Finite volume analysis on systematics of the derivative expansion in HAL QCD method

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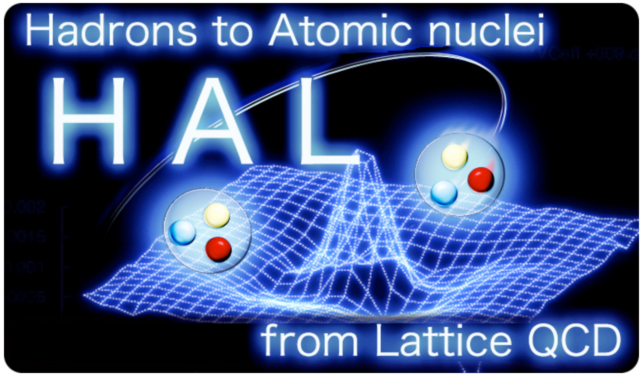


Based on arXiv:2102.00181 (to appear in PRL)

by **Y. Lyu, H. Tong**, T. Sugiura, S. Aoki,
TD, T. Hatsuda, J. Meng, T. Miyamoto

[HAL QCD method]

- Nambu-Bethe-Salpeter (NBS) wave function

$$\boxed{\psi(\vec{r}) = \langle 0 | N(\vec{x}+\vec{r}) N(\vec{x}) | N(k) N(-k); W \rangle}$$

M.Luscher, NPB354(1991)531

C.-J.Lin et al., NPB619(2001)467

N.Ishizuka, PoS LAT2009 (2009) 119

CP-PACS Coll., PRD71(2005)094504

S. Aoki et al., PRD88(2013)014036

Gongyo-Aoki, PTEP2018(2018) 093B03

  - phase shift at asymptotic region

$$\psi(r) \simeq A \frac{\sin(kr - l\pi/2 + \boxed{\delta(k)})}{kr}$$

- "Potential" in QFT ←→ faithful to phase shift !

$$(\nabla^2 + k^2)\psi(\boldsymbol{r}) = m \int d\boldsymbol{r}' U(\boldsymbol{r}, \boldsymbol{r}')\psi(\boldsymbol{r}')$$

  - U(r,r'): E-independent & non-local potential
    - "Proof of Existence" : Explicit form can be given as

$$U(\boldsymbol{r}, \boldsymbol{r}') = \frac{1}{m} \sum_{n,n'}^{n_{\rm th}} (\nabla_{\boldsymbol{r}}^2 + k_n^2)\psi_n(\boldsymbol{r}) \mathcal{N}_{nn'}^{-1} \psi_{n'}^*(\boldsymbol{r}') \qquad \mathcal{N}_{nn'} = \int d\boldsymbol{r} \psi_n^*(\boldsymbol{r}) \psi_{n'}(\boldsymbol{r})$$

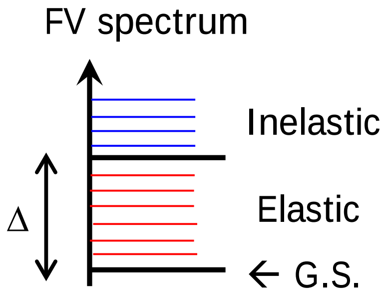
[Time-dependent HAL QCD method]

E-indep of potential U(r,r') ➔ *(excited) scatt states share the same U(r,r')*

*They are **not contaminations**, **but signals***

$$\int d\boldsymbol{r}' U(\boldsymbol{r},\boldsymbol{r}')R(\boldsymbol{r}',t) = (-\frac{\partial}{\partial t} + \frac{1}{4m}\frac{\partial^2}{\partial t^2} - H_0)R(\boldsymbol{r},t)$$

	Ground state	Excited (elastic)	Excited (inelastic)
HAL method	Signal	Signal	Noise
Direct method	Signal	Noise	Noise



Non-locality of U(r,r') ← derivative expansion

Okubo-Marshak(1958)

$U(\boldsymbol{r},\boldsymbol{r}') = \sum_n V_n(\boldsymbol{r})\nabla^n \delta(\boldsymbol{r}-\boldsymbol{r}')$

Expansion w.r.t. $\nabla/\Lambda, \ (\Lambda \simeq \Lambda_{\mathrm{QCD}}, \Delta)$

