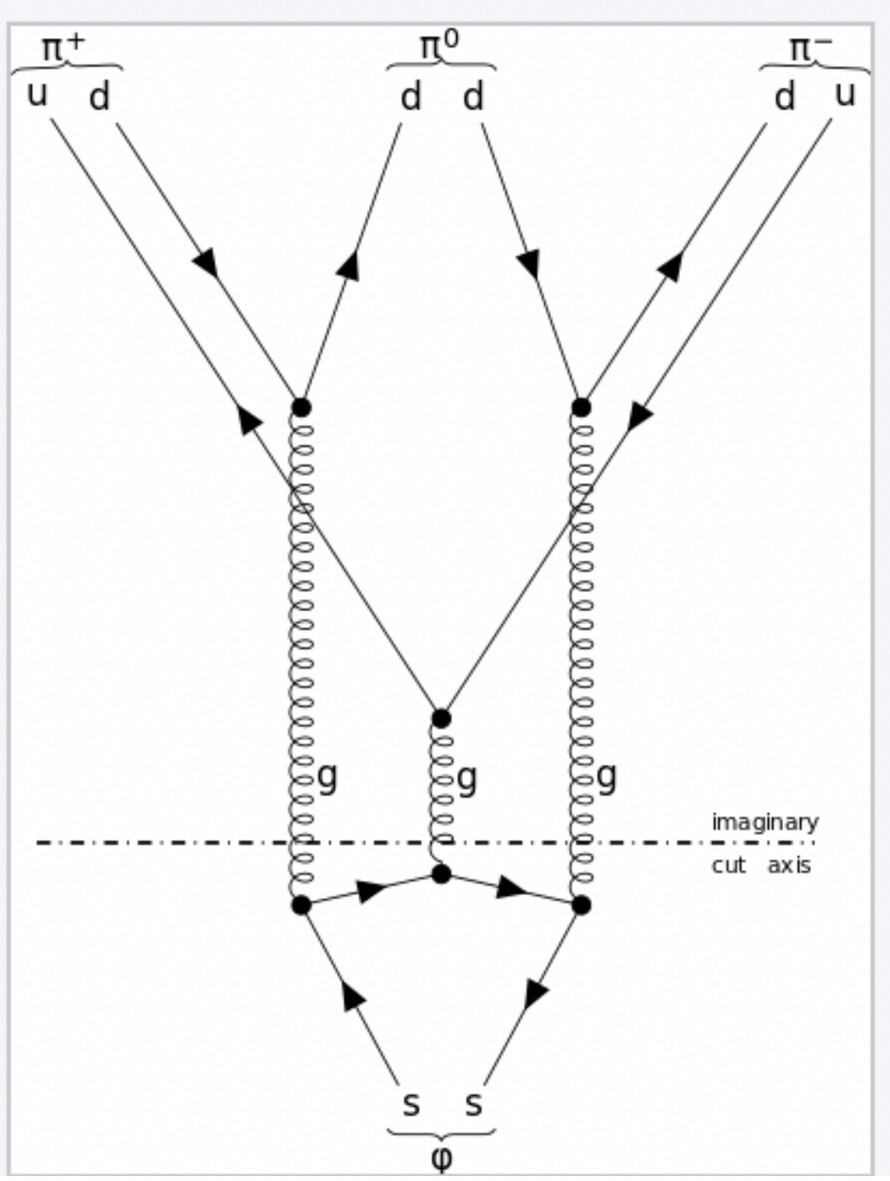
Lattice 2021

Excited J^{--} resonances from meson-meson scattering at the SU(3) flavor point in lattice QCD.

CHRIS JOHNSON

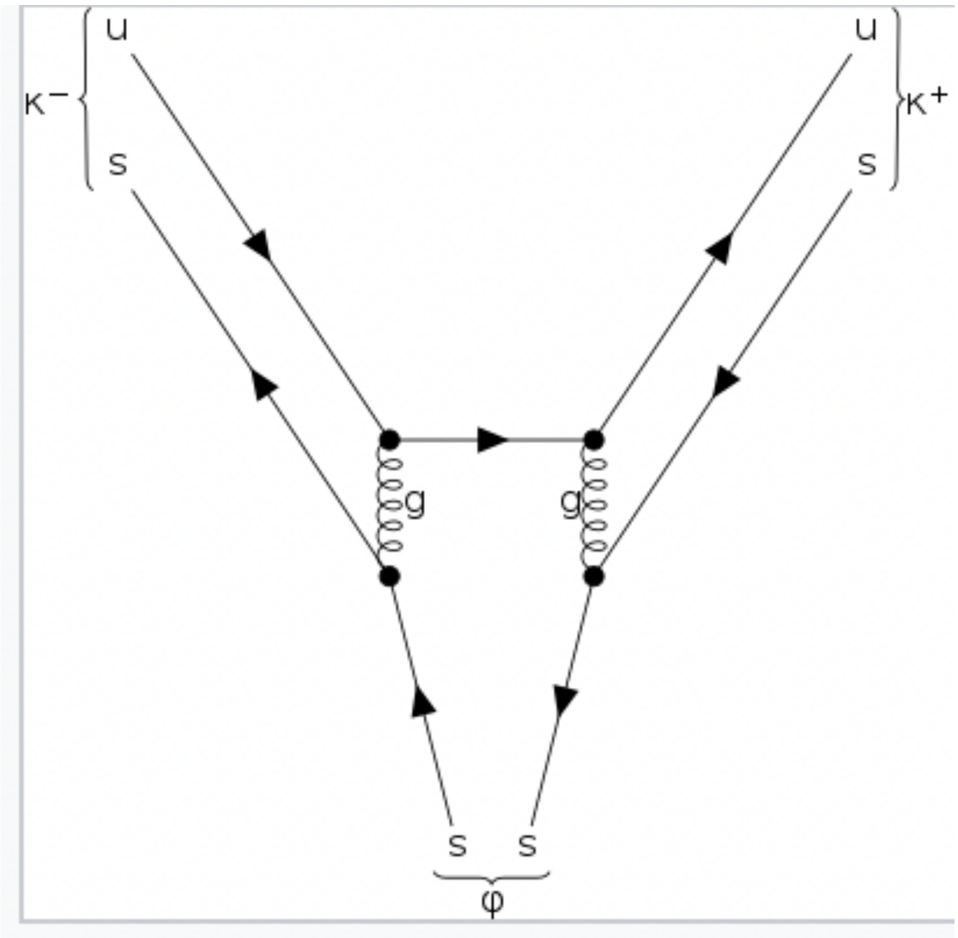
Experimental Status



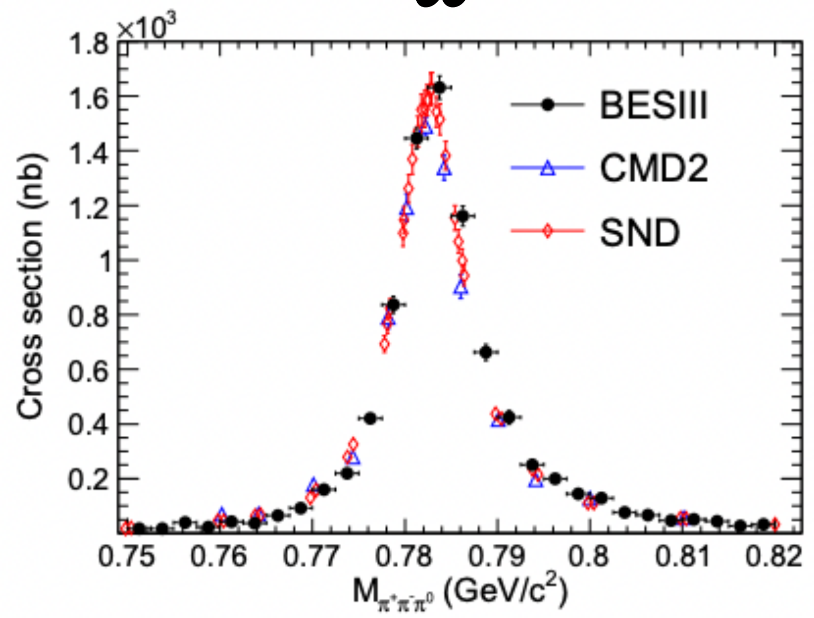
The lightest vector ($J^{PC} = 1^{--}$) mesons are the $\rho(770)$, $\omega(782)$, $\phi(1020)$

States are well understood in e⁺e⁻ annihilation due to their narrow widths and little background into decay into simple states like $\pi\pi$, $\pi\pi\pi$, $K\bar{K}$.

ω and ϕ states separated via decay channels $\pi\pi\pi$ vs $K\bar{K}$ (OZI)

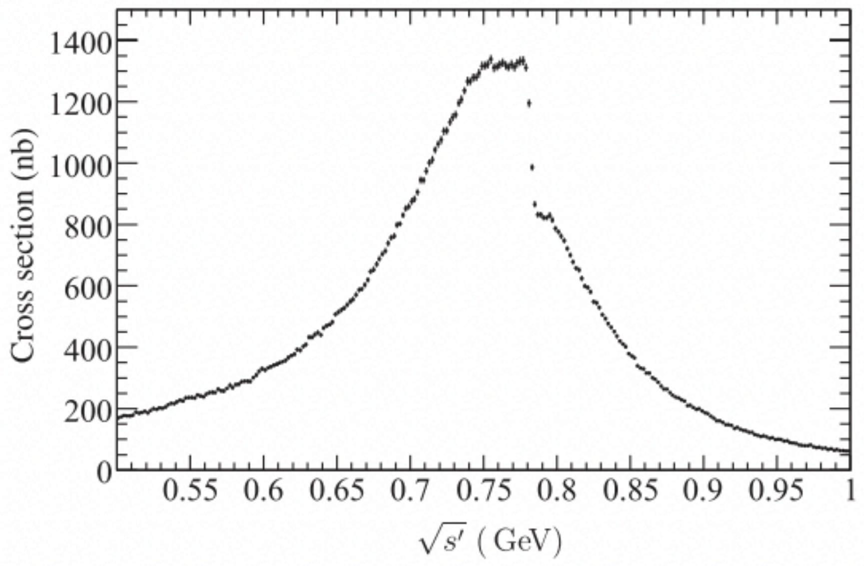


ω



arXiv:1912.11208v1 BES III

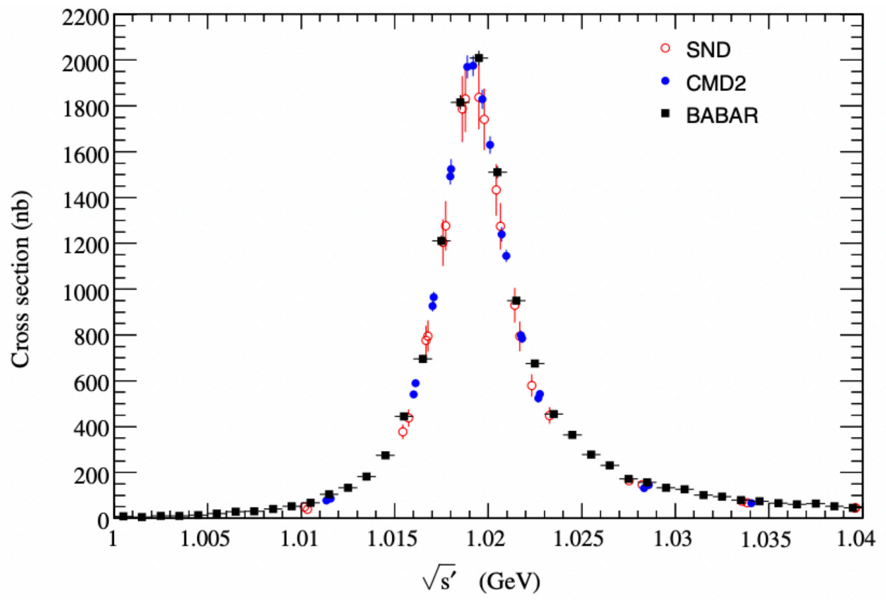
ρ



BaBar

PR D86 032013 J.P. Lees et al.

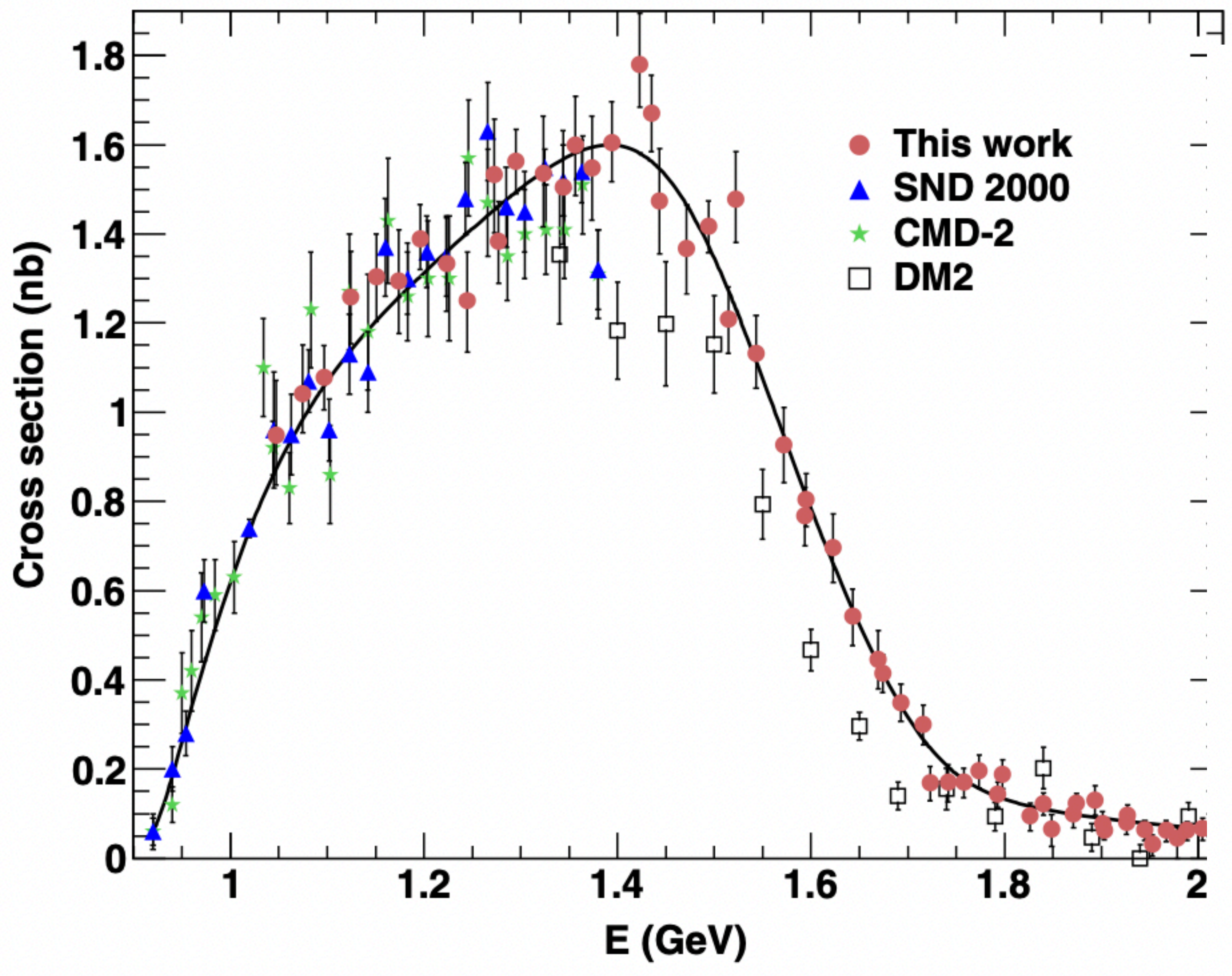
ϕ



PR D88, 032013 J.P. Lees et al.

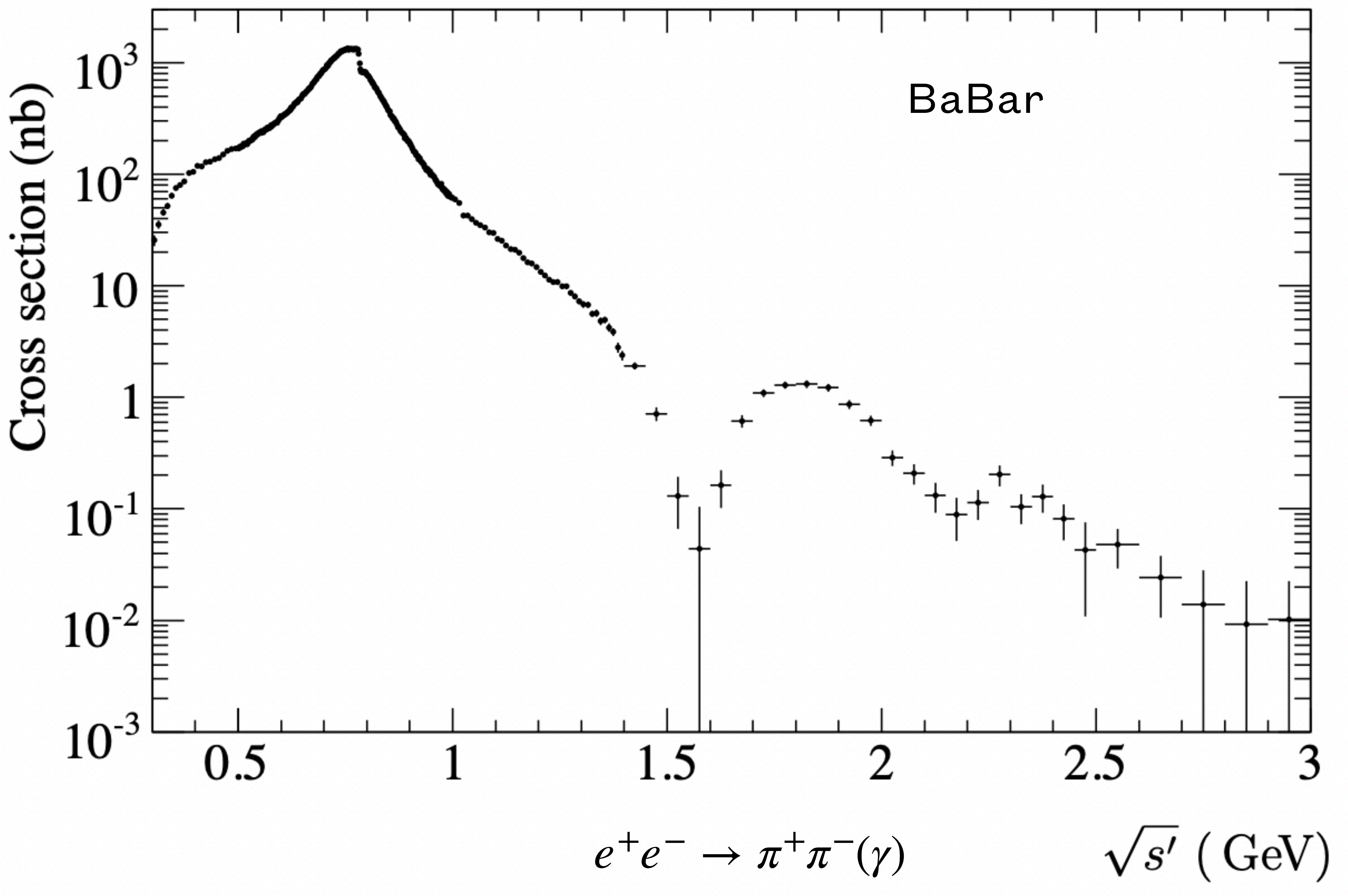
Excited light vector mesons (I=1)

Phys. Rev. D 94, 11201 (2016)



