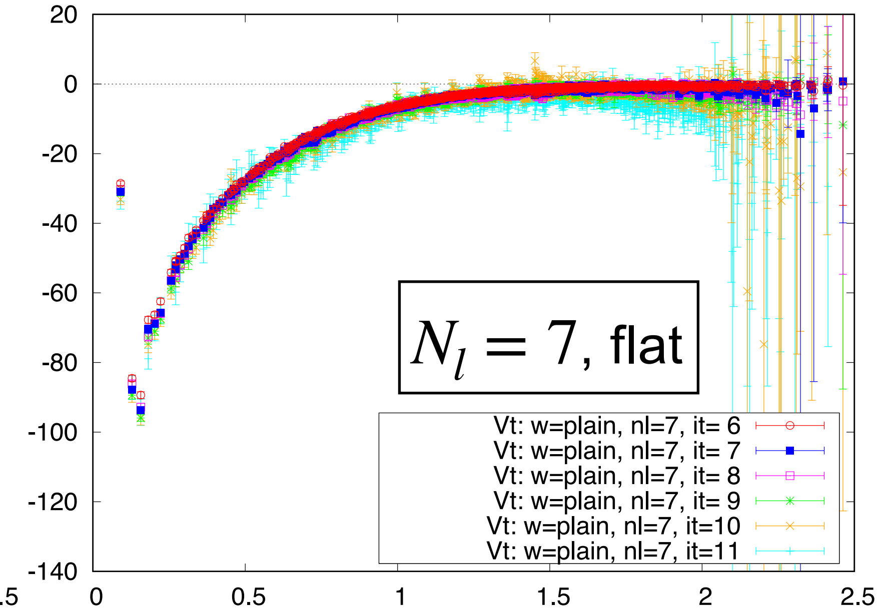


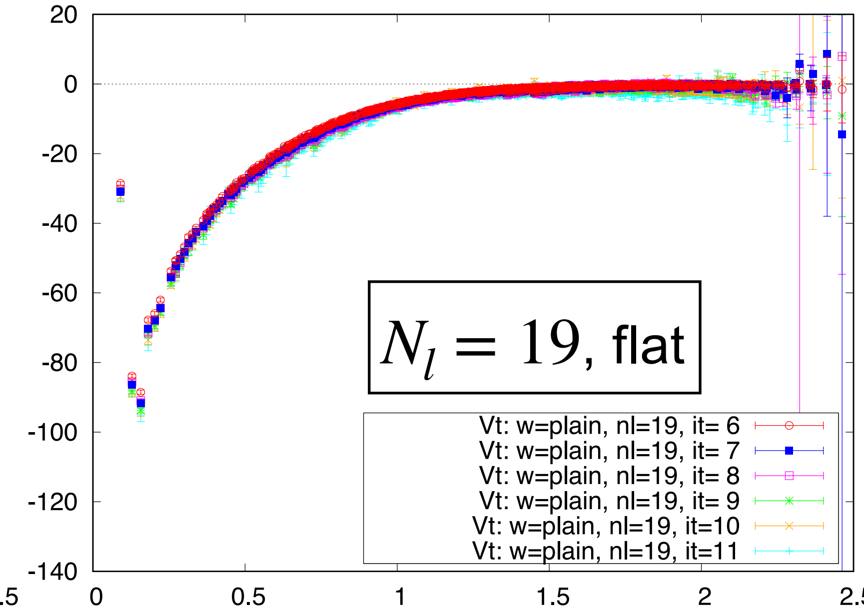
Results: NN Potential ($^3S_1$+$^3D_1$ central, I=0)

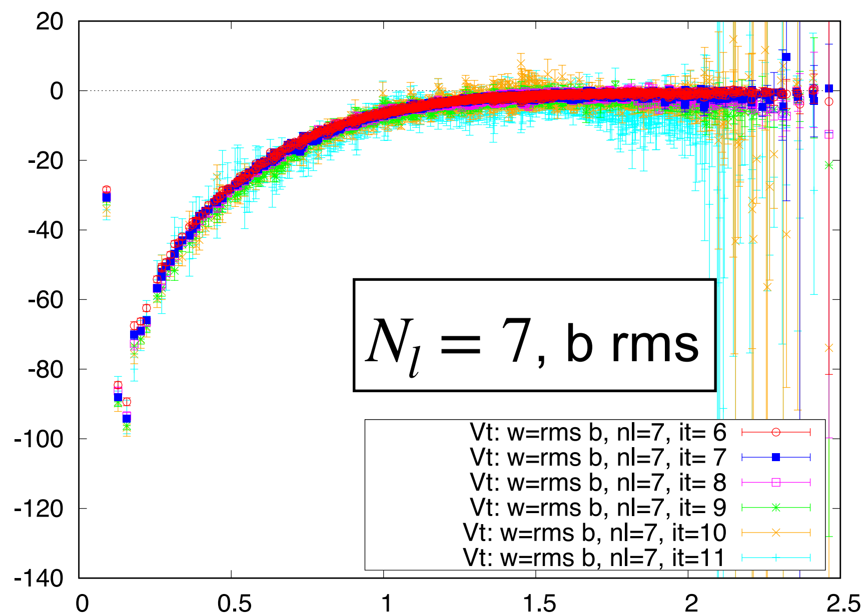


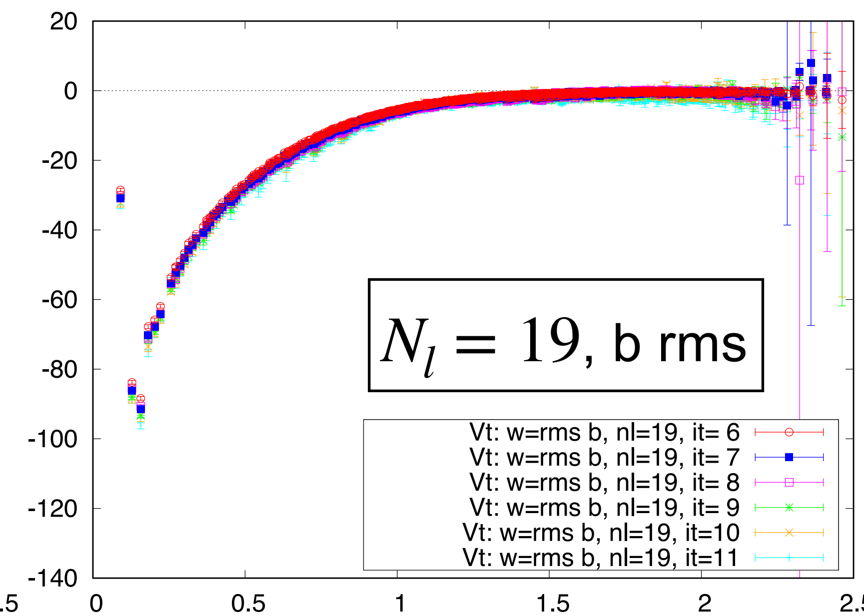
Results: NN Potential ($^3S_1$+$^3D_1$ tensor, I=0)











Summary and Outlook

- The nuclear force can be determined with improved accuracy by combining the HAL QCD method and the LapH smearing
- We use the free Laplacian instead of the covariant one: the computational cost is reduced as $\mathcal{O}(N_l^4) \to \mathcal{O}(N_l^3)$
- As the number of modes $N_l = 1,7,9$ increases, S/N becomes drastically better
- By tuning the weight factors, the effective mass can be improved while the potential cannot
- We will further check the systematics of this method & use this for parity-odd nuclear forces