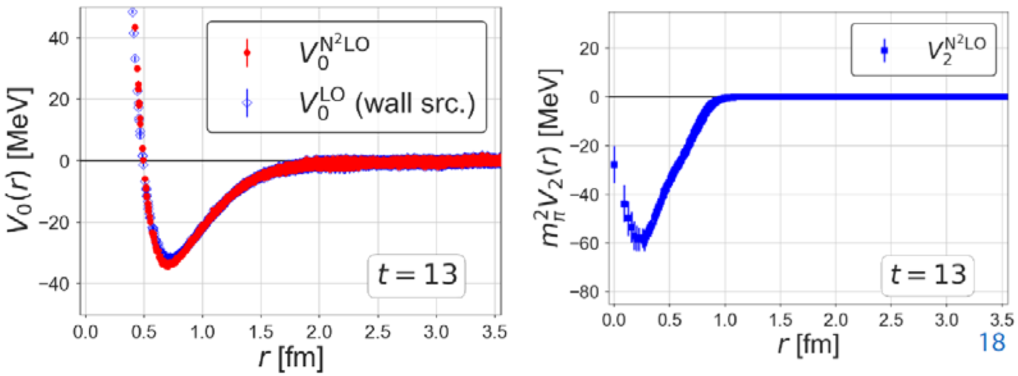
$$U(\boldsymbol{r},\boldsymbol{r}') = \left[\underset{\text{LO}}{V_c(r)} + \underset{\text{LO}}{S_{12}V_T(r)} + \underset{\text{NLO}}{\boldsymbol{L}\cdot\boldsymbol{S}V_{LS}(r)} + \underset{\text{NNLO}}{\mathcal{O}(\nabla^2)}\right]\delta(\boldsymbol{r}-\boldsymbol{r}')$$

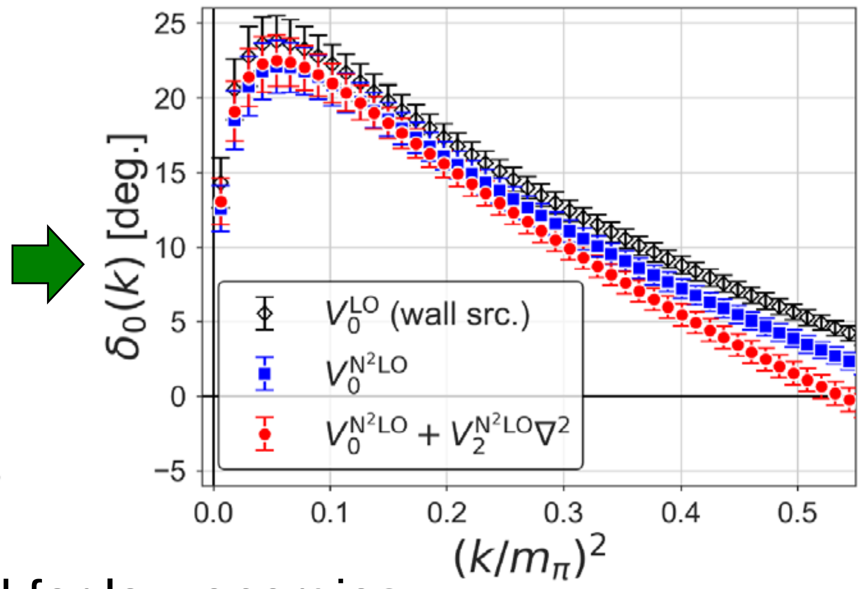
For phase shifts at energy region close to (far from) region which correlator couples to, truncation err of the expansion is expected to be small (large)

Control/Check sys err in the derivative expansion

- Calc correlators w/ two different source operators
  - Indep info for NLO pot (←→ Info at different energy regions)
  - Check the consistency of pot, or determine LO, NLO pot explicitly

wall src. –> small $V_2\nabla^2$ correction
smeared src. –> large $V_2\nabla^2$ correction





➔ Convergence is good for low energies

T. Iritani et al. (HAL) PRD99(2019)014514; See also K.Murano et al., PTP125(2011)1225

- Due to calc cost, check on t-dep of pot is often employed
  - Cost is small, but t-dep may be hidden in increasing stat noises

New method using FV spectrum w/ HAL pot

T. Iritani et al. (HAL), JHEP03(2019)007

- FV spectrum from temporal corr (w/ Luscher's formula)
  - No issue on the derivative expansion of potential
  - Naïve plateau fitting is unreliable ("pseudo-plateau issue")