$e^+e^- \to \omega\pi^0$

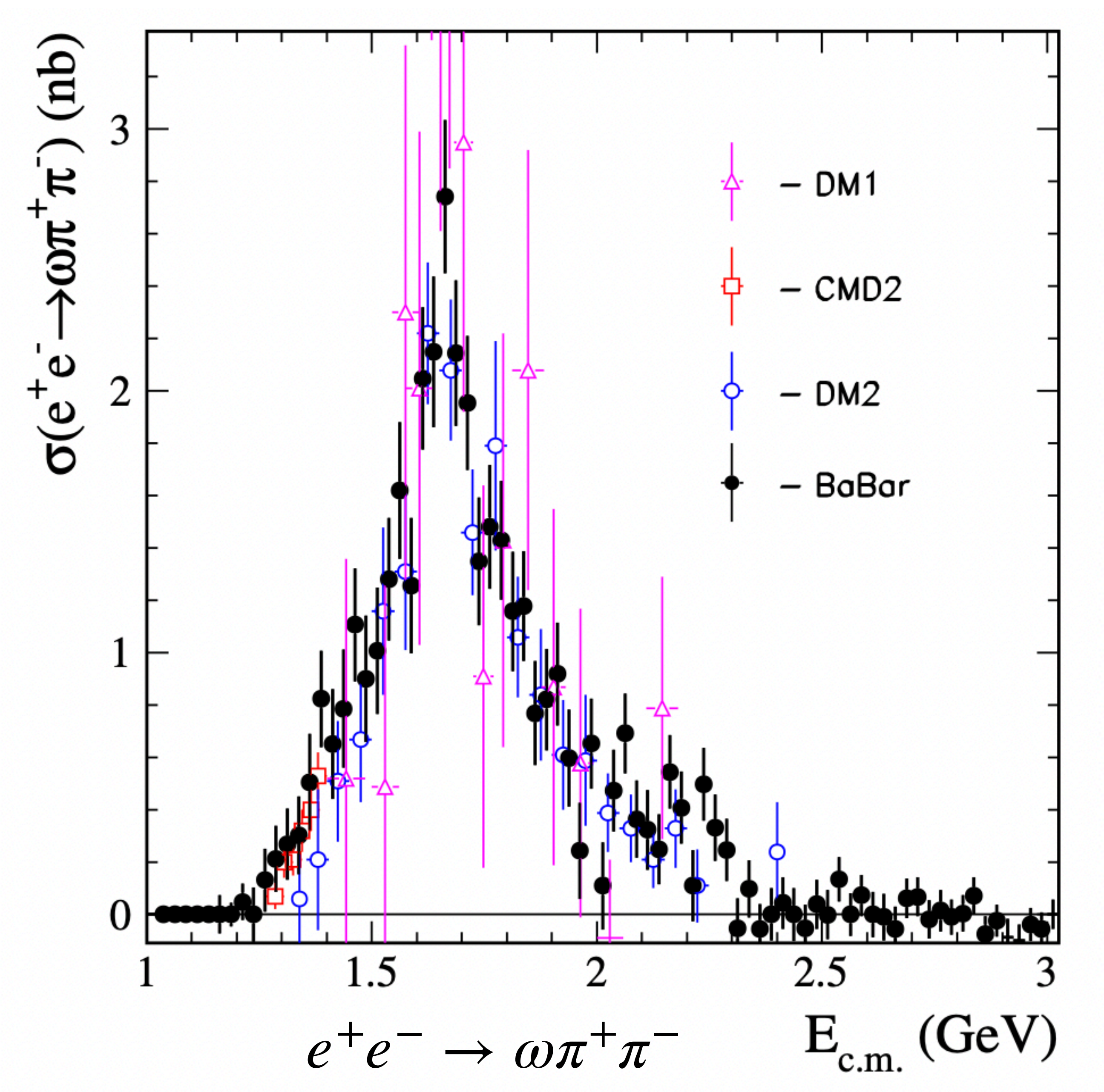
Phys. Rev. D 86, 032013 (2012)



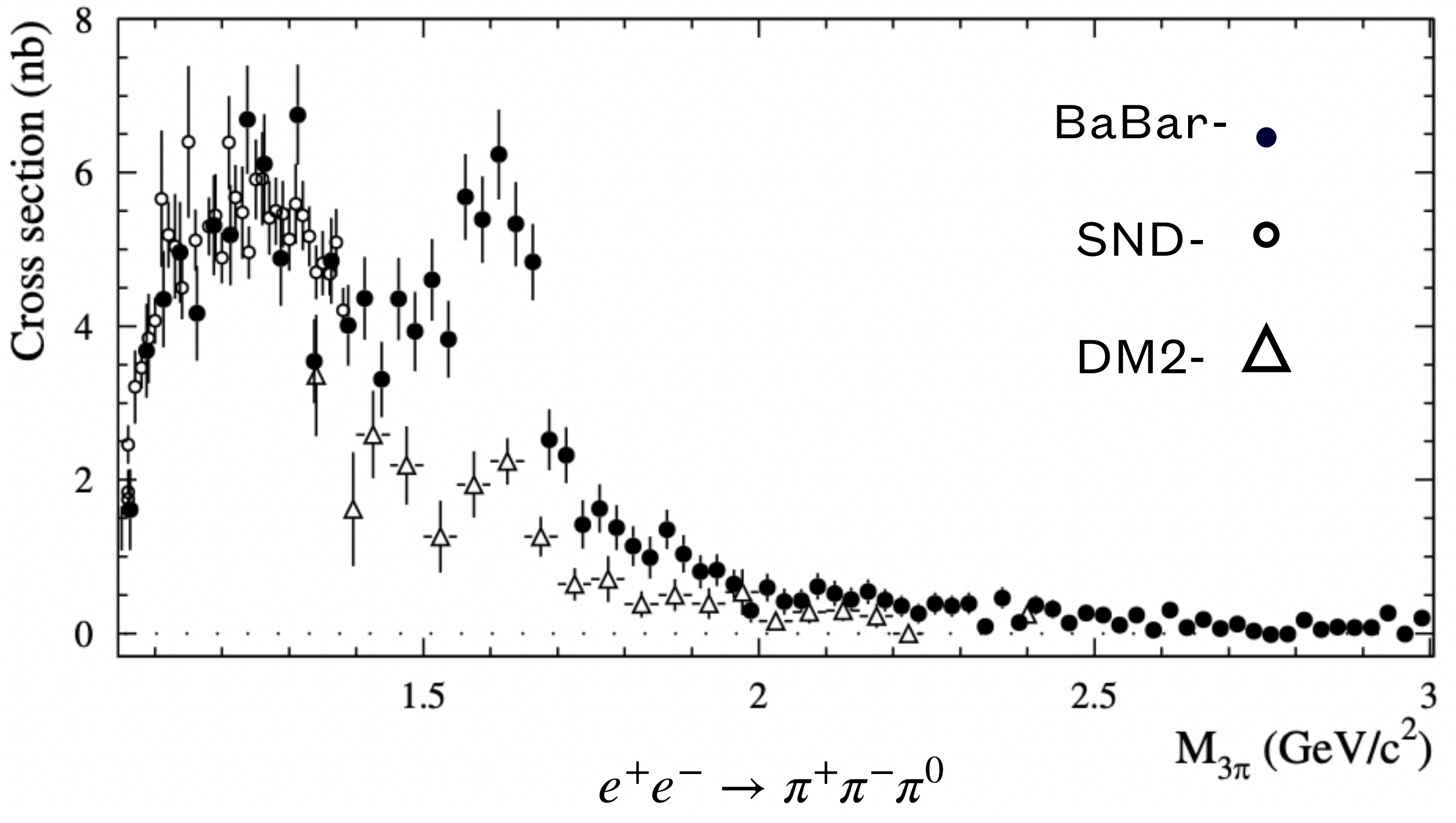
The $\rho(1450)$ and the $\rho(1700)$

Excited light vector mesons (I=0)

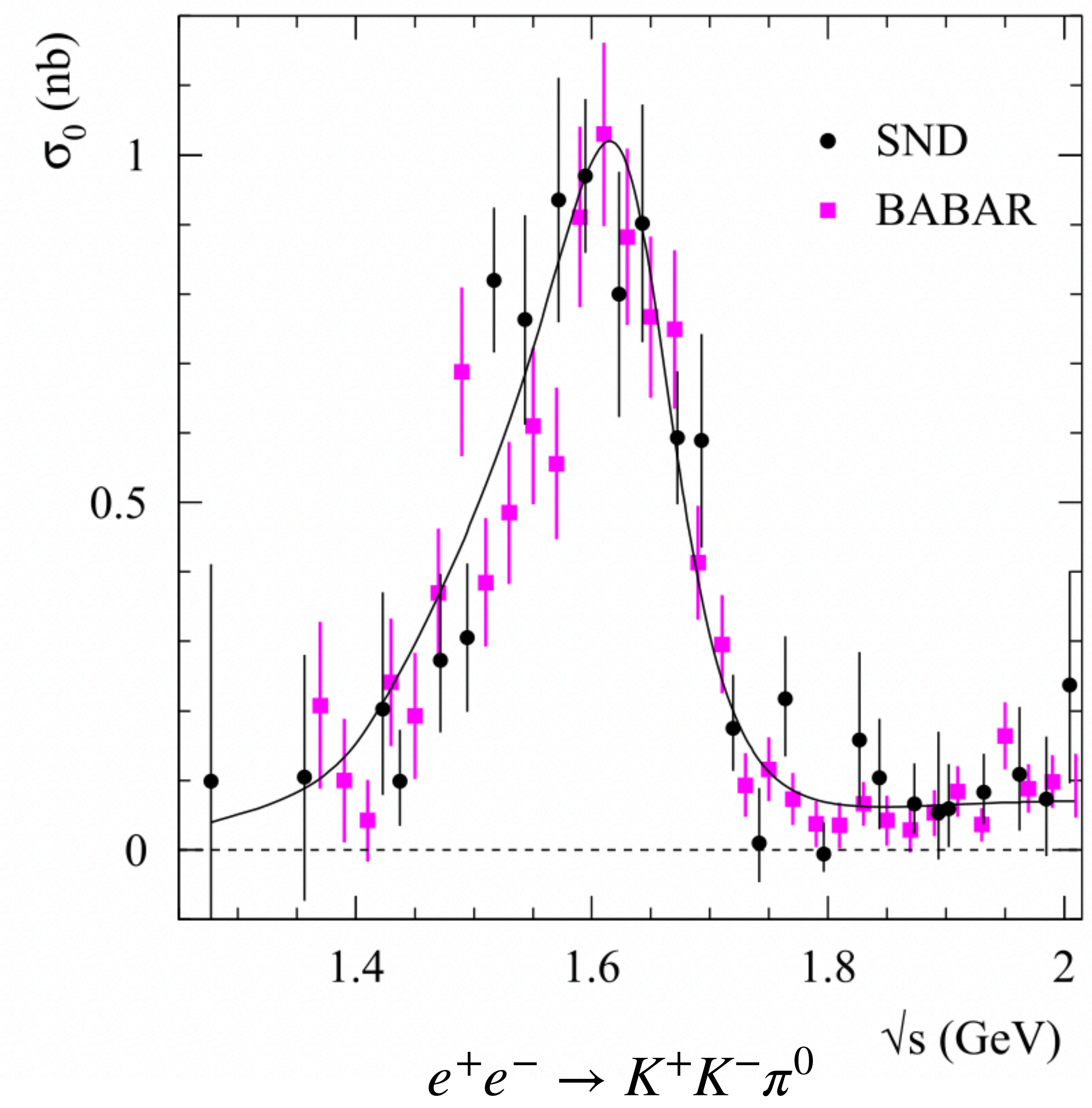
Phys. Rev. D 76, 092005 (2007)



Phys. Rev. D 70, 072004 (2004)



Eur. Phys. J. C 80, 1139 (2020)



The $\omega(1420)$, $\omega(1650)$, and the $\phi(1680)$

A Place to start

Presence of two states in 1^{--} from quark model it is natural to interpret these states as a radial excitation in S-wave $[2^3S_1]$, and an orbital excitation in D-wave $[^3D_1]$ (or some linear combination of the two).

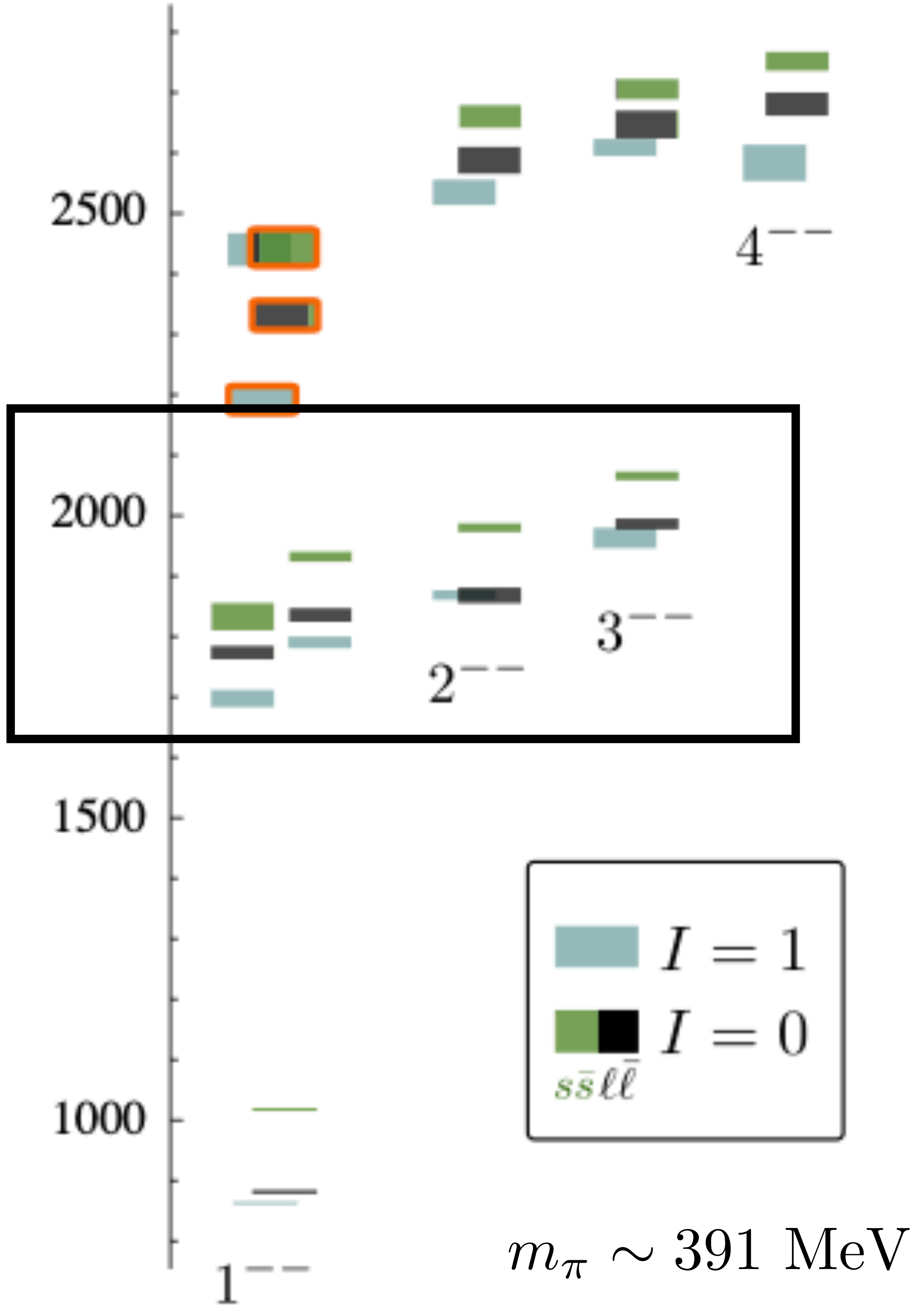
	J^P
$\ell = 0$	1^-
$\ell = 1$	$(0,1,2)^+$
$\ell = 2$	$(1,2,3)^-$
$\dots$	$\dots$

What the PDG says:

Isovector : $\rho(1450), \rho(1700), \rho_3(1690)$

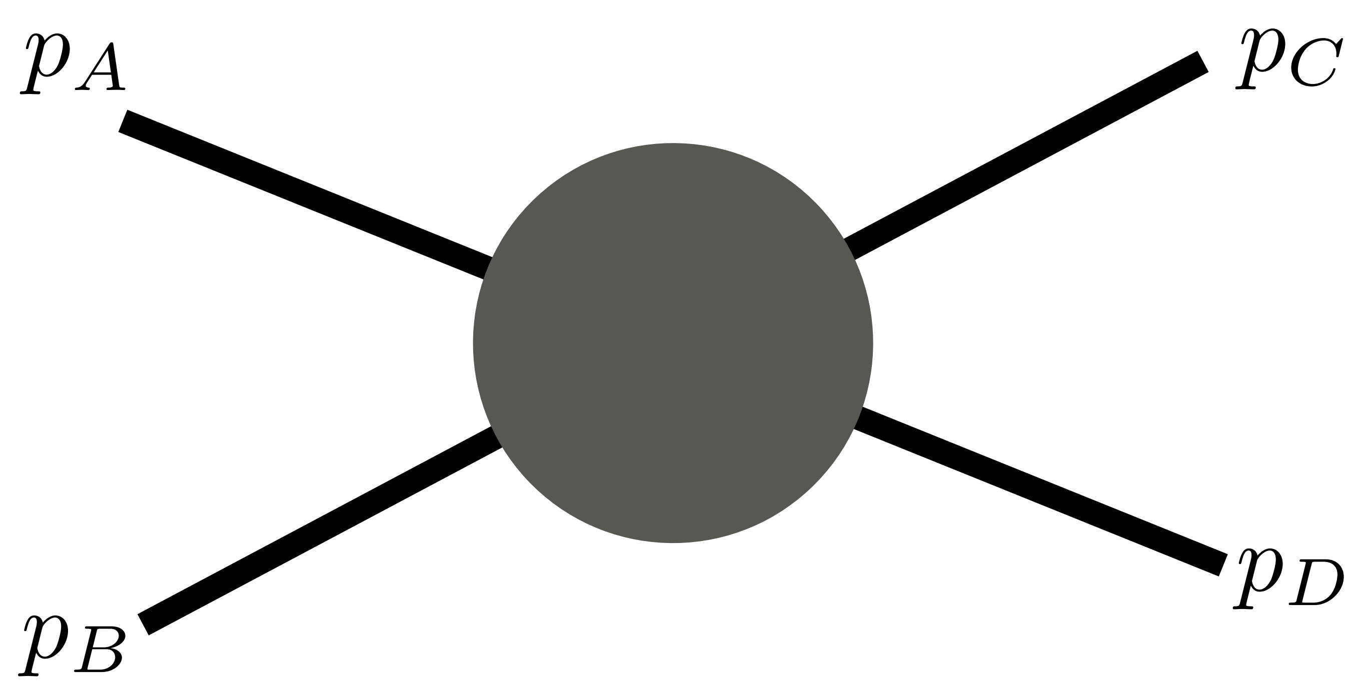
Isoscalar: $\omega(1420), \omega(1650), \omega_3(1670)$ / $\phi(1680), \phi_3(1850)$.

Lattice: $C_{ij}(t) = \sum_\alpha \langle 0|O_i|\alpha\rangle\langle\alpha|O_j|0\rangle e^{-E_\alpha t}$



J. J. Dudek, R. G. Edwards, P. Guo, and C. E. Thomas (Hadron Spectrum), Phys. Rev. D88, 094505 (2013), arXiv:1309.2608 [hep-lat].