T. Iritani et al., (HAL) ('16, '17, '19, '19)

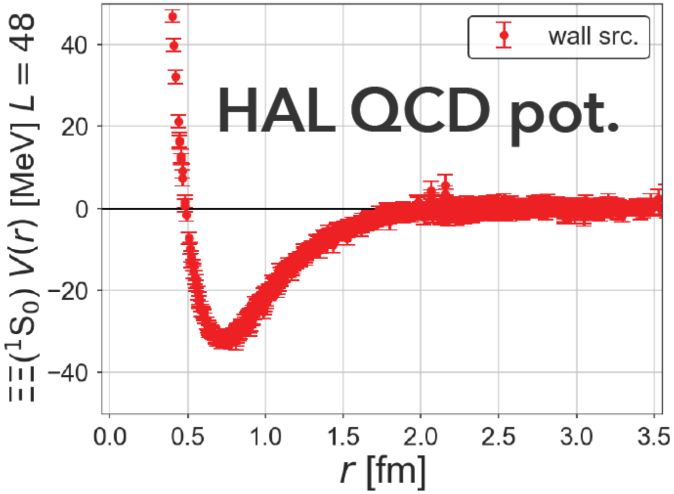
See Mainz('19,'21), CalLat('21) for variational study

[Utilize HAL pot to overcome the pseudo-plateau issue]

- Construct optimized op for each FV eigen state by HAL pot
  ➔ optimized temporal corr for each FV eigen state

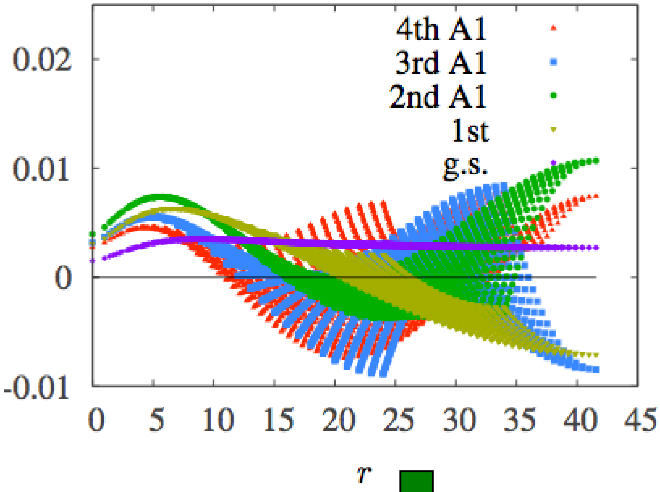
- Consistency check between
  (1) FV spectrum from optimized temporal corr
  (2) FV spectrum from HAL QCD pot
  - If consistent ➔ sys err in HAL pot is well under control

New method using FV spectrum w/ HAL pot



Solve Schrodinger eq. in Finite V

Eigen wave functions



Solve in FV

Eigen energies = FV spectrum

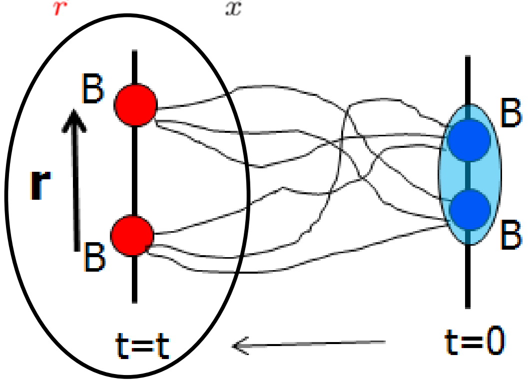
n-th A1	ΔE_n [MeV]
0	-2.58(1)
1	52.49(2)
2	112.08(2)
3	169.78(2)
4	224.73(1)

Consistency Check!

FV spectrum from Optimized temporal corr

Optimized (sink) op

$$\mathcal{J}^{2B}_{\rm sink} = \sum_{\vec r} \psi^\dagger(r) \sum_{\vec x} B(\vec r + \vec x) B(\vec x)$$

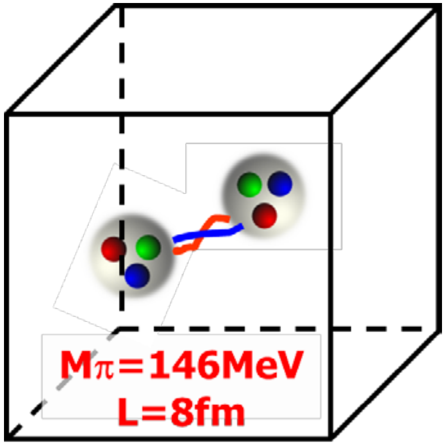


Eigen w.f. can be also used to analyze non-optimized temporal corr

Lattice QCD Setup

- **Nf = 2 + 1 gauge configs**
  - clover fermion + Iwasaki gauge w/ stout smearing
  - $V=(8.1\mathrm{fm})^4$, a=0.085fm (1/a = 2.3 GeV)
  - m(pi) ~= 146 MeV, m(K) ~= 525 MeV
  - #traj ~= 2000 generated

PACS Coll., PoS LAT2015, 075



  - (Quenched) Charm quark w/ RHQ action

Y. Namekawa (PACS), PoS LAT2016, 125

  - **Wall quark source ➔ LO potential in the derivative expansion**

- **Channels in this study**
  - $\Omega\Omega(=\Omega_{sss}\Omega_{sss})$ ($^1S_0$)
  - $\Omega_{ccc}\Omega_{ccc}$ ($^1S_0$)

⇨ Bound states near unitary limit in QCD

S. Gongyo et al. (HAL), PRL120(2018)212001

Y. Lyu, H. Tong et al., arXiv:2102.00181 (PRL)

➔ Talk by Y. Lyu

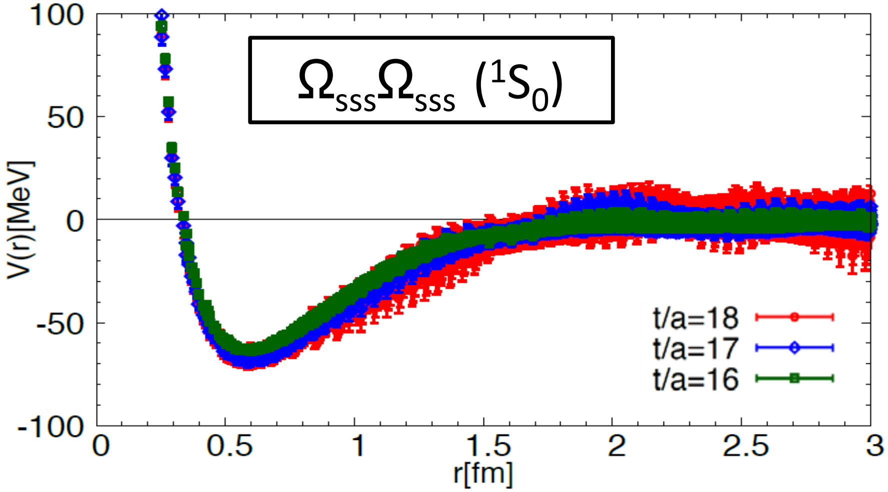
c.f.

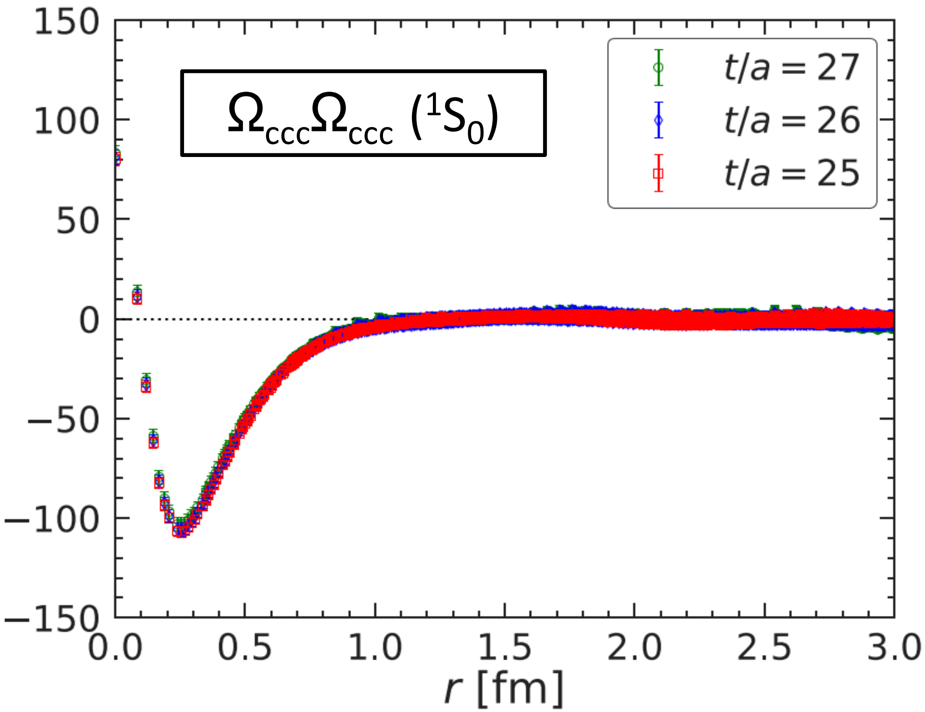
$N\Omega$ T. Iritani et al. (HAL), PLB792(2019)284