Outline



These J^{--} states are resonances which can be accessed from scattering amplitudes.

$$t \sim \frac{g^2}{s - s_0} \qquad \sqrt{s_0} = m_R + \frac{i}{2}\Gamma_R$$

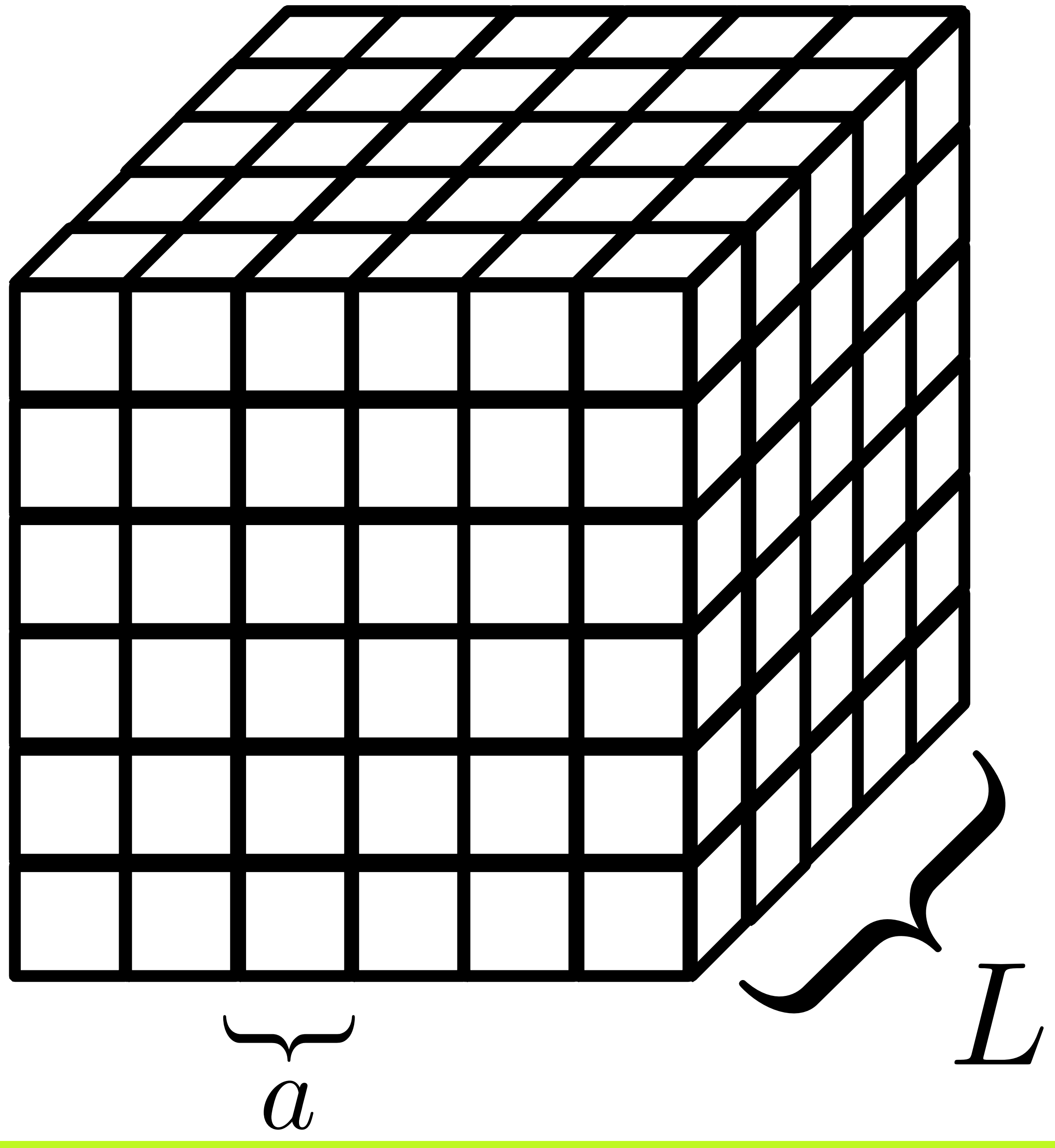
Finite-volume spectrum $\leftrightarrow$ scattering amplitude $(2 \to 2)$

$$\det\left[1 + i\boldsymbol{\rho}(E)\cdot\mathbf{t}(E)\cdot\left(\mathbf{1} + i\mathcal{M}(E,L)\right)\right] = 0$$

$$\mathbf{t}^{-1}(E) = \mathbf{K}^{-1}(E) - i\boldsymbol{\rho}(E) \qquad \rho_i = \frac{2k_i}{E}$$

Compute correlation functions on the lattice to obtain finite-volume spectrum.

$$\Rightarrow C_{ij}(t) = \sum_{\alpha} \langle 0 | O_i | \alpha \rangle \langle \alpha | O_j | 0 \rangle e^{-E_\alpha t}$$



SU(3) Flavor Ensembles

J^{--}excited mesons at the SU(3) flavor point in the singlet representation

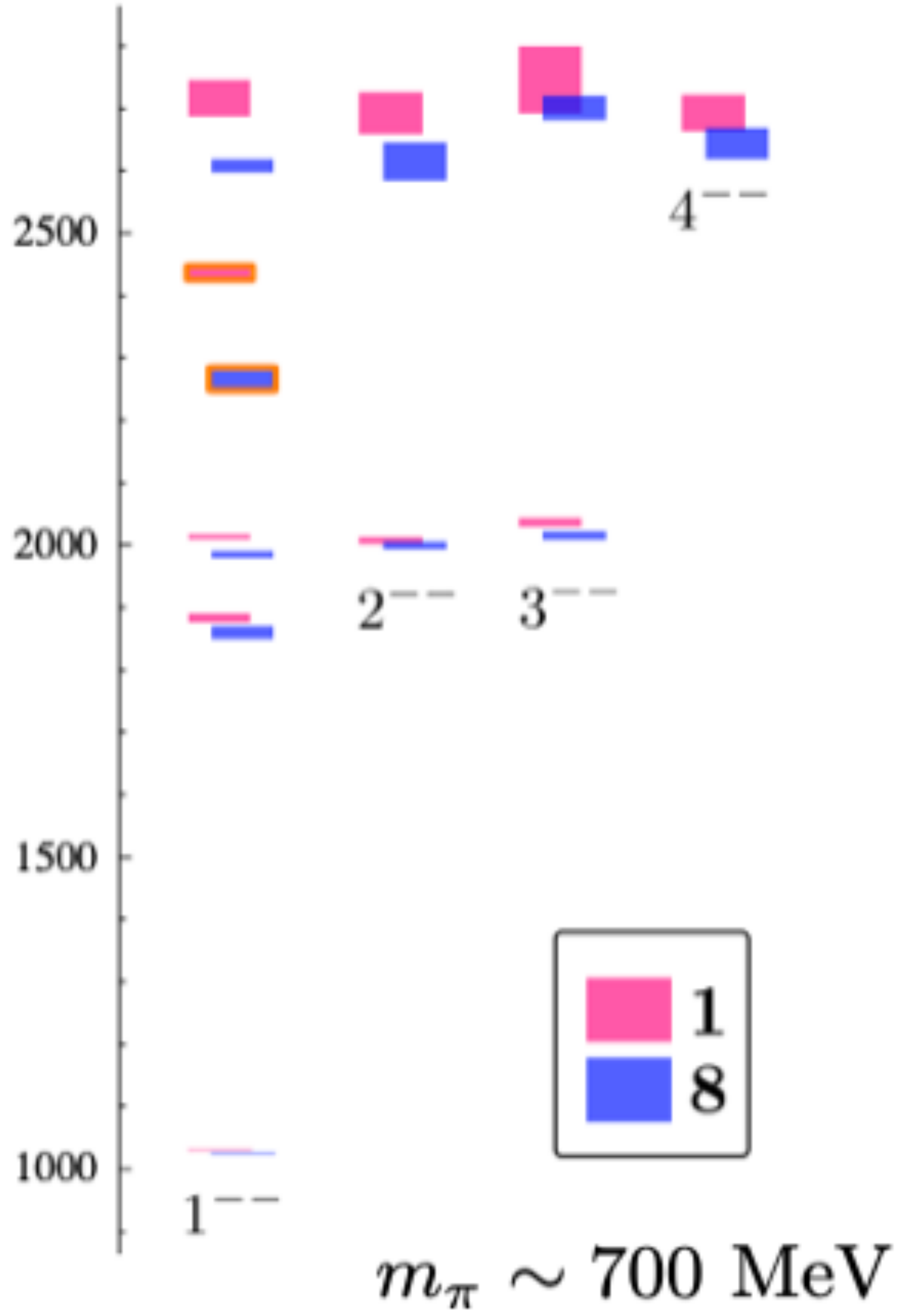
Advantages:

⇒Heavier light quark masses allow us to probe higher energy regions:

first three-particle threshold gets moved higher up

resonant states at lighter quark masses feature as stable particles

⇒Fewer channels (ex. $\pi, K, \bar{K}, \eta$ are all just η^8)



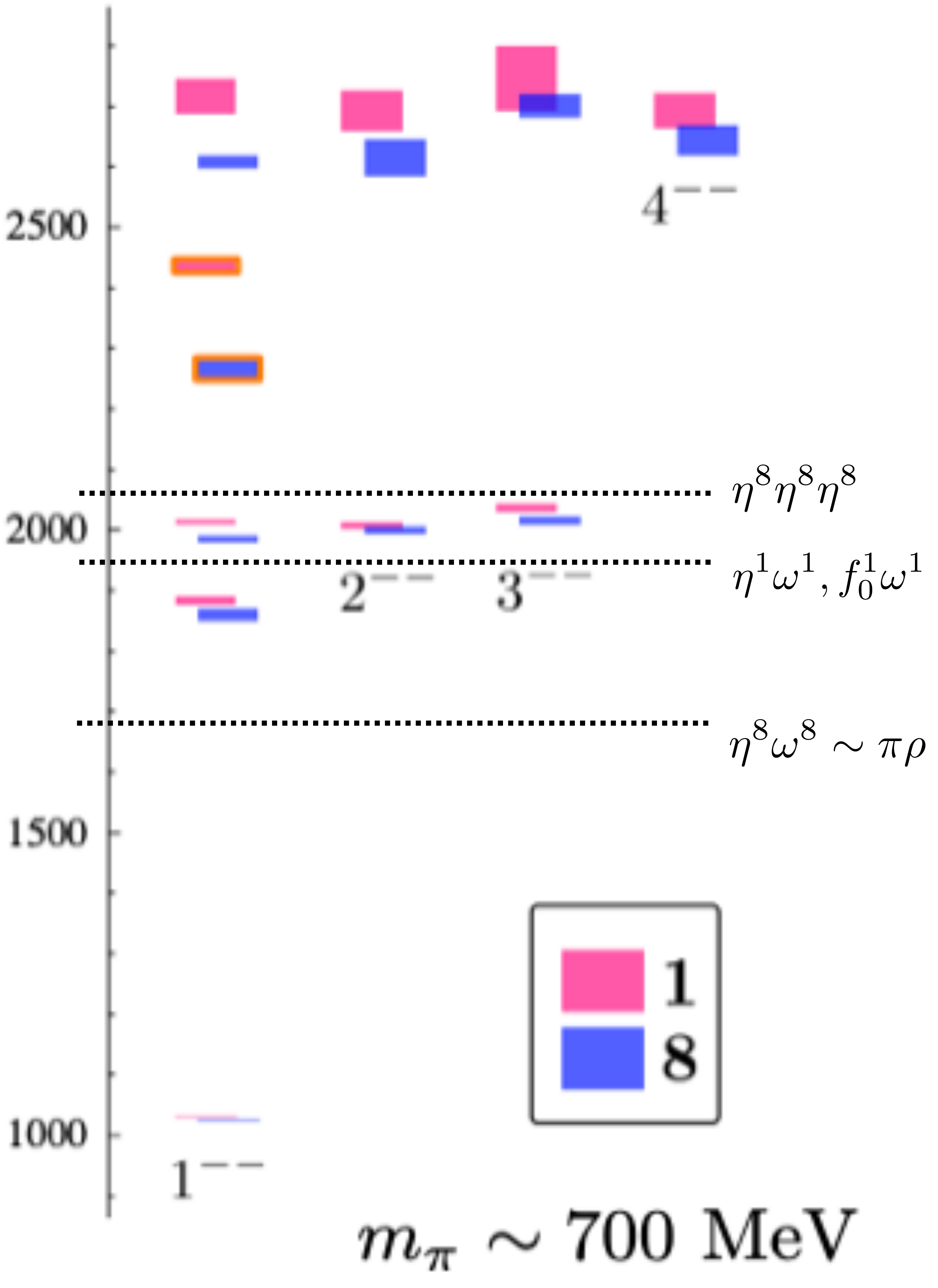
J. J. Dudek, R. G. Edwards, P. Guo, and C. E. Thomas (Hadron Spectrum), Phys. Rev. D88, 094505 (2013), arXiv:1309.2608 [hep-lat].

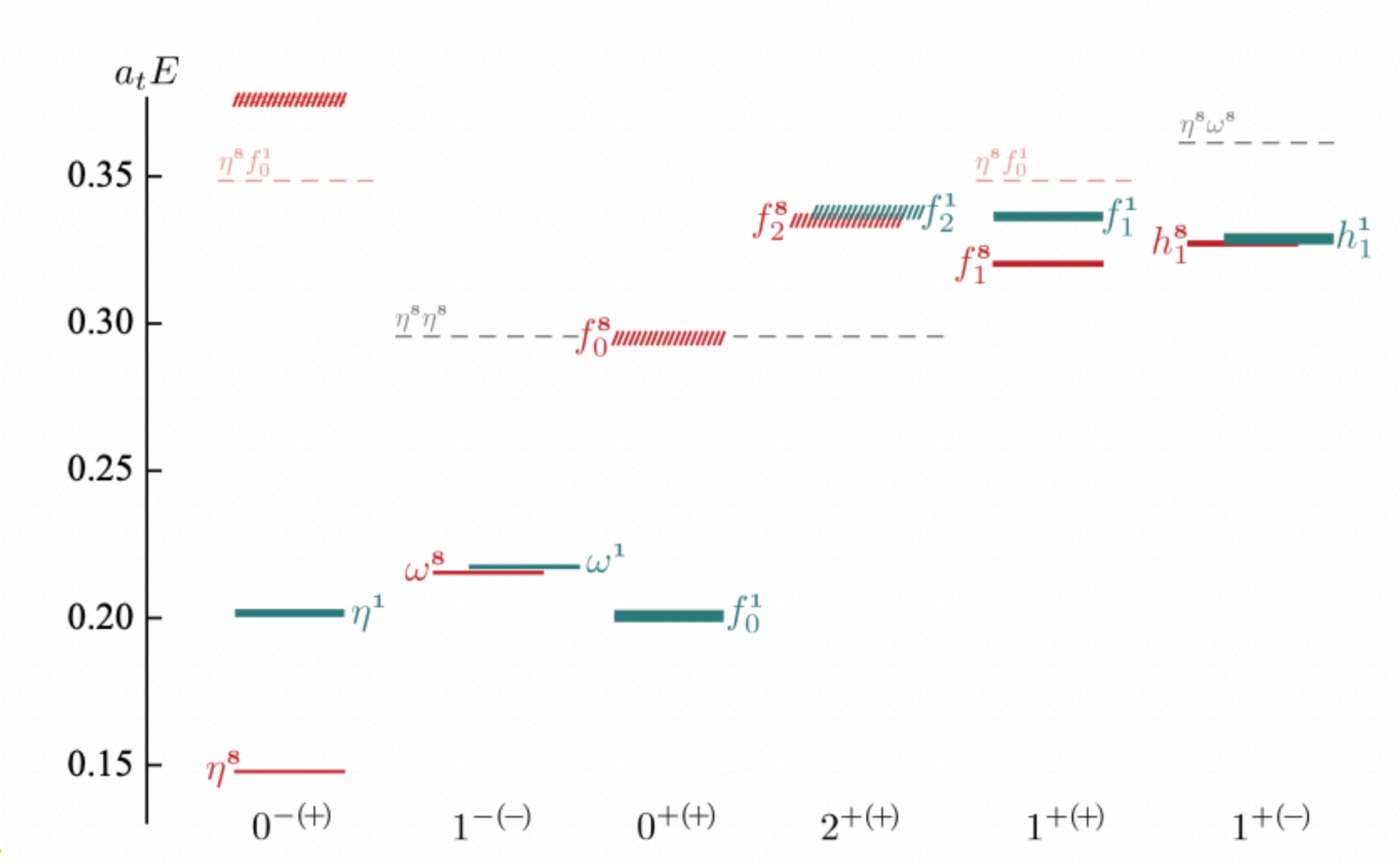
Channels $SU(3)_F$

J=1: $\eta^8\omega^8\{^3P_1\}, f_0^1\omega^1\{^3S_1, {}^3D_1\}, \eta^1\omega^1\{^3P_1\}$

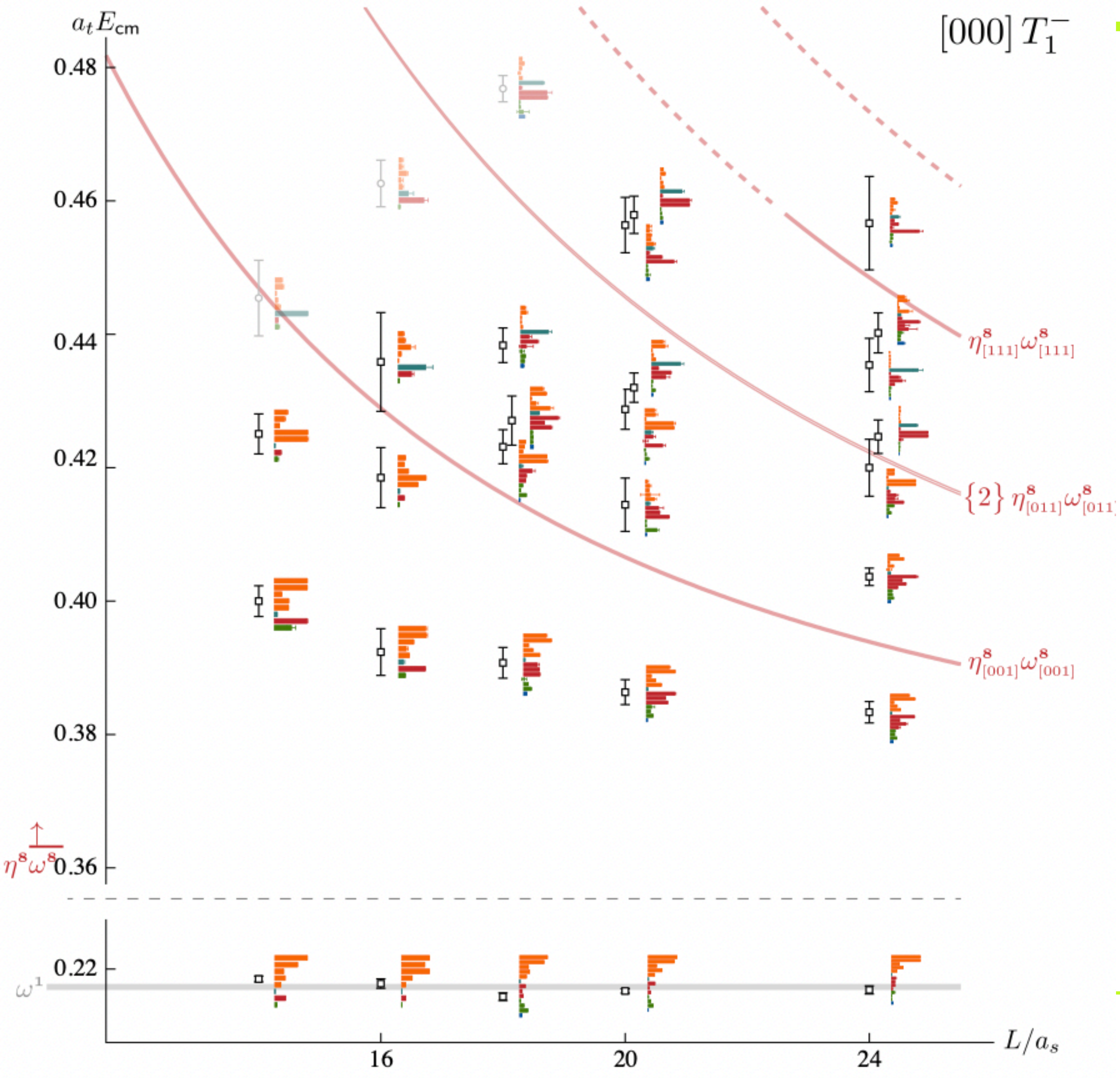
J=2: $\eta^8\omega^8\{^3P_2, {}^3F_2\}, f_0^1\omega^1\{^3D_2\}, \eta^1\omega^1\{^3P_2, {}^3F_2\}$

J=3: $\eta^8\omega^8\{^3F_3\}, f_0^1\omega^1\{^3D_3, {}^3G_3\}, \eta^1\omega^1\{^3F_3\}$





J. J. Dudek, R. G. Edwards, P. Guo, and C. E. Thomas (Hadron Spectrum), Phys. Rev. D88, 094505 (2013), arXiv:1309.2608 [hep-lat].