$\Lambda\Lambda-N\Xi$ K. Sasaki et al. (HAL), NPA998(2020)121737

$N\Lambda-N\Sigma$ ➔ Talks by H. Nemura / T. M. Doi

HAL QCD Potential





If we solve Schrodinger equation in infinite V, systems are bound

$$B^{(\mathrm{QCD})} = 1.6(0.6)\left({}^{+0.7}_{-0.6}\right)\ \mathrm{MeV}$$

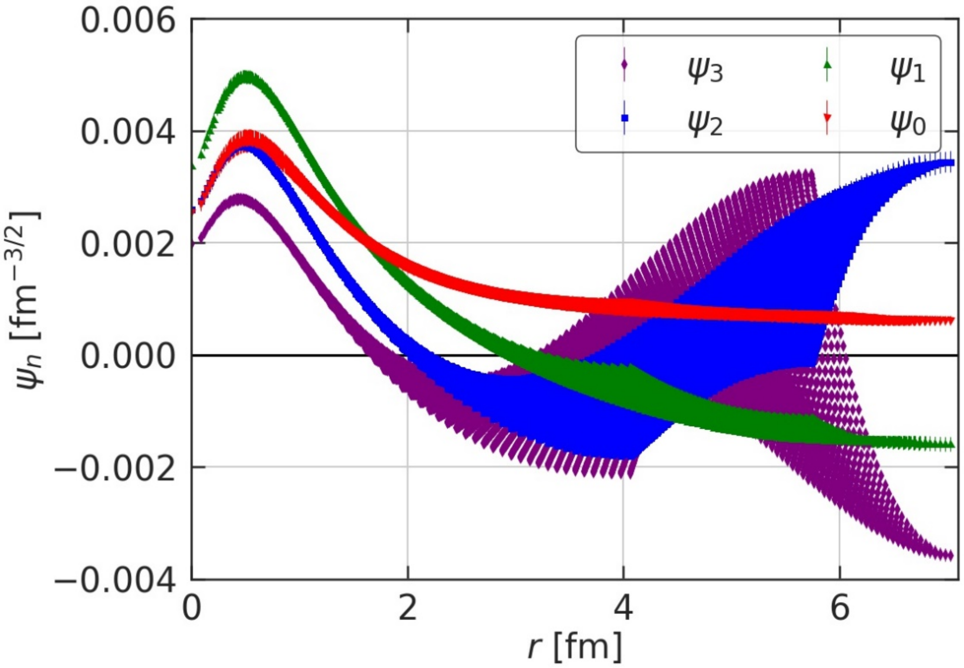
$$\sqrt{\langle r^2\rangle}^{(\mathrm{QCD})} = 3.3(0.5)\left({}^{+0.8}_{-0.3}\right)\ \mathrm{fm}$$

$$B^{(\mathrm{QCD})} = 5.68(0.77)\left({}^{+0.46}_{-1.02}\right)\ \mathrm{MeV}$$

$$\sqrt{\langle r^2\rangle}^{(\mathrm{QCD})} = 1.13(0.06)\left({}^{+0.08}_{-0.03}\right)\ \mathrm{fm}$$