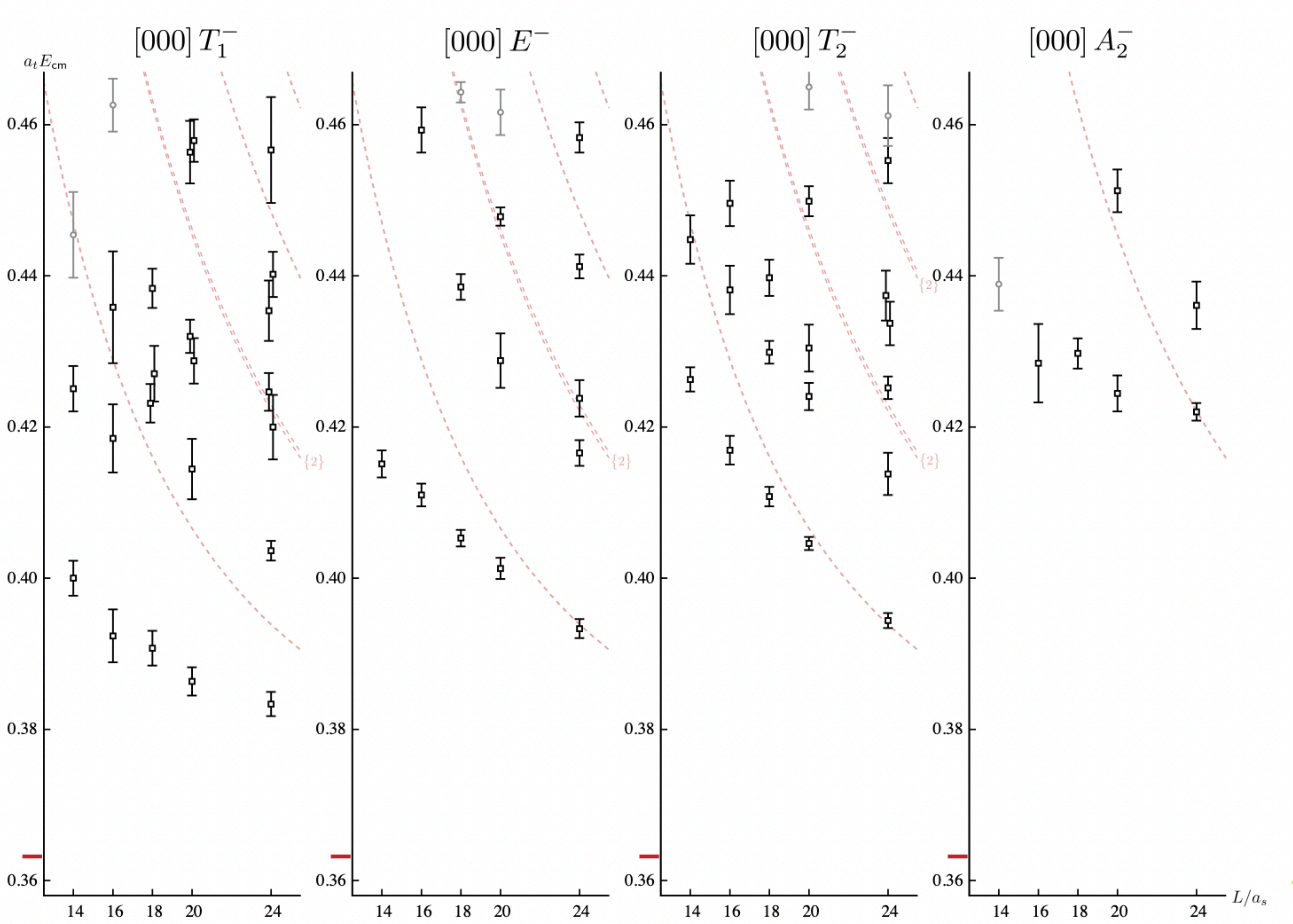


$J^P = (1,3,\ldots)^-$

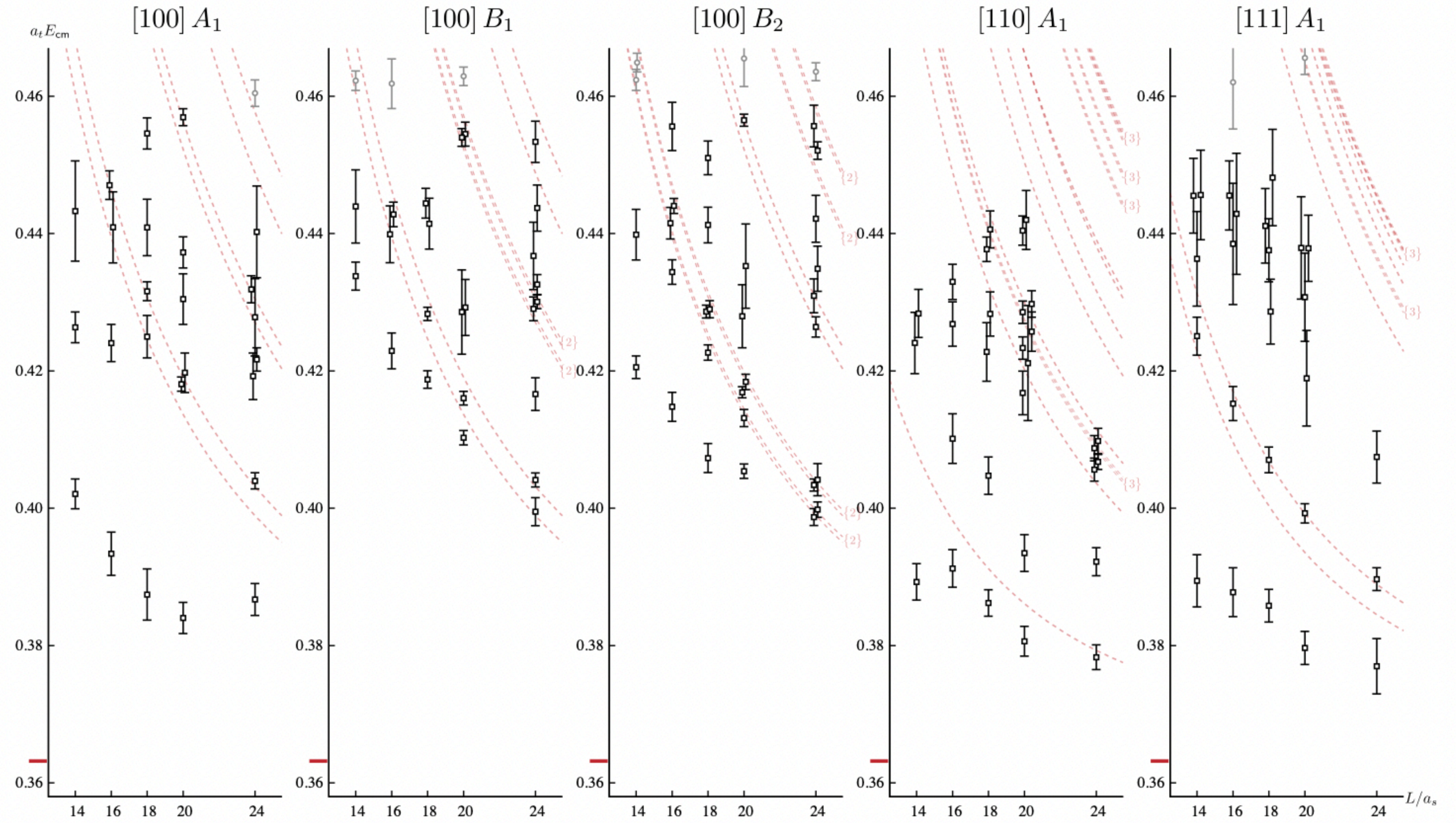
Three resonances in a single irrep.

$\Rightarrow \rho\{^3 2S_1\}, \rho\{^3D_1\}, \rho\{^3D_3\}$

Very dense in energy levels.



$J^P = (1, 3, ...)^-$ $J^P = (2, ...)^-$ $J^P = (2, 3, ...)^-$ $J^P = (3, ...)^-$



$J^P = 0^+, 1^-, 2^+, 3^-, \ldots$ $J^P = 2^\pm, 3^\pm, \ldots$ $J^P = 2^\pm, 3^\pm, \ldots$ $J^P = 0^+, 1^-, 2^\pm, 3^\pm, \ldots$ $J^P = 0^+, 1^-, 2^+, 3^\pm, \ldots$

Parameterizations, J=2,3

J=2 dynamically coupled in P- and F-waves

$$K_{ij} \to (2k_i)^{\ell} K_{ij}^{\ell\ell'} (2k_j)^{\ell'}$$

Can handle this with the K-matrix $\boldsymbol{t}^{-1} = \boldsymbol{K}^{-1} + \boldsymbol{I}$

$$K_{J=2} = \begin{bmatrix} ({}^3P_2|{}^3P_2) & ({}^3P_2|{}^3F_2) \\ ({}^3P_2|{}^3F_2) & ({}^3F_2|{}^3F_2) \end{bmatrix}$$

	J^P
$\ell = 0$	1^+
$\ell = 1$	$(0, 1, 2)^-$
$\ell = 2$	$(1, 2, 3)^+$
$\ell = 3$	$(2, 3, 4)^-$
$\ldots$	$\ldots$

J=3 Breit-Wigner parameterization

$$I(s) = I(s_0) - \frac{s - s_0}{\pi} \int_{s_{thr}}^{\infty} \frac{\rho(s')}{(s' - s_0)(s' - s - i\epsilon)} ds'$$

$\mathrm{Im}\boldsymbol{I} = -\boldsymbol{\rho}$

$\eta^8\omega^8$ elastic scattering in $2^{--}, 3^{--}$

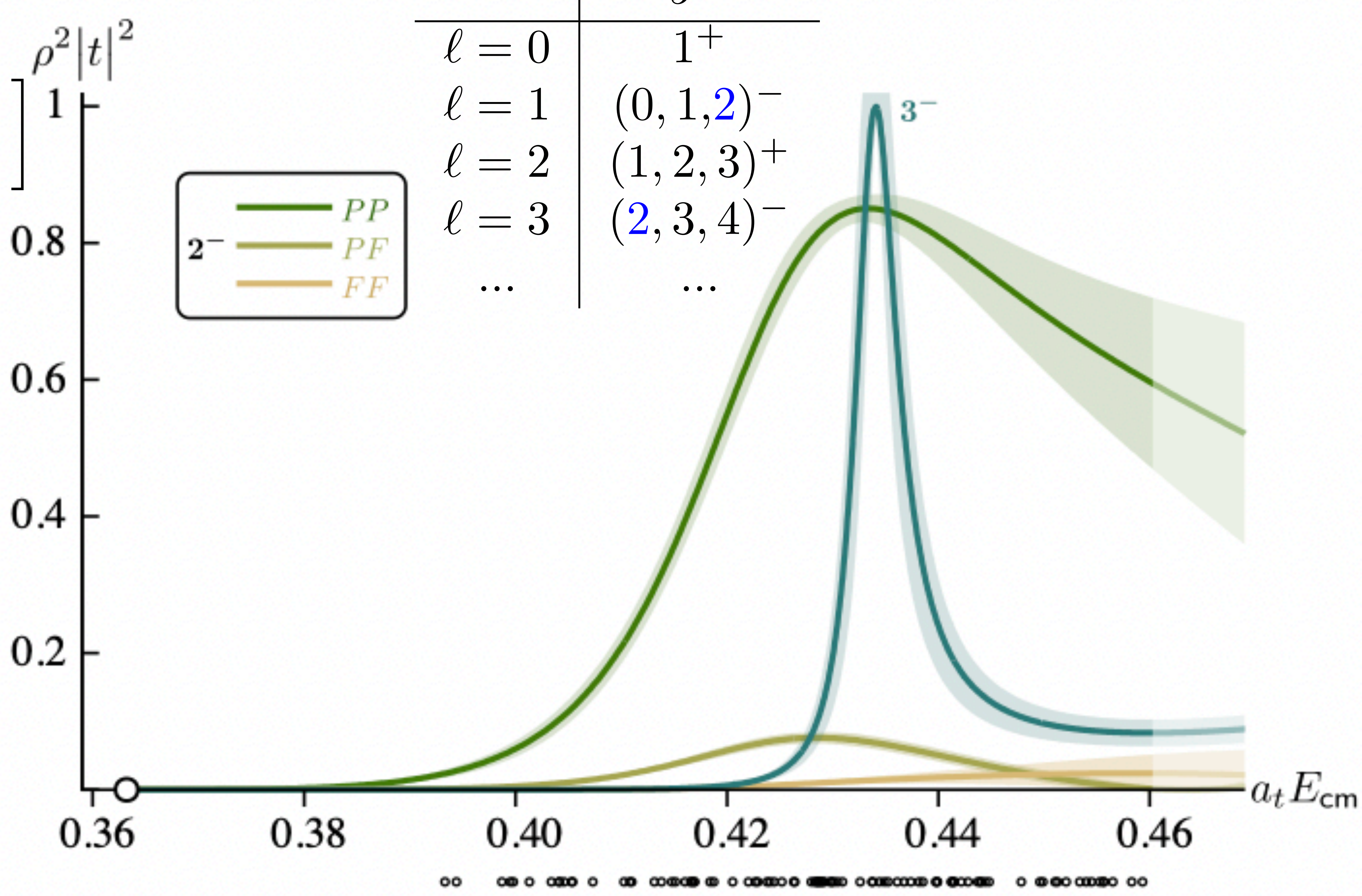
$$K_{J=2} = \frac{1}{m_R^2 - s}\begin{bmatrix} g_P^2 & g_P g_F \\ g_P g_F & g_F^2 \end{bmatrix} + \begin{bmatrix} \gamma_{PP} & \gamma_{PF} \\ \gamma_{PF} & \gamma_{FF} \end{bmatrix}$$

$$K_{J=3} = \frac{g_F^2}{m_R^2 - s}$$

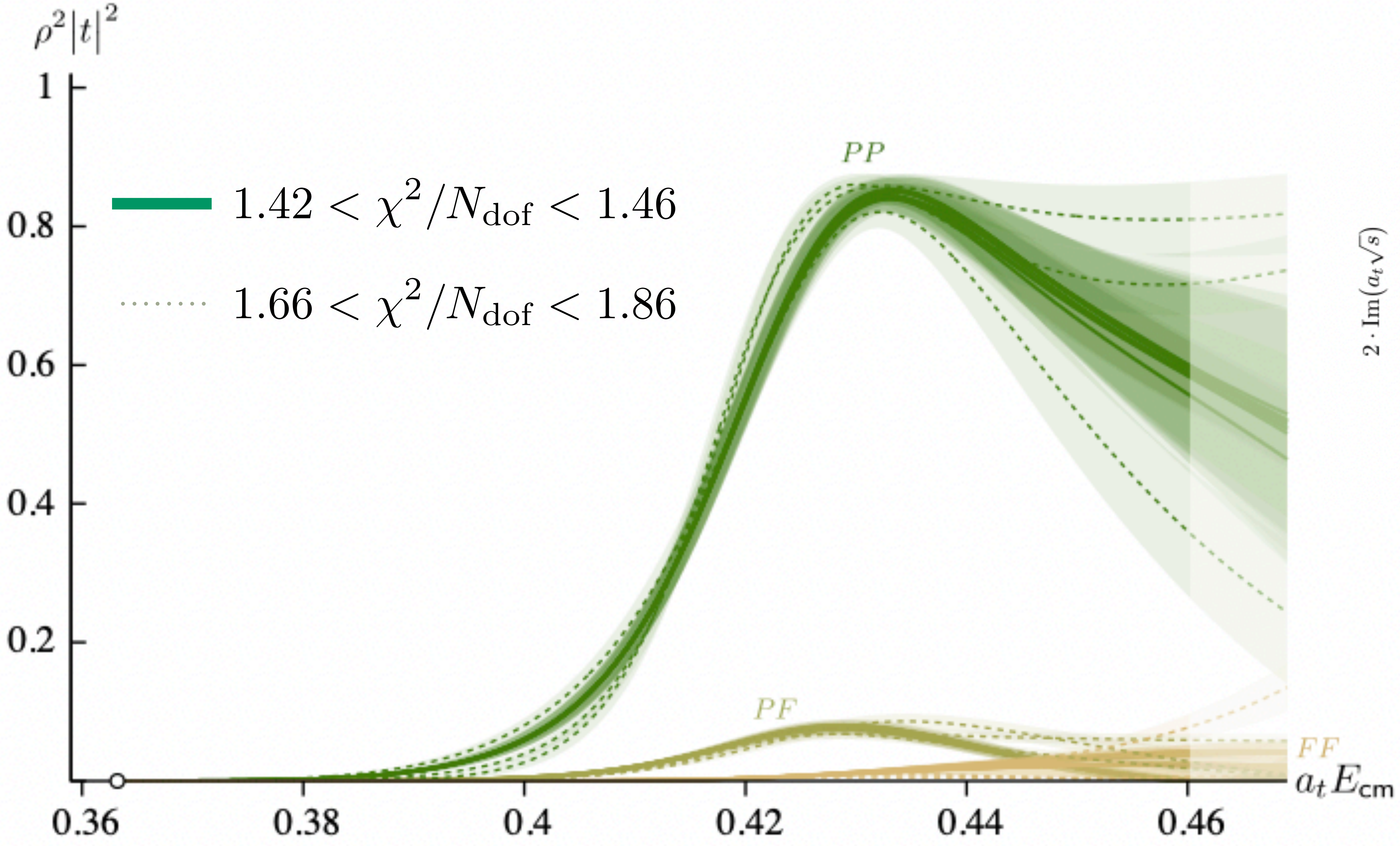
$$\begin{array}{ll}
J=2 \left\{ \begin{array}{rl} m =& 0.4322(15)\cdot a_t^{-1} \\ g_P =& 0.753(37) \\ g_F =& -4.13(29)\cdot a_t^2 \\ \gamma_{PP} =& 0.1(33)\cdot a_t^2 \\ \gamma_{PF} =& -110(17)\cdot a_t^4 \\ \gamma_{FF} =& 143(322)\cdot a_t^6 \end{array}\right. \\
J=3 \left\{ \begin{array}{rl} m =& 0.4341(9)\cdot a_t^{-1} \\ g =& 4.85(28)\cdot a_t^2 \end{array}\right.
\end{array}
\begin{bmatrix}
1 & 0.31 & 0.29 & 0.13 & -0.37 & 0.31 & 0.19 & 0.07 \\
 & 1 & -0.08 & -0.70 & 0.04 & 0.48 & 0.07 & -0.23 \\
 & & 1 & 0.21 & -0.15 & -0.18 & -0.01 & -0.12 \\
 & & & 1 & -0.34 & -0.34 & -0.16 & 0.23 \\
 & & & & 1 & -0.23 & -0.03 & -0.05 \\
 & & & & & 1 & 0.02 & 0.05 \\
 & & & & & & 1 & -0.04 \\
 & & & & & & & 1
\end{bmatrix}$$

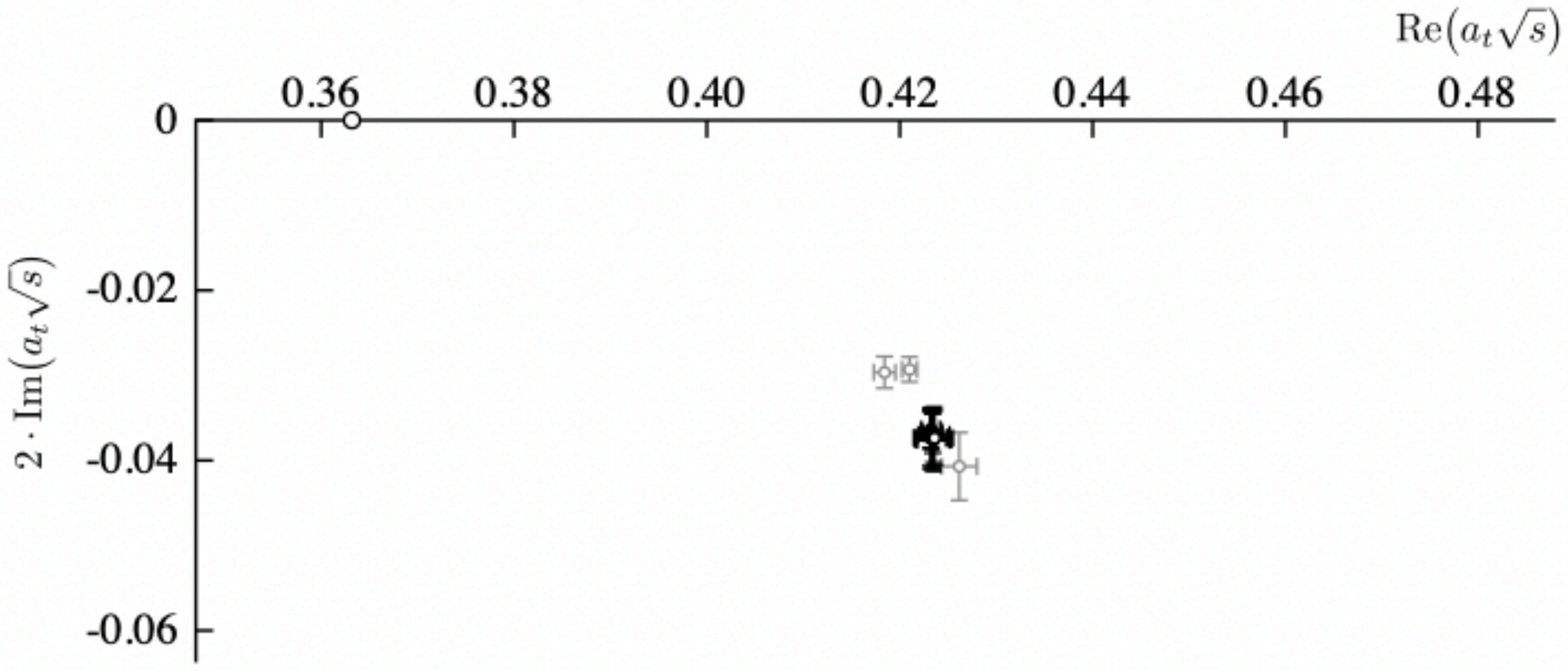
$$\chi^2/N_{\rm dof} = \frac{120.3}{91-8} = 1.45$$

	J^P
$\ell = 0$	1^+
$\ell = 1$	$(0,1,2)^-$
$\ell = 2$	$(1,2,3)^+$
$\ell = 3$	$(2,3,4)^-$
…	…



$\eta^8\omega^8$ elastic scattering in 2^{--}



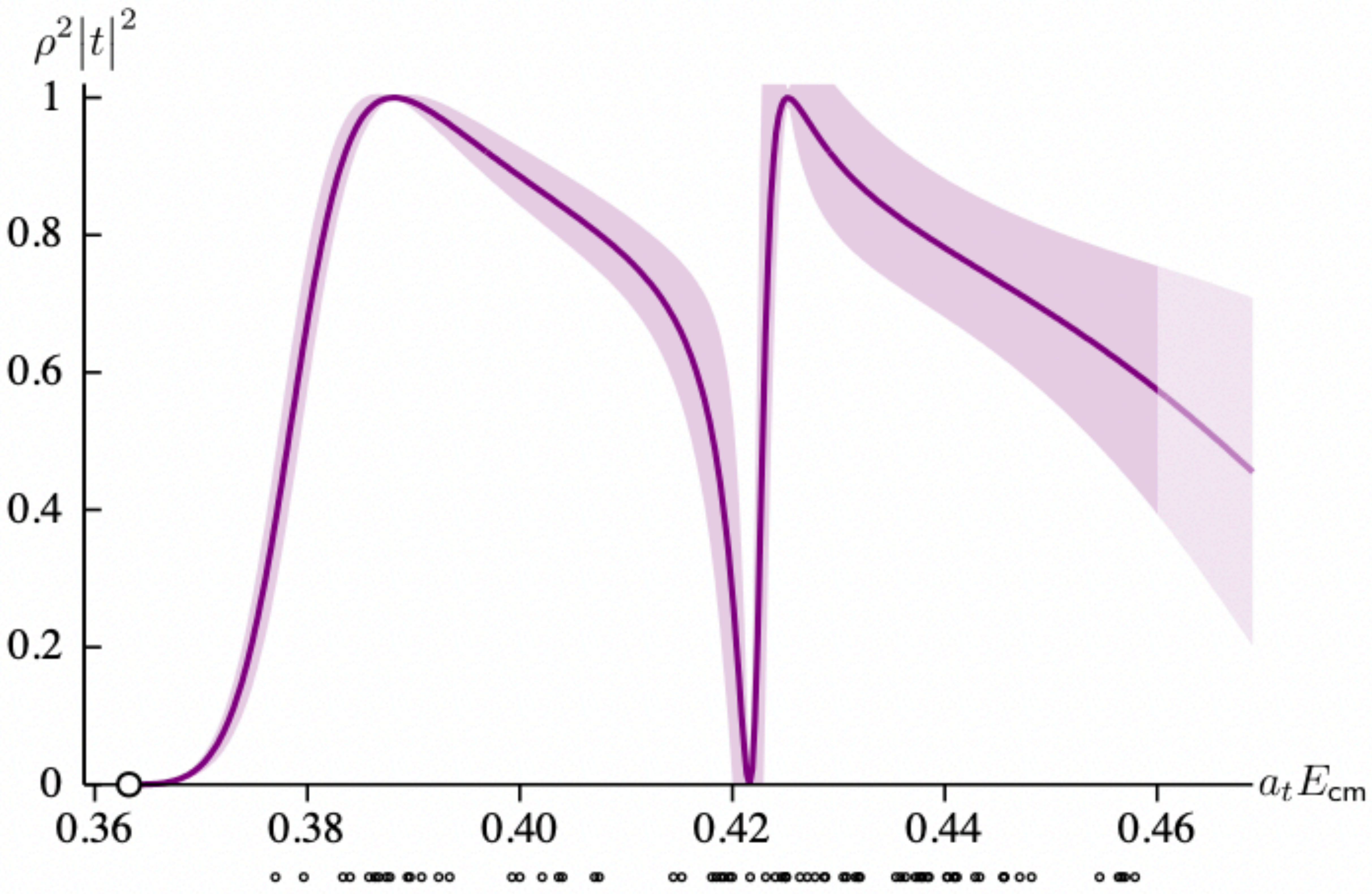


$\eta^8\omega^8$ elastic scattering in 1^{--}

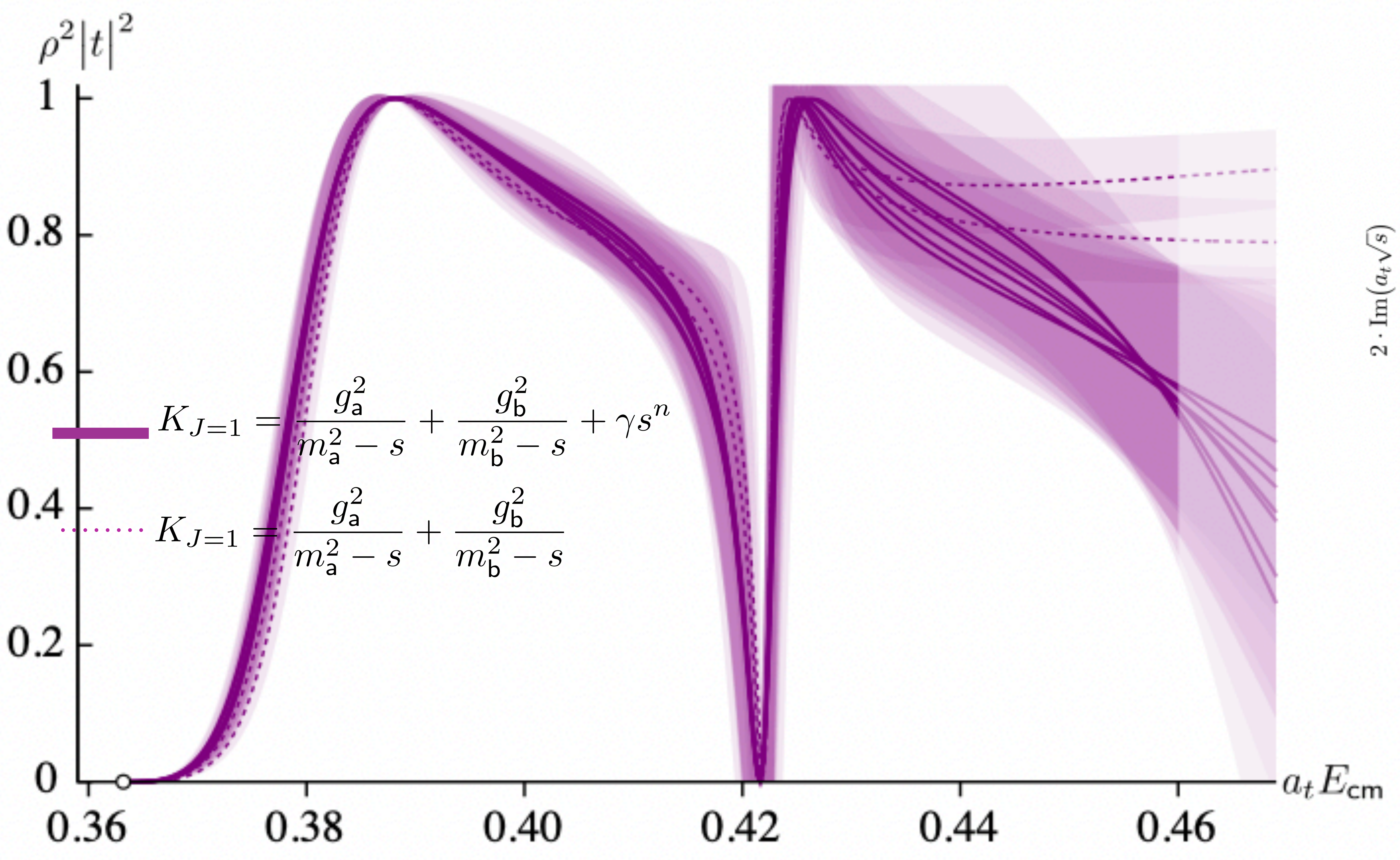
$$K_{J=1} = \frac{g_\mathsf{a}^2}{m_\mathsf{a}^2 - s} + \frac{g_\mathsf{b}^2}{m_\mathsf{b}^2 - s} + \gamma$$

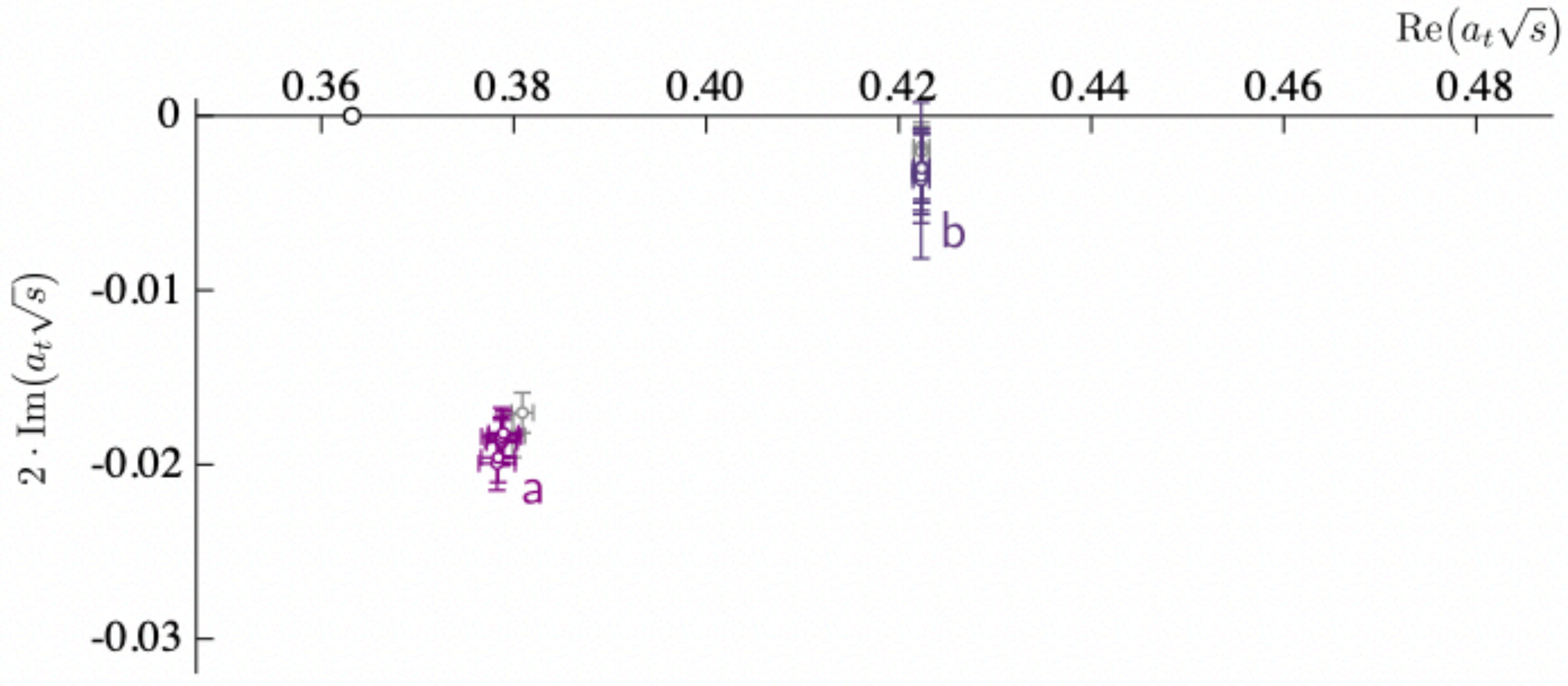
$$\begin{aligned} m_\mathsf{a} &= 0.3881(14) \cdot a_t^{-1} \\ g_\mathsf{a} &= 1.46(10) \\ m_\mathsf{b} &= 0.4242(17) \cdot a_t^{-1} \\ g_\mathsf{b} &= -0.36(13) \\ \gamma &= 20.9(86) \cdot a_t^2 \end{aligned} \qquad \begin{bmatrix} 1 & 0.08 & 0.43 & -0.33 & 0.19 \\ & 1 & 0.37 & -0.46 & 0.81 \\ & & 1 & -0.86 & 0.49 \\ & & & 1 & -0.57 \\ & & & & 1 \end{bmatrix}$$

$$\chi^2/N_\mathrm{dof} = \tfrac{91.3}{72-5} = 1.36$$

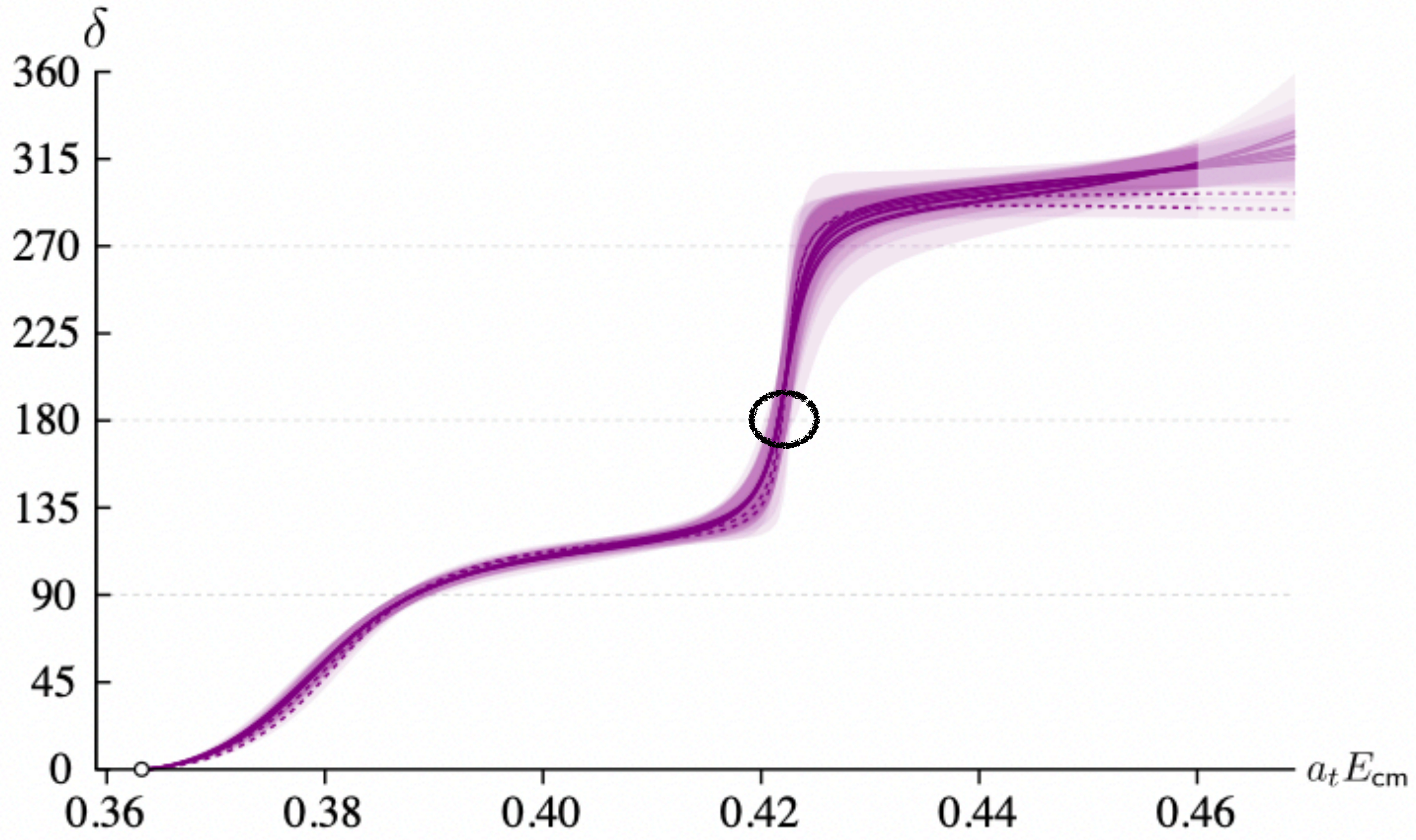


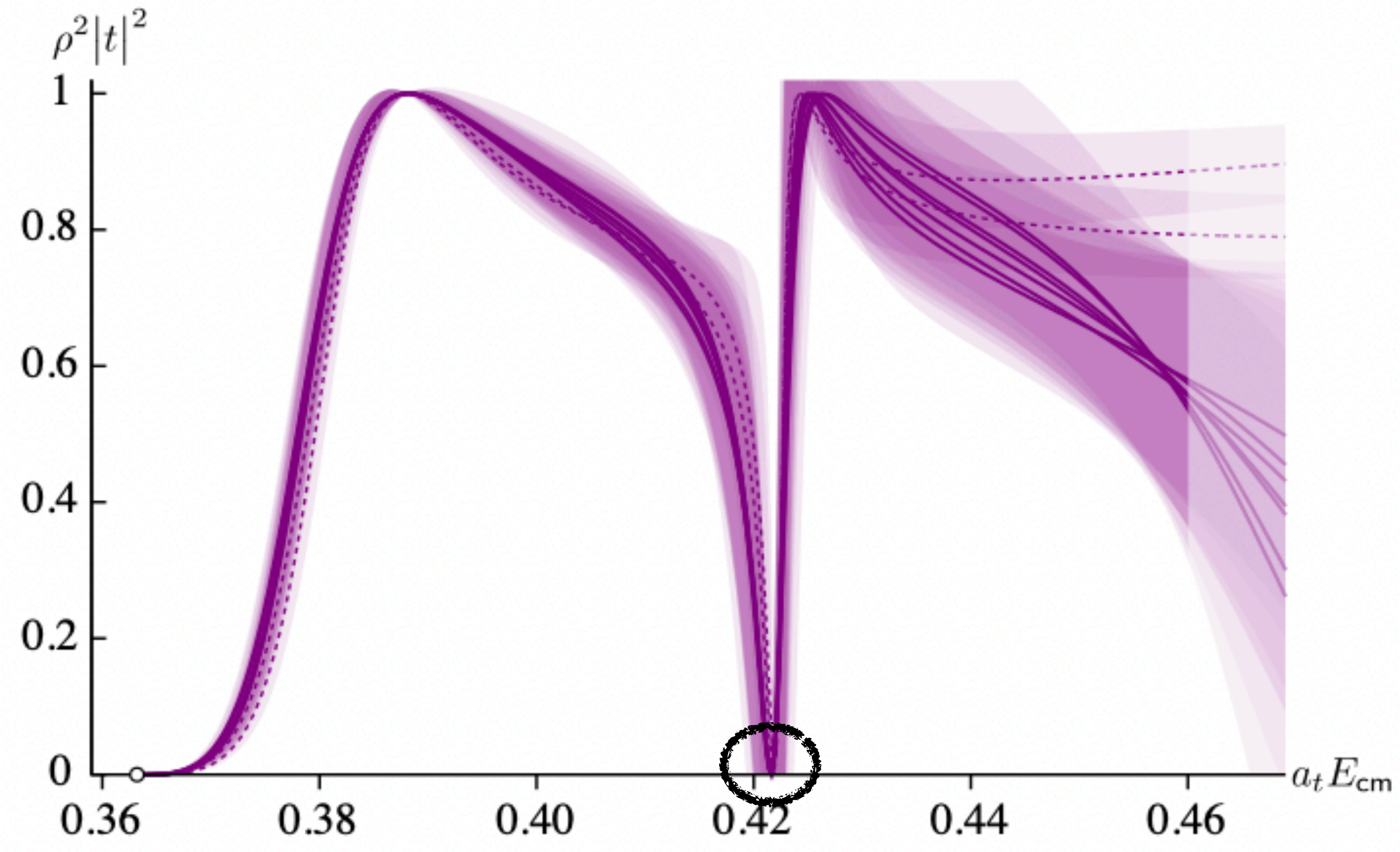
$\eta^8\omega^8$ elastic scattering in 1^{--}