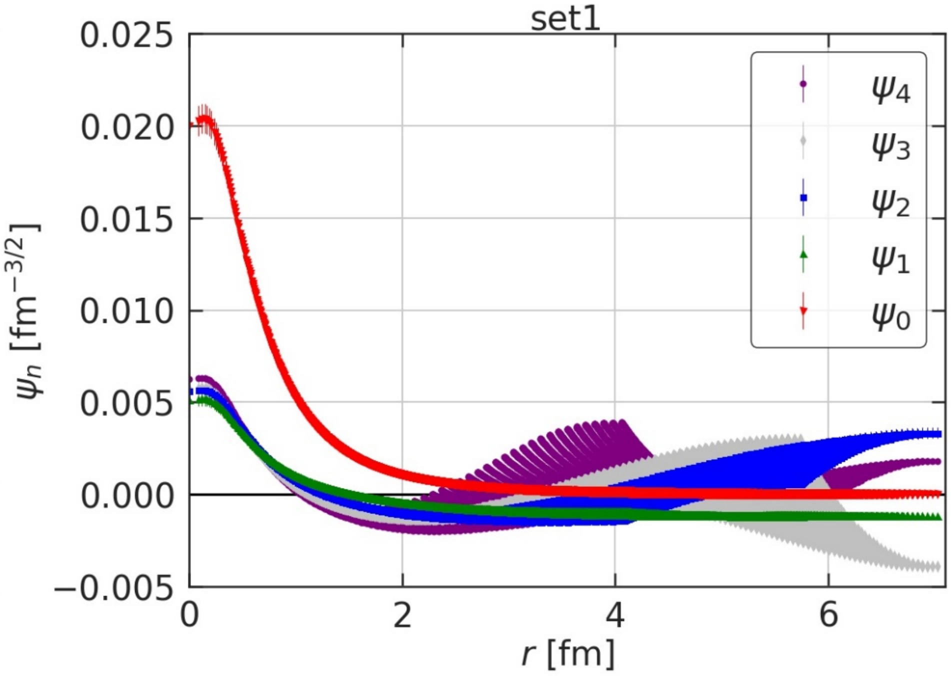
Eigen wave functions and energies in finite V

$\Omega_{sss}\Omega_{sss}$ ($^1S_0$)



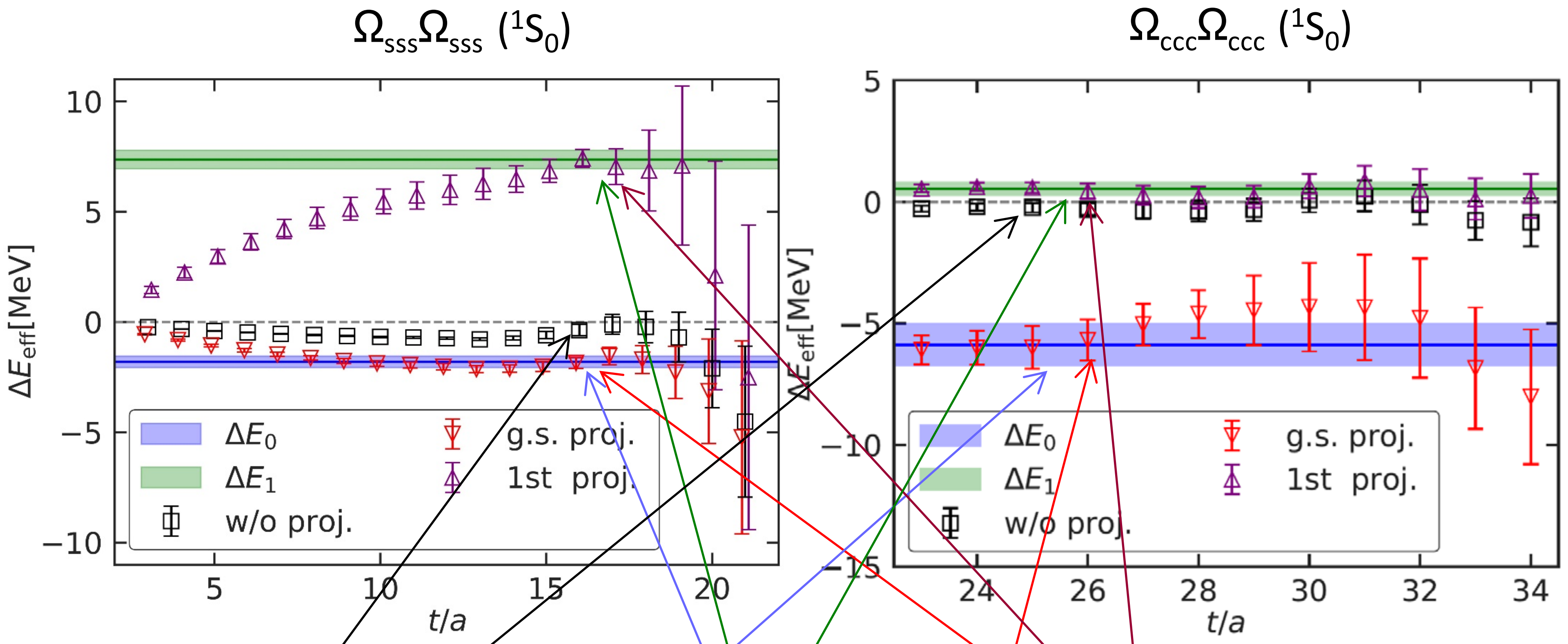
Level	ΔE_n [MeV]
G.S.	-1.81(0.26)
1st	7.35(0.42)
2nd	22.55(0.54)
3rd	39.34(0.62)