Elasticity





Zero is a feature of elastic unitarity

$$t = \frac{1}{\rho(\cot\delta - i)}$$

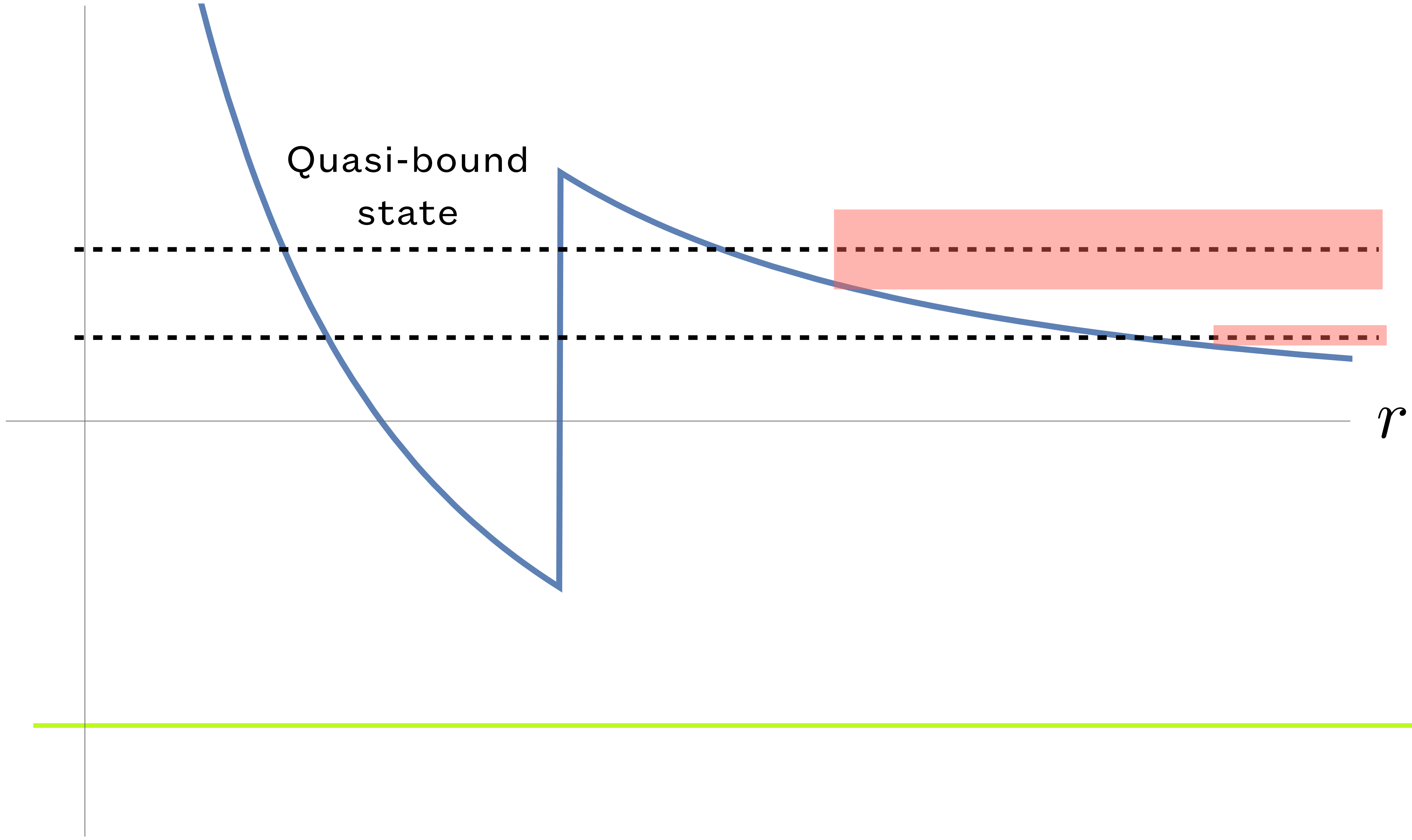
Cannot generate with an effective range

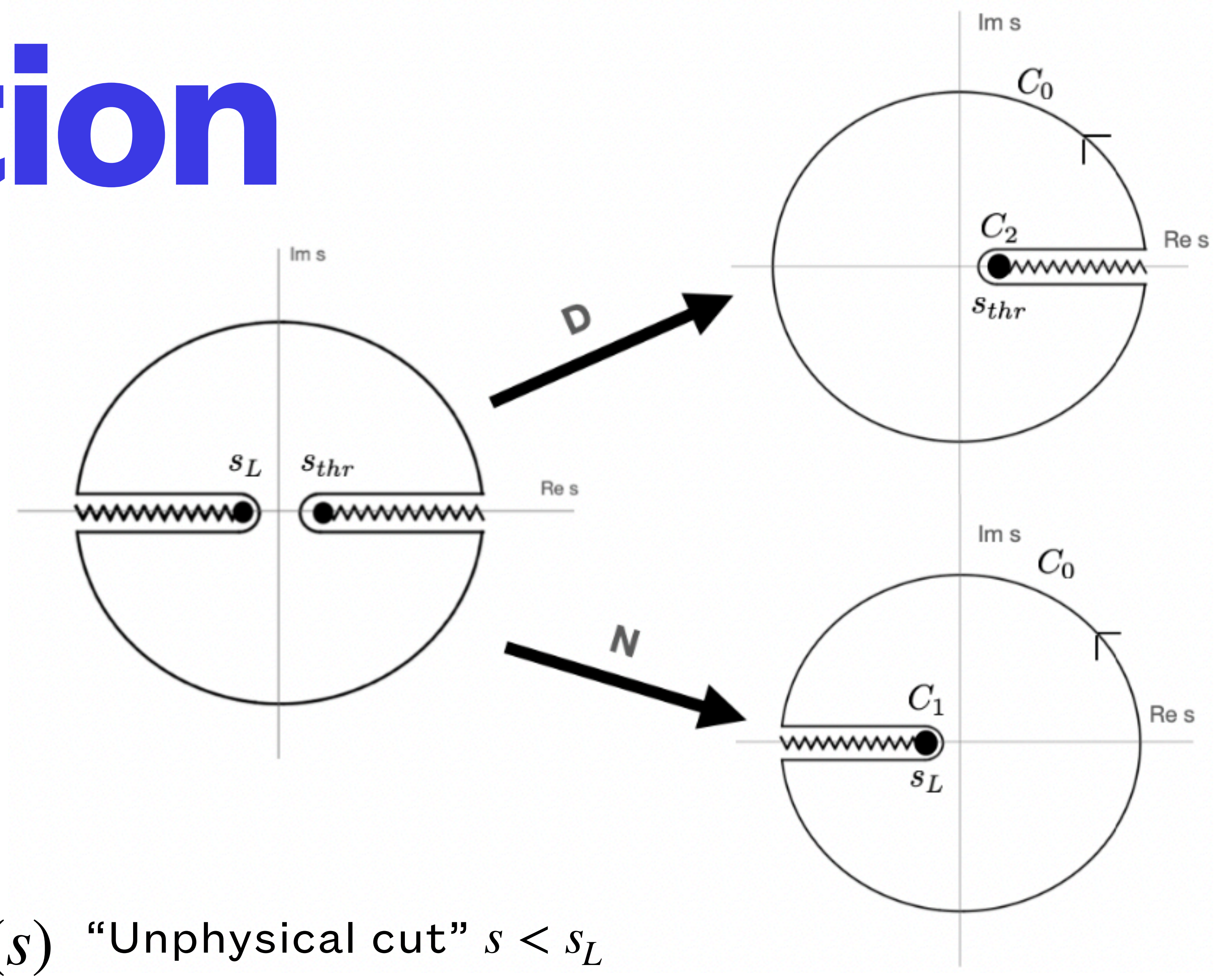
$$k^3 \cot\delta = \frac{1}{a} + \frac{1}{2} r k^2 + \ldots$$

Resonance interpretation

In N.R. scattering, the scattering amplitude is completely determined by the potential.

$$V_{eff}(r) = V(r) + \frac{\ell(\ell+1)}{r^2}$$





$$t(s) = \frac{N(s)}{D(s)}$$

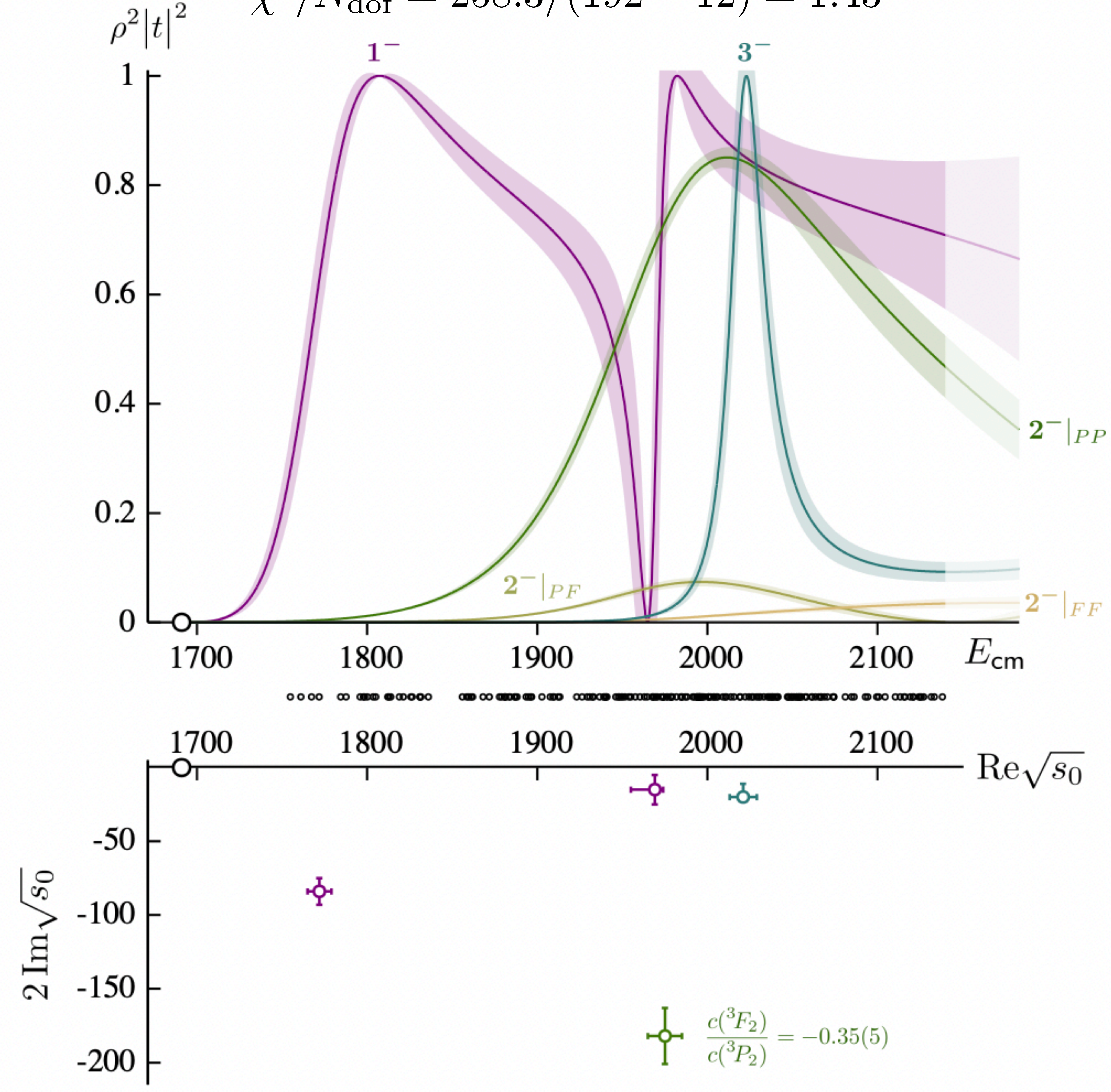
"Unphysical cut" $s < s_L$

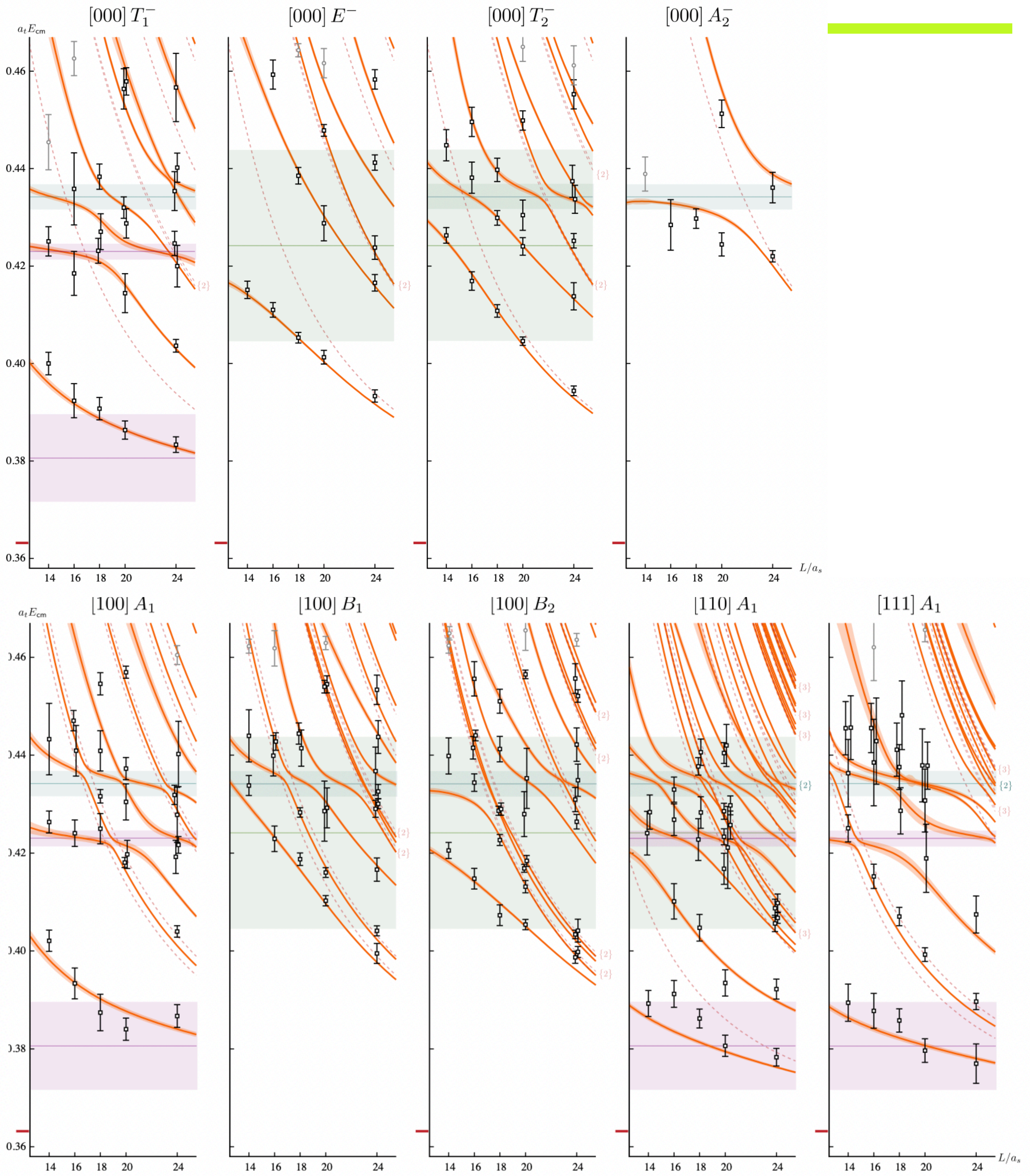
"Physical cut" $s > s_{thr}$

$$D(s) = D(s_0) - \frac{s - s_0}{\pi} \int_{s_{thr}}^{\infty} \frac{N(s')\rho(s')}{(s'-s)(s'-s_0)} ds' + \sum_\alpha \frac{g_\alpha^2}{m_\alpha^2 - s}$$

Can add poles to D(s) that produce zeros in t(s)

$\chi^2/N_{\text{dof}} = 258.3/(192-12) = 1.43$

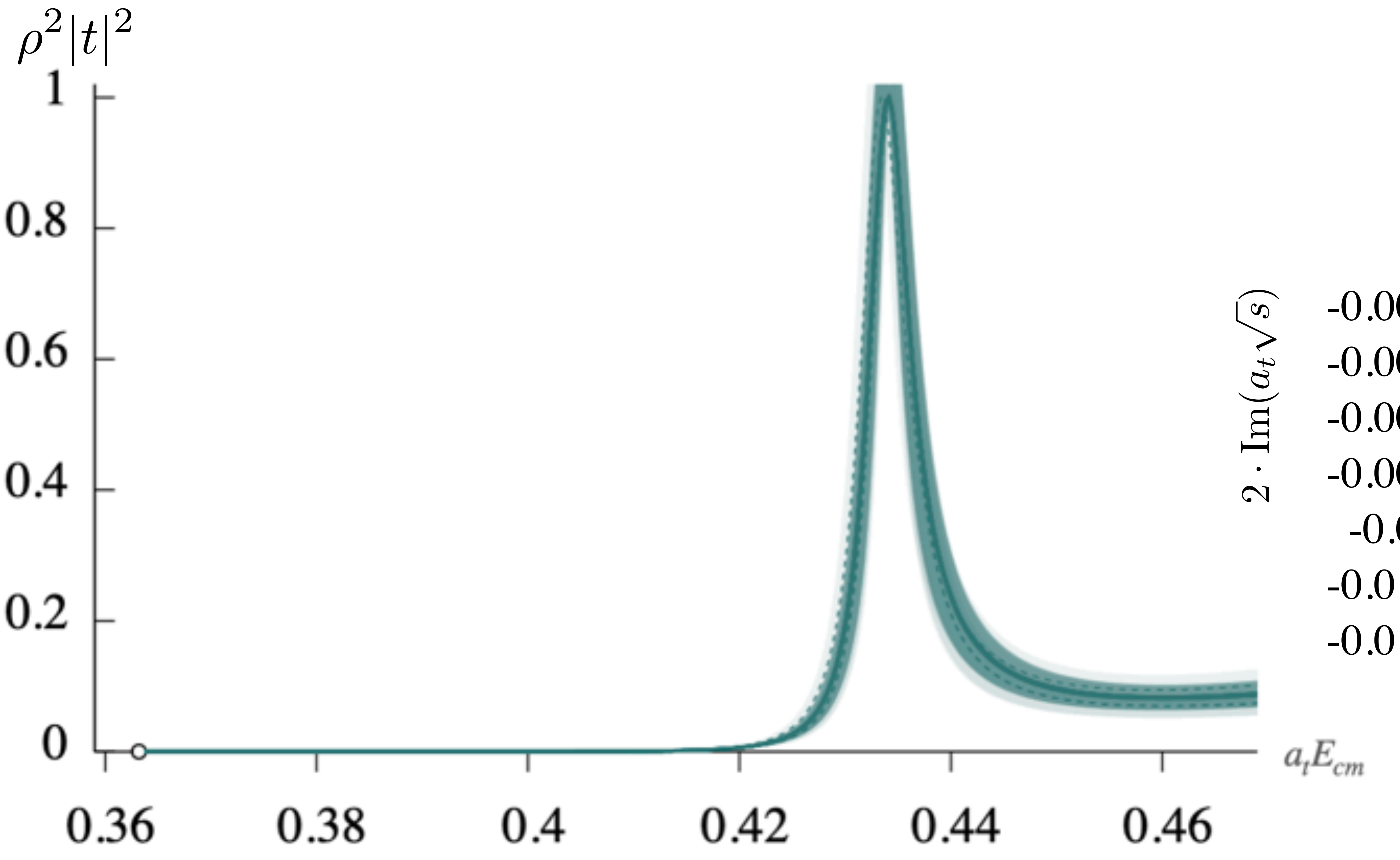


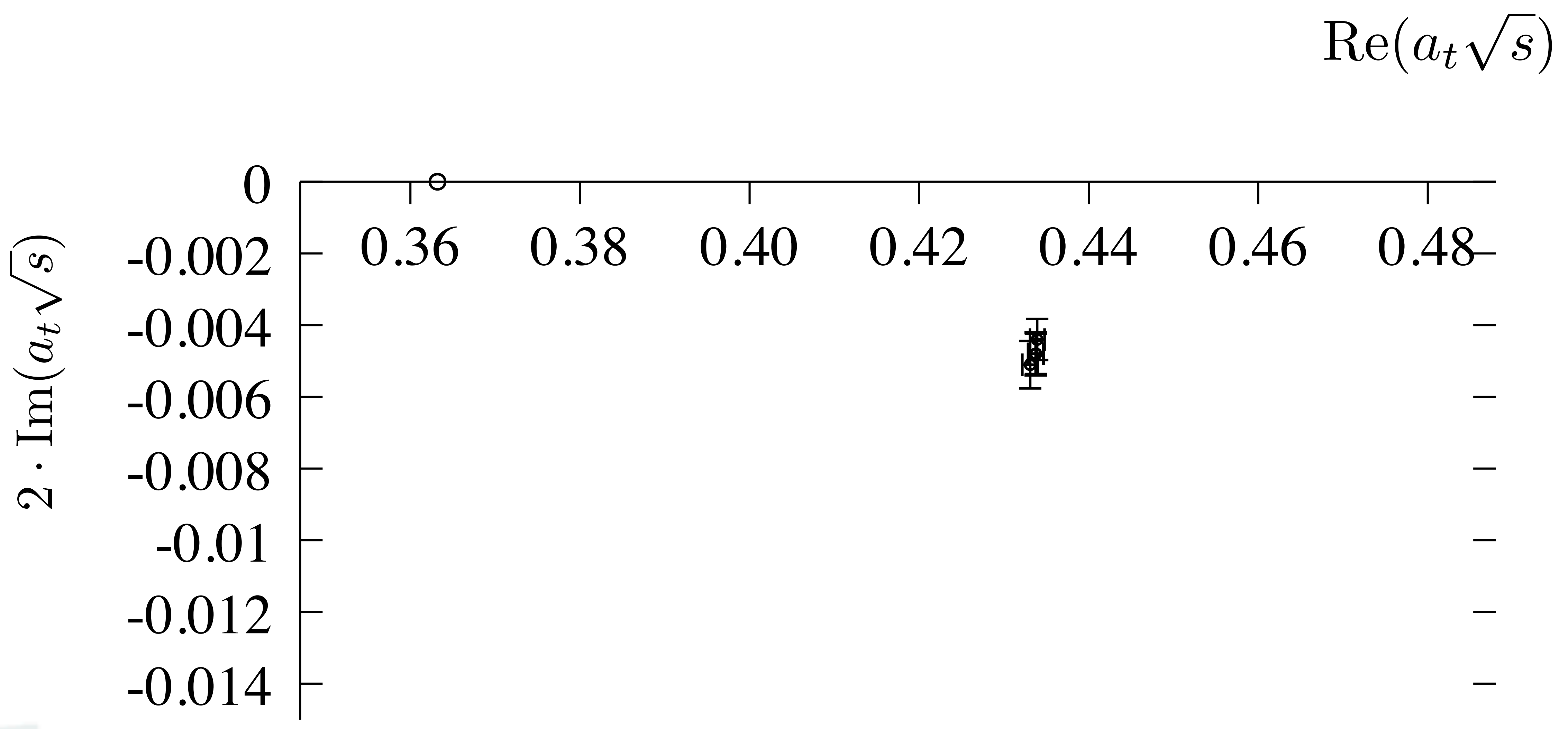


Thanks

Extra

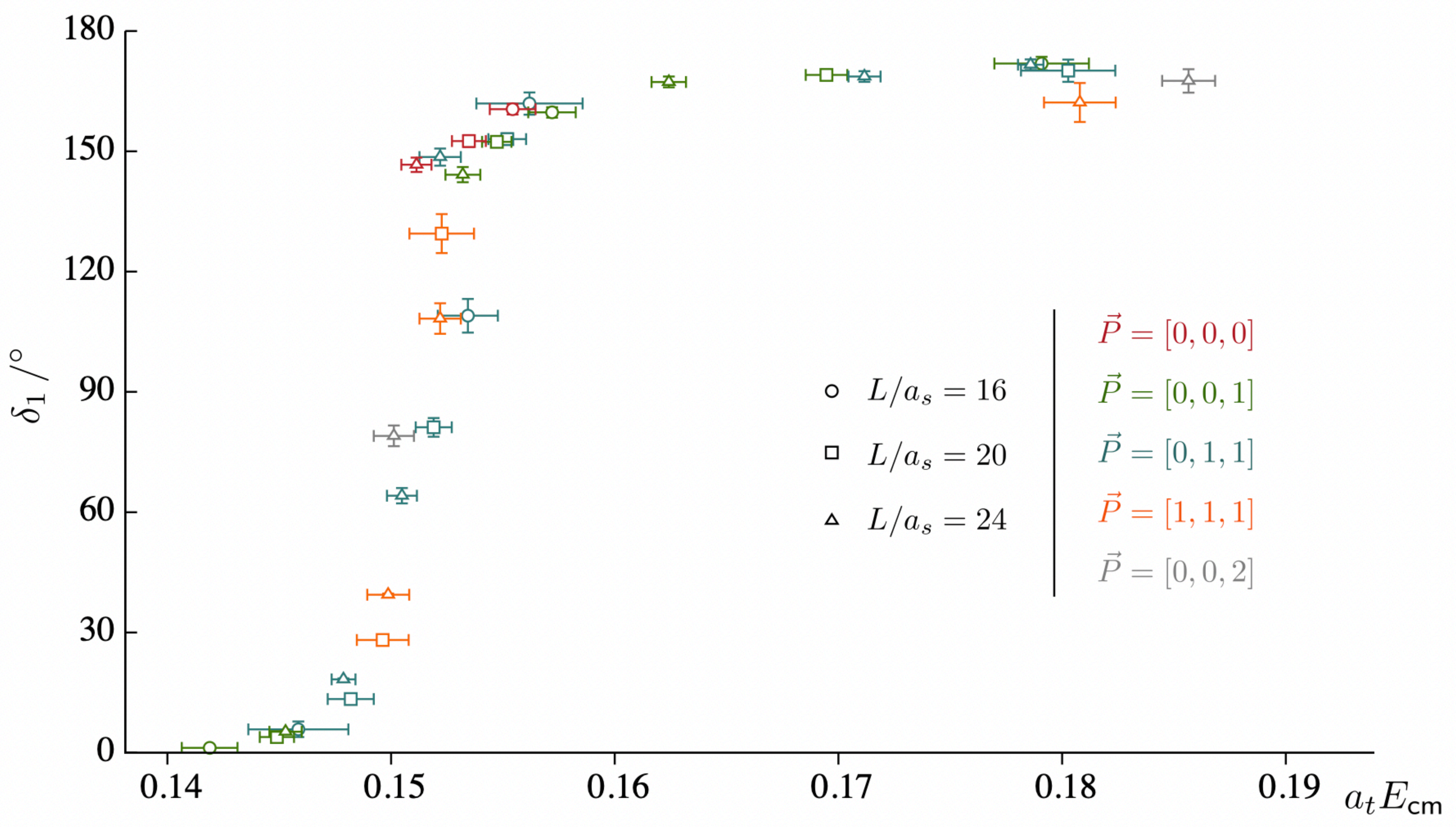
$\eta^8\omega^8$ elastic scattering in 3^{--}





Scattering in a finite volume

Luscher's quantization condition: $\det\left[1 + i\boldsymbol{\rho}\cdot\mathbf{t}\cdot\left(\mathbf{1}+i\mathbf{M}\right)\right] = 0$

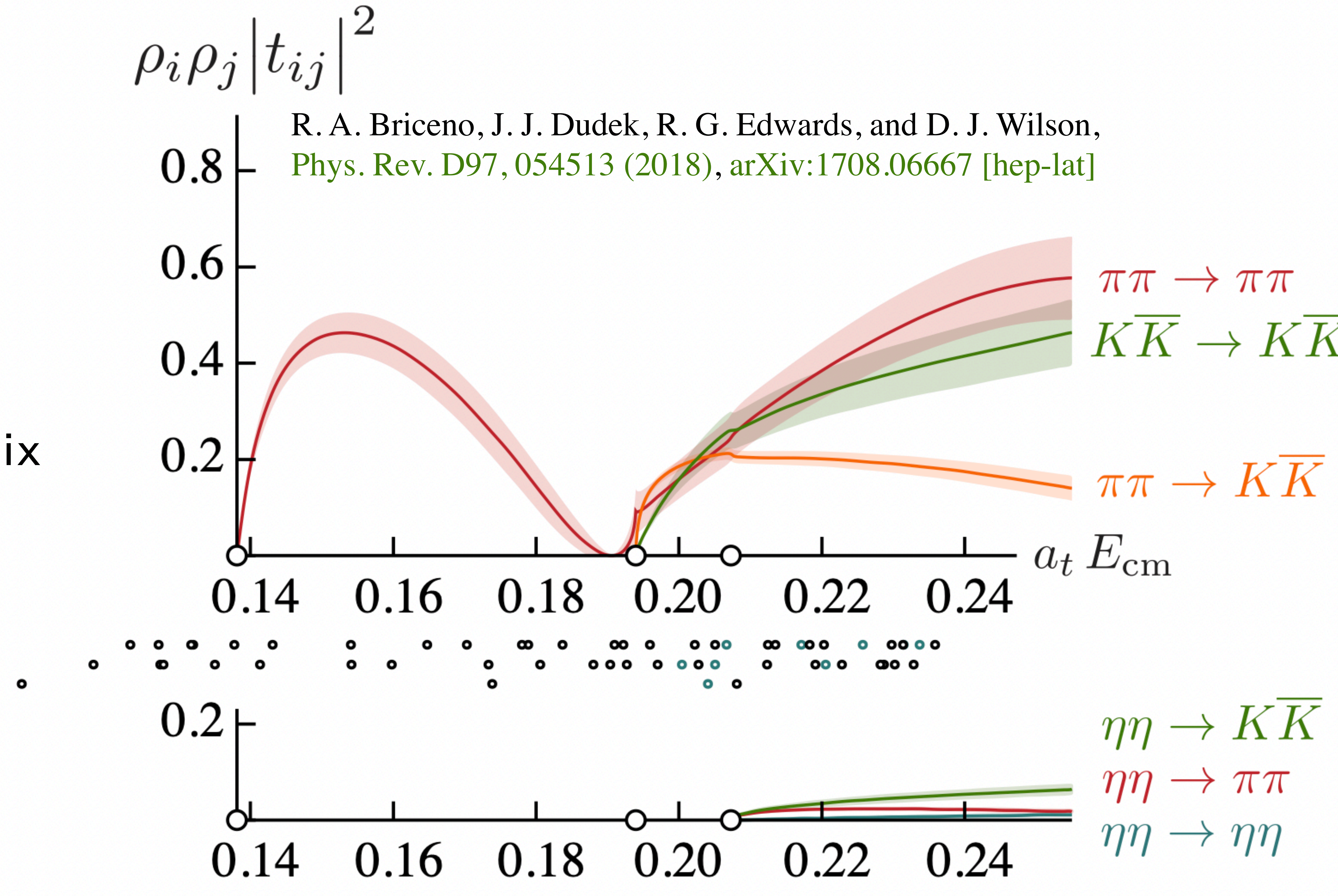


J. J. Dudek, R. G. Edwards, and C. E. Thomas, Phys. Rev. D86, 034031 (2012), arXiv:1203.6041 [hep-ph]

$$\mathrm{Im}\,(t(E))^{-1}_{ij} = -\delta_{ij}\rho_i(E)$$

$$\rho_i(E) = \frac{2k_i}{E}$$

$M_{ij}(E,L)$ is a known matrix



Elastic scattering for spin-zero particles

$$\Rightarrow t_\ell(E) = \frac{1}{\rho(\cot\delta_\ell(E) - i)}$$

Coupled channel systems:

$$\Rightarrow \mathbf{t}^{-1}(E) = \mathbf{K}^{-1}(E) - i\boldsymbol{\rho}(E)$$

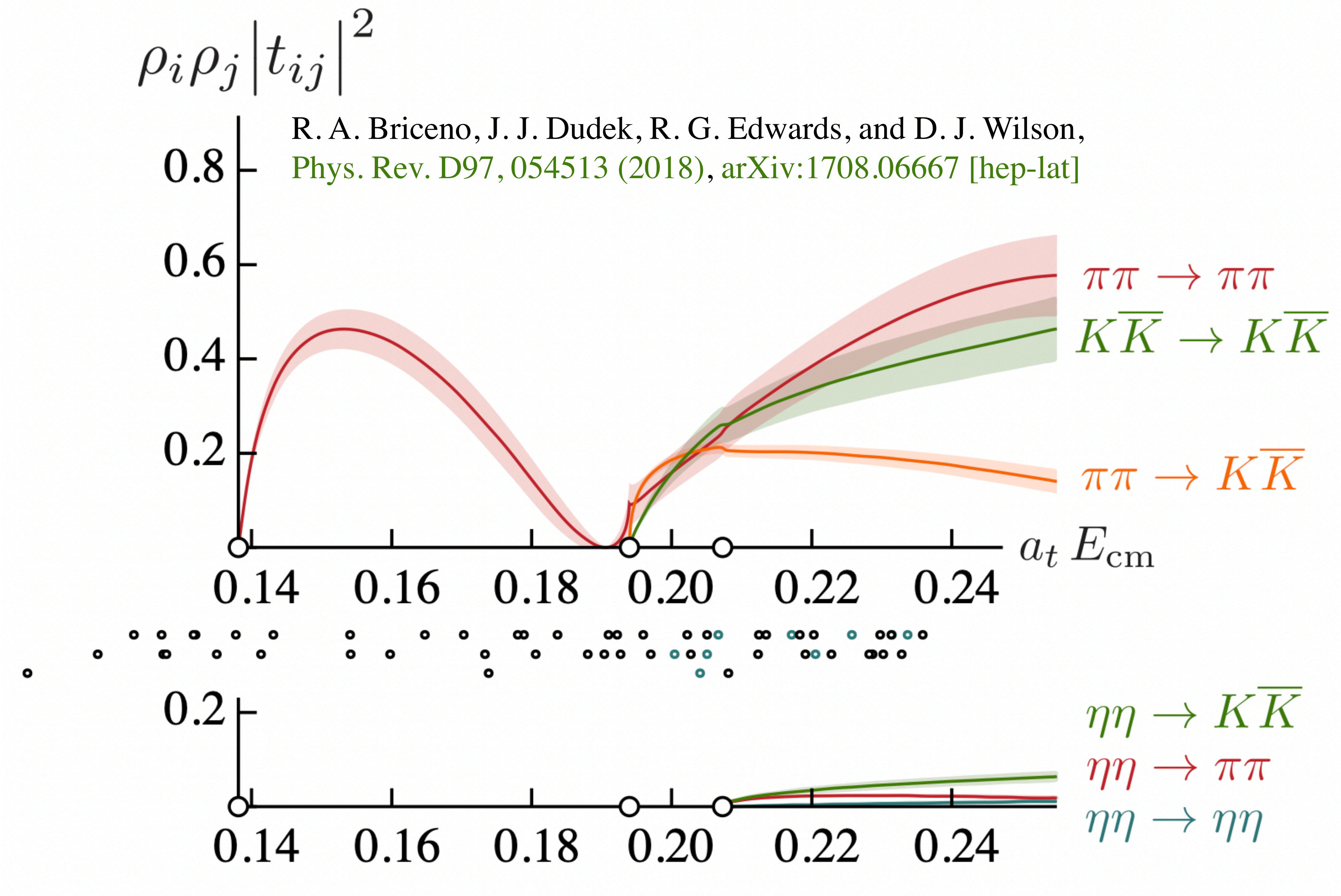
Real and symmetric

Coupled-channel

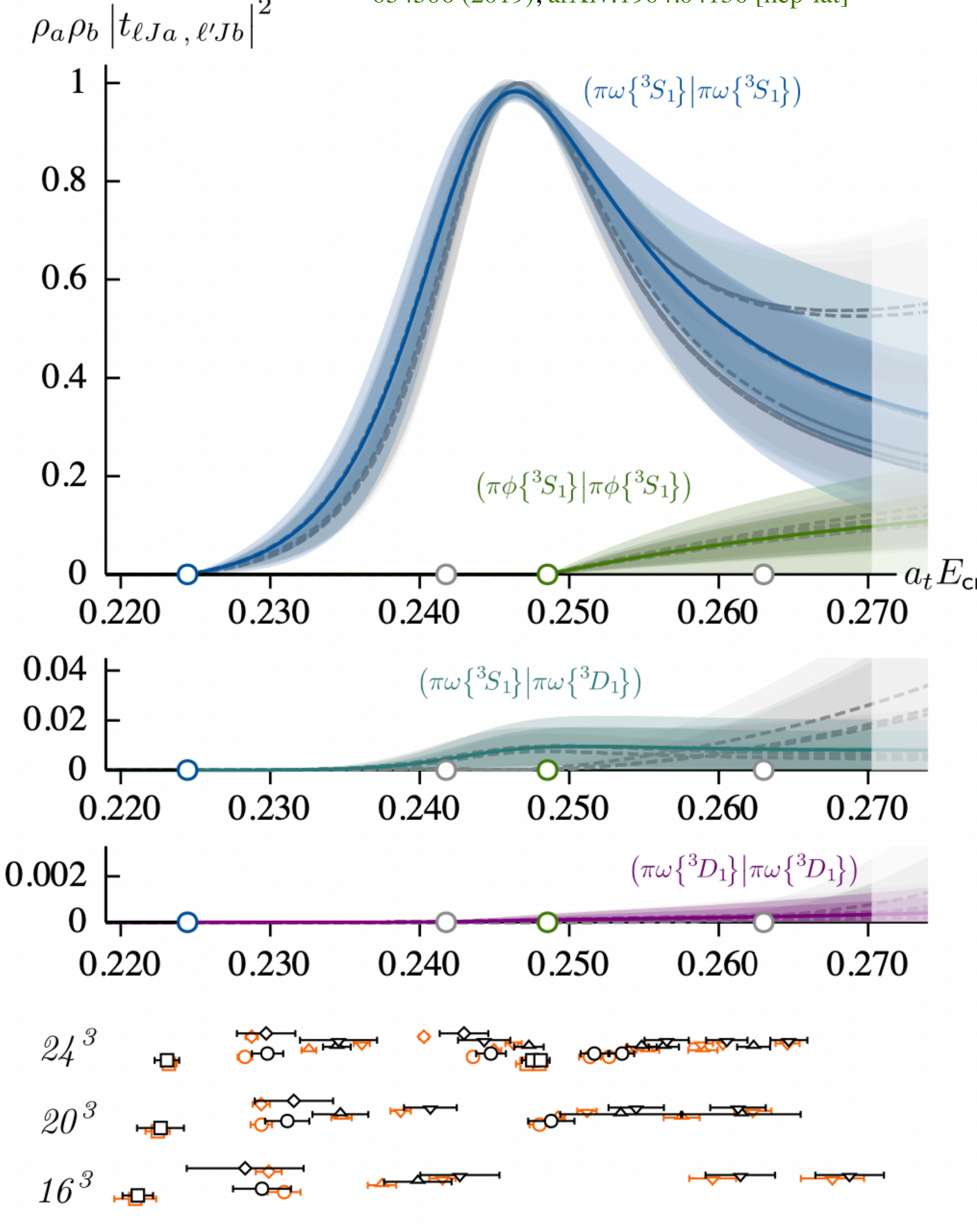
$$\det\left[1 + i\boldsymbol{\rho}\cdot\mathbf{t}\cdot\left(\mathbf{1} + i\mathbf{M}\right)\right] = 0$$