$\Omega_{ccc}\Omega_{ccc}$ ($^1S_0$)



Level	ΔE_n [MeV]
G.S.	-5.87(0.85)
1st	0.53(0.25)
2nd	6.58(0.29)
3rd	12.44(0.36)

Comparison of FV spectrum (effective energy shift)


$\Omega_{sss}\Omega_{sss}$ ($^1S_0$)

$\Omega_{ccc}\Omega_{ccc}$ ($^1S_0$)

ΔE of g.s. from HAL pot

ΔE of 1st from HAL pot

⟷

ΔE from opt. t-corr for g.s.

ΔE from opt. t-corr for 1st.

Good agreement!

Sys err in HAL pot under control

ΔE from un-optimized t-corr
(usual one in direct method)

➔ Significant deviation from the correct ΔE of g.s.

Decomposition of correlator

In the HAL QCD method,

the following R-corr is used to obtain the potential

$$\begin{aligned} R(\boldsymbol{r},t) &= \langle 0|\mathrm{T}[B(\boldsymbol{r},t)B(\boldsymbol{0},t)\overline{\mathcal{J}_{\mathrm{src}}(0)}]|0\rangle/(G_{2\mathrm{pt}}(t))^2 \\ &= \sum_n a_n \psi_n(\boldsymbol{r}) e^{-\Delta E_n t} \end{aligned}$$

Since we know ψ_n, ΔE_n from FV eigenmodes, a_n can be determined

In the direct method,

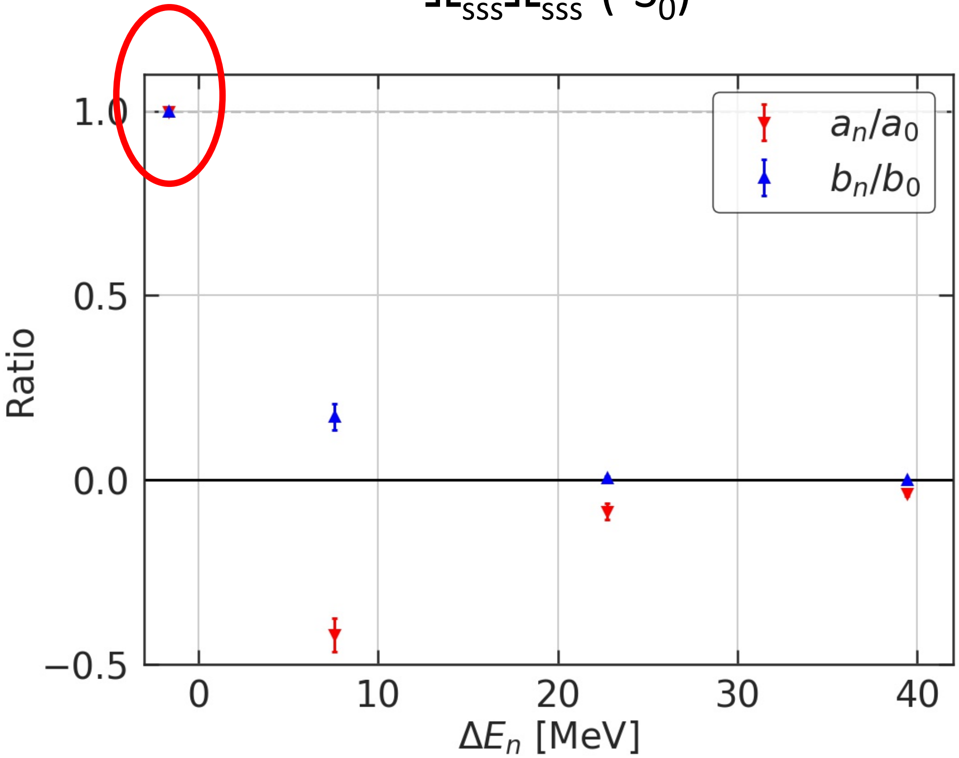
the following (non-optimized) temporal corr is usually used

$$R(\boldsymbol{r},t) = \sum_{\boldsymbol{r}} R(\boldsymbol{r},t) = \sum_n b_n e^{-\Delta E_n t}$$

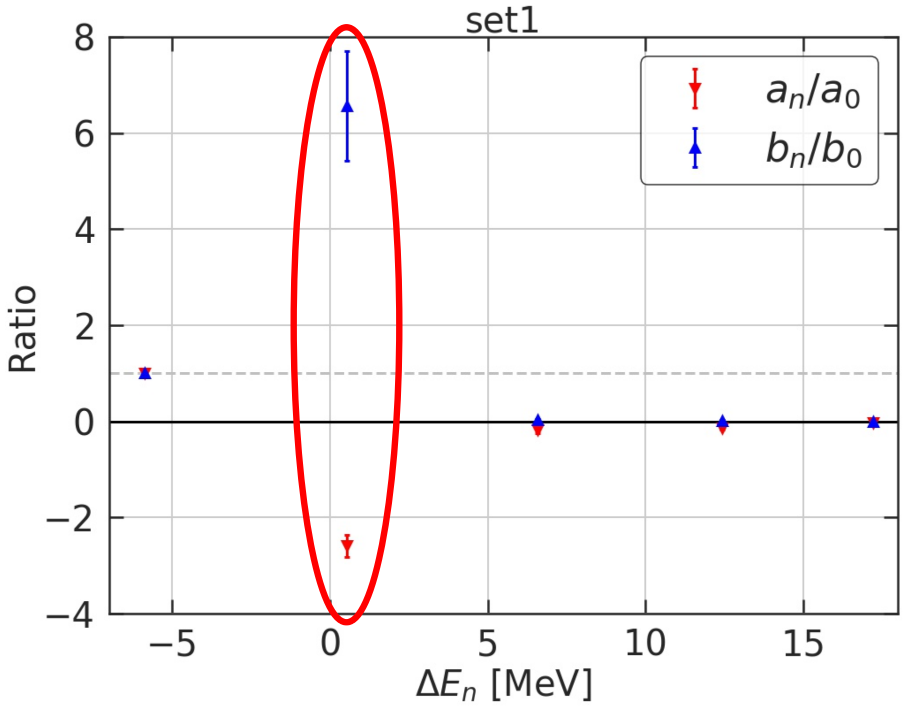
$$b_n = \sum_{\boldsymbol{r}} a_n \psi_n(\boldsymbol{r})$$

← Each baryon op @ sink is projected to zero-momentum

Decomposition of correlator


$\Omega_{sss}\Omega_{sss}$ $(^1S_0)$

Both of R(r,t) and R(t): G.S. dominant


$\Omega_{ccc}\Omega_{ccc}$ $(^1S_0)$

Both of R(r,t) and R(t): 1st. dominant

Potential can be reliably extracted regardless
whether R-corr is dominated by G.S. or 1st excited state

Decomposition params can be also compared w/ output from variational study

Summary

- HAL QCD method
  - Powerful method to extract hadron-hadron interactions w/o requiring ground state saturation
  - Derivative expansion for non-locality of the potential ➔ truncation error should be studied

- Self-consistent check using FV spectrum
  - HAL potential → FV eigenmodes → optimized operator
    - “pseudo-plateau issue” can be resolved
  - Consistency check between FV spectrum from temporal correlator and that from potential ➔ convergence of derivative expansion can be studied
  - $\Omega_{sss}\Omega_{sss}$ and $\Omega_{ccc}\Omega_{ccc}$ near phys point ➔ good convergence confirmed
  - Clear evidence that HAL potential can be reliably extracted regardless correlator is dominated by G.S. or excited states

- Fugaku (2021-)
  - Hadron interactions on the physical point w/ self-consistent check