Solutions follow from K-matrix parameterizations of the amplitude :

$$\mathbf{t}^{-1} = \mathbf{K}^{-1} - i\boldsymbol{\rho} \qquad K_{ij}(s) = \sum_{\alpha}\frac{g_i^{(\alpha)}g_j^{(\alpha)}}{m_\alpha^2 - s} + \sum_{\beta} s^\beta \gamma_{ij}$$



A. J. Woss, C. E. Thomas, J. J. Dudek, R. G. Edwards, and D. J. Wilson, Phys. Rev. D100, 054506 (2019), arXiv:1904.04136 [hep-lat]



Future

Calculation of the octet is underway:

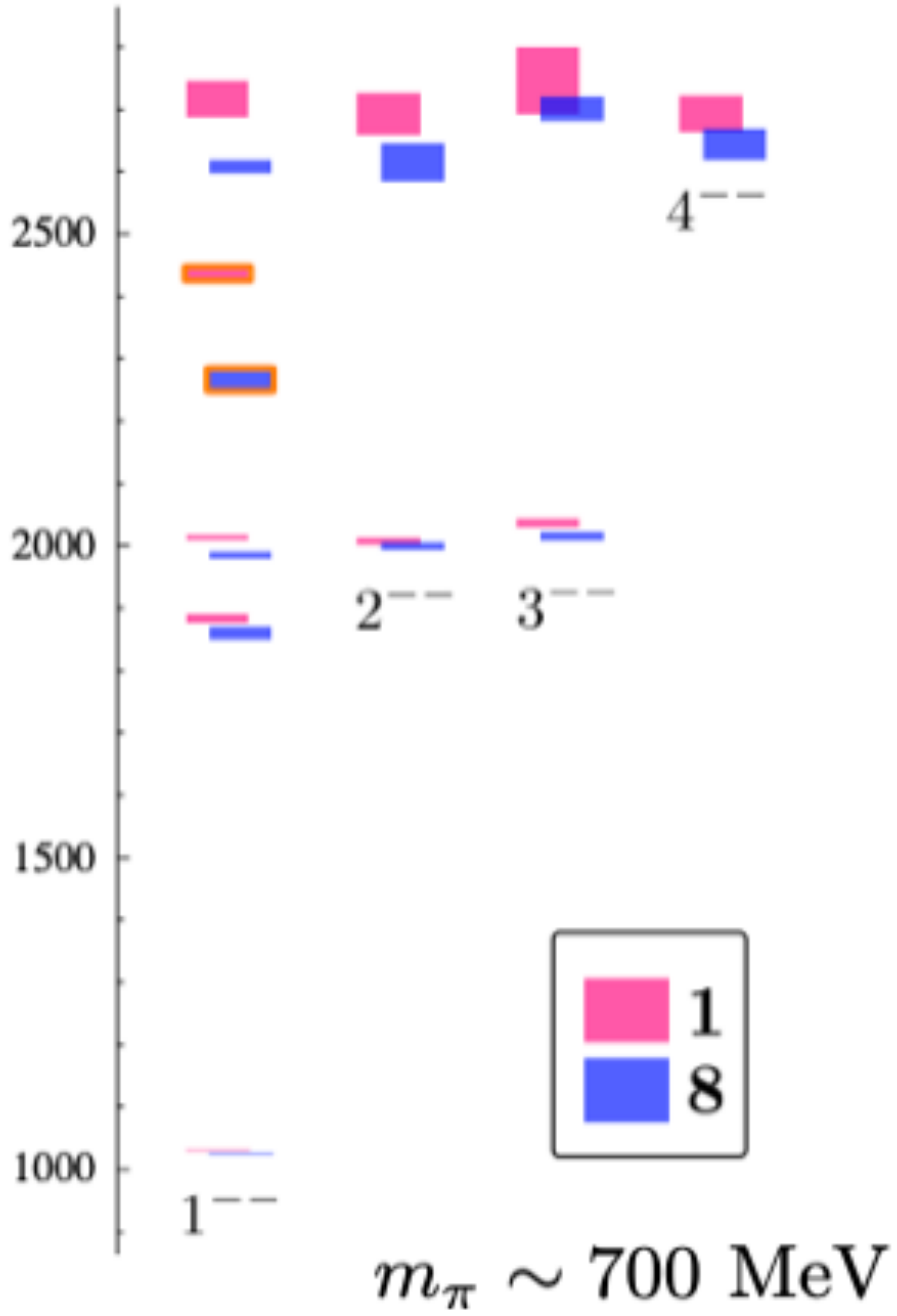
⇒ more channels

⇒ identical particles $\eta^8\eta^8, \omega^8\omega^8$

⇒ nearly degenerate thresholds in $\eta^8\omega^8, \eta^8\omega^1$

Would like to be able to study the hybrid candidate that lies slightly above in 1^{--}

⇒ likely requires three-particle formalism



A crude extrapolation

$$\omega = \sqrt{\frac{2}{3}}\omega_1 + \sqrt{\frac{1}{3}}\omega_8\,;\,\phi = \sqrt{\frac{1}{3}}\omega_1 - \sqrt{\frac{2}{3}}\omega_8$$

Assume an exact OZI symmetry to get the couplings to the octet

Assume width scales with the angular momentum $\sim k^{\ell}$

$$g^1 = \left|\frac{k^{phys}(M^{phys})}{k(M)}\right|^{\ell} |c_{\eta^8\omega^8}|$$

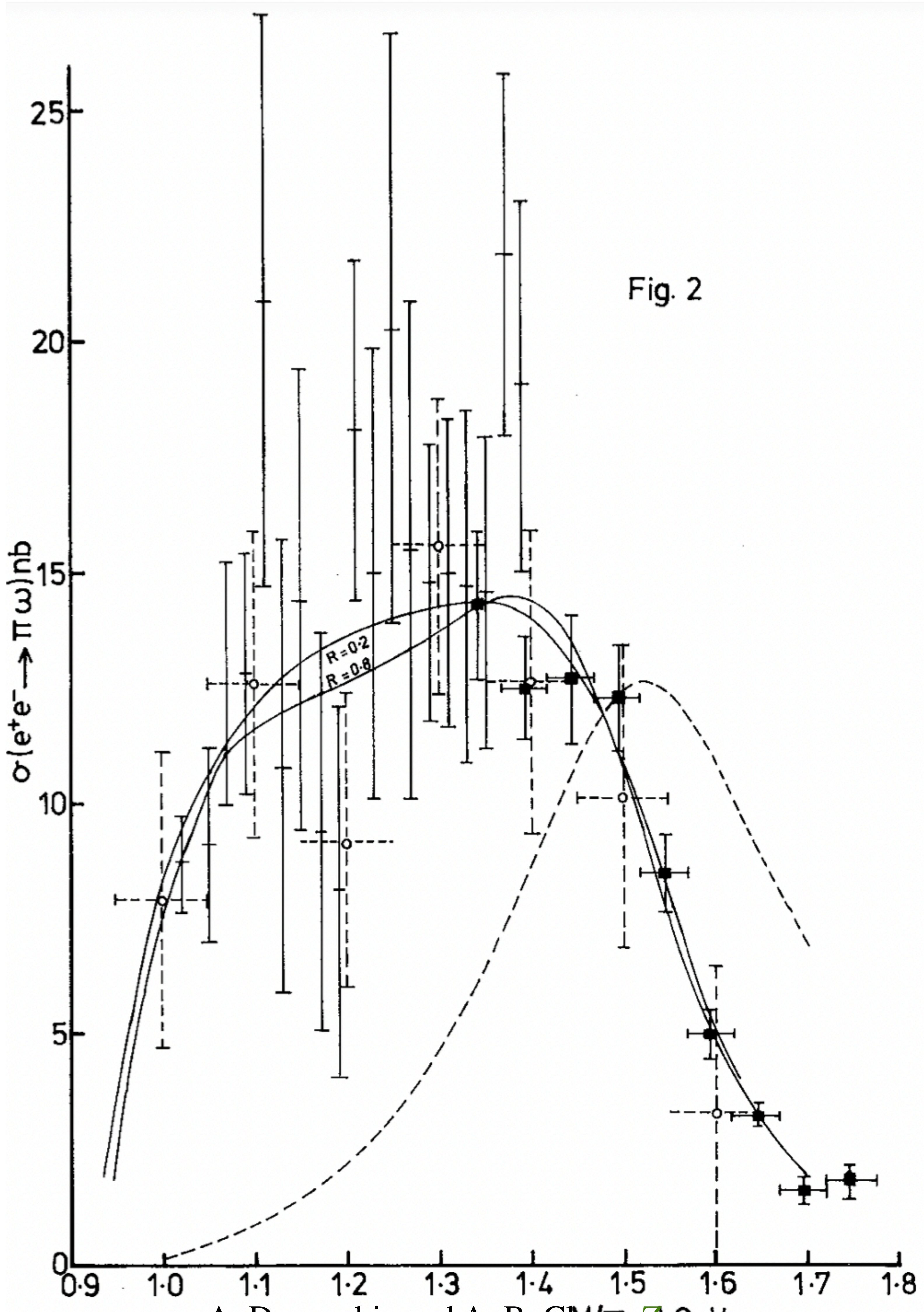
Octet calculation is underway

Calculation	PDG
$\Gamma^{\pi\rho}_{\omega_a} \sim 384$ MeV	$\Gamma^{\pi\rho}_{\omega(1420)} \sim 240$ MeV
$\Gamma^{K\bar{K}^*}_{\omega_a} \sim 4$ MeV	$\Gamma^{tot}_{\omega(1420)} \sim 290(120)$ MeV
$\Gamma^{\eta\omega}_{\omega_a} \sim 5$ MeV	
$\Gamma^{K\bar{K}^*}_{\phi_a} \sim 154$ MeV	$\Gamma^{tot}_{\phi(1680)} \sim 150(50)$ MeV
$\Gamma^{\eta\omega}_{\phi_a} \sim 25$ MeV	
$\Gamma^{\pi\omega}_{\rho_a} \sim 133$ MeV	$\Gamma^{\pi\omega}_{\rho(1450)} \sim$ 52-78 MeV
$\Gamma^{K\bar{K}^*}_{\rho_a} \sim 9$ MeV	$\Gamma^{tot}_{\rho(1450)} \sim 400(60)$ MeV

Calculation	PDG
$\Gamma^{\pi\rho}_{\omega_3} \sim 62$ MeV	$\Gamma^{tot}_{\omega_3(1670)} \sim 168(10)$ MeV
$\Gamma^{K\bar{K}^*}_{\omega_3} \sim 2$ MeV	
$\Gamma^{\eta\omega}_{\omega_3} \sim 1$ MeV	
$\Gamma^{K\bar{K}^*}_{\phi_3} \sim 20$ MeV	$\Gamma^{tot}_{\phi_3(1850)} \sim 87(25)$ MeV
$\Gamma^{\eta\phi}_{\phi_3} \sim 3$ MeV	
$\Gamma^{\pi\omega}_{\rho_3} \sim 22$ MeV	$\Gamma^{\pi\omega}_{\rho_3(1690)} \sim 30(10)$ MeV
$\Gamma^{K\bar{K}^*}_{\rho_3} \sim 2$ MeV	$\Gamma^{K\bar{K}\pi}_{\rho_3(1690)} \sim 7$ MeV

Calculation	PDG
$\Gamma^{\pi\rho}_{\omega_b} \sim 25$ MeV	$\Gamma^{\pi\rho}_{\omega(1650)} \sim 84$ MeV
$\Gamma^{K\bar{K}^*}_{\omega_b} \sim 3$ MeV	$\Gamma^{tot}_{\omega(1650)} \sim 315(35)$ MeV
$\Gamma^{\eta\omega}_{\omega_b} \sim 1$ MeV	
$\Gamma^{\pi\omega}_{\rho_b} \sim 9$ MeV	$\Gamma^{\pi\omega}_{\rho(1700)} \sim 0$ MeV
$\Gamma^{K\bar{K}^*}_{\rho_b} \sim 3$ MeV	$\Gamma^{tot}_{\rho(1700)} \sim 250(100)$ MeV

A. B. Clegg and A. Donnachie, Z. Phys. C 62, 455 (1994).



A. Donnachie and A. B. Clegg, Z. Phys. C 51, 689 (1991)

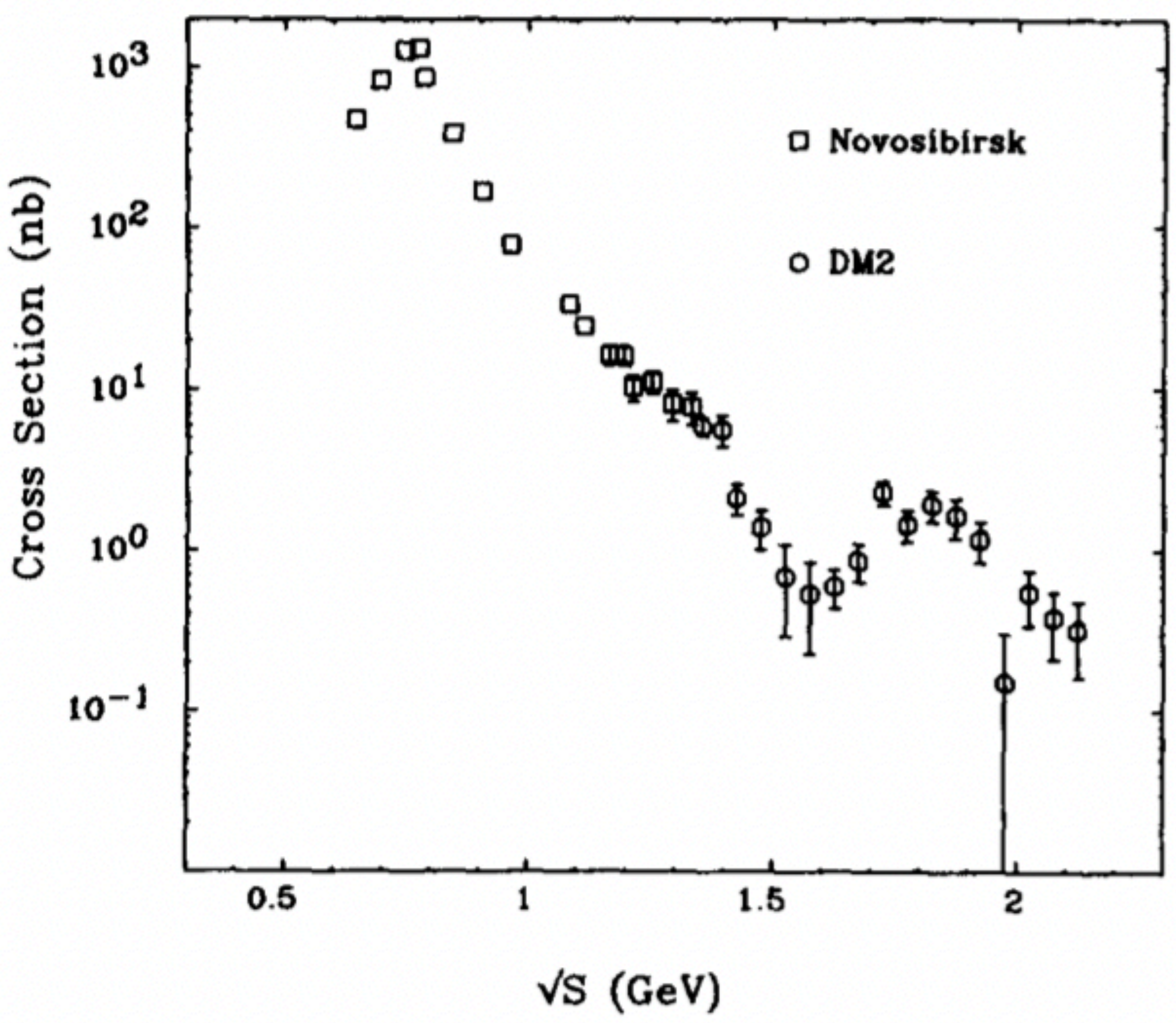
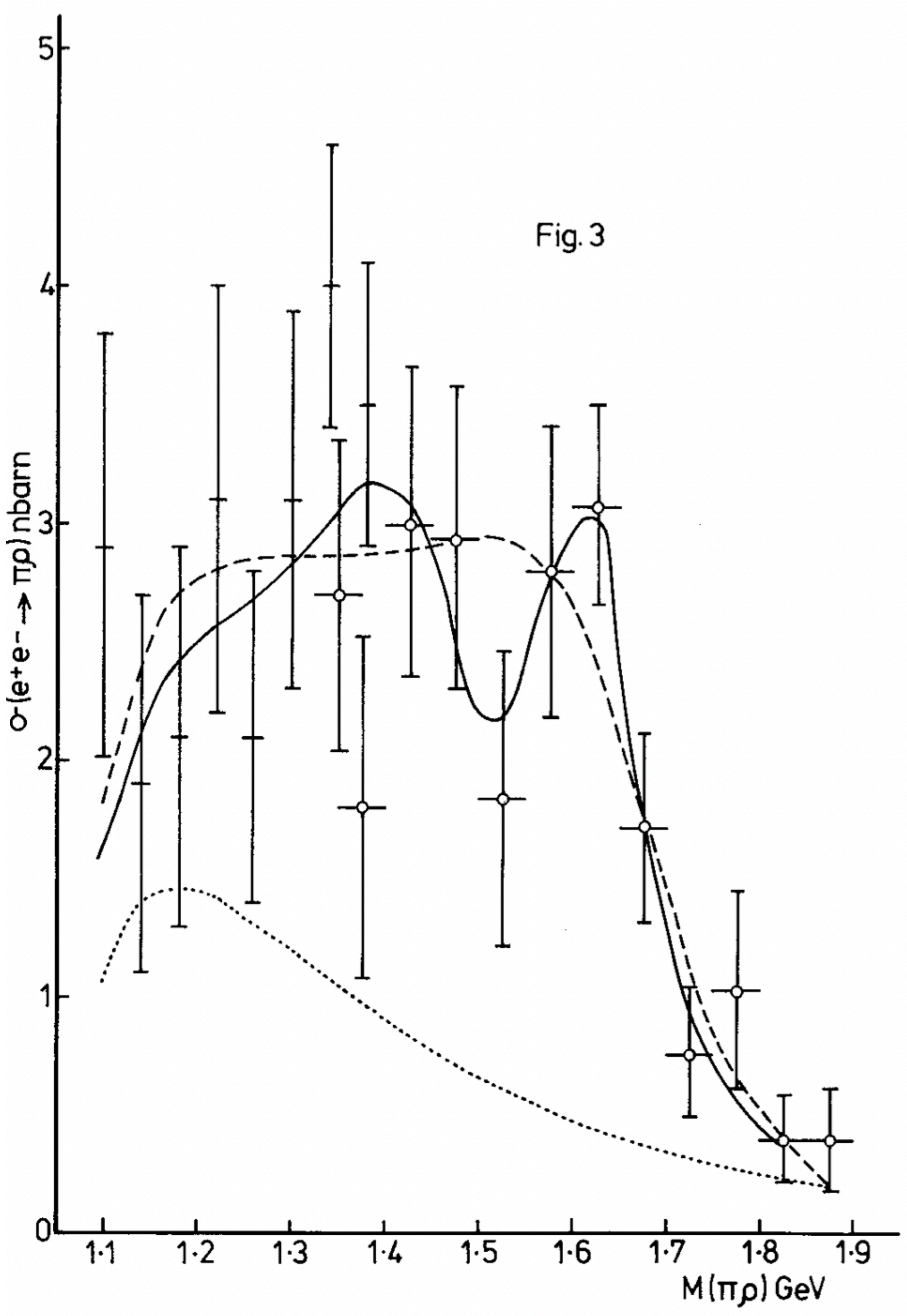


Fig. 5. $e^+e^- \to \pi^+\pi^-$ cross section versus $\sqrt{s}$. The Novosibirsk points are from ref. [2].

D. Bisello et al. (DM2), Phys. Lett. B 220, 321 (1989)



A. Donnachie and A. B. Clegg, Z. Phys. C 51, 689 (1991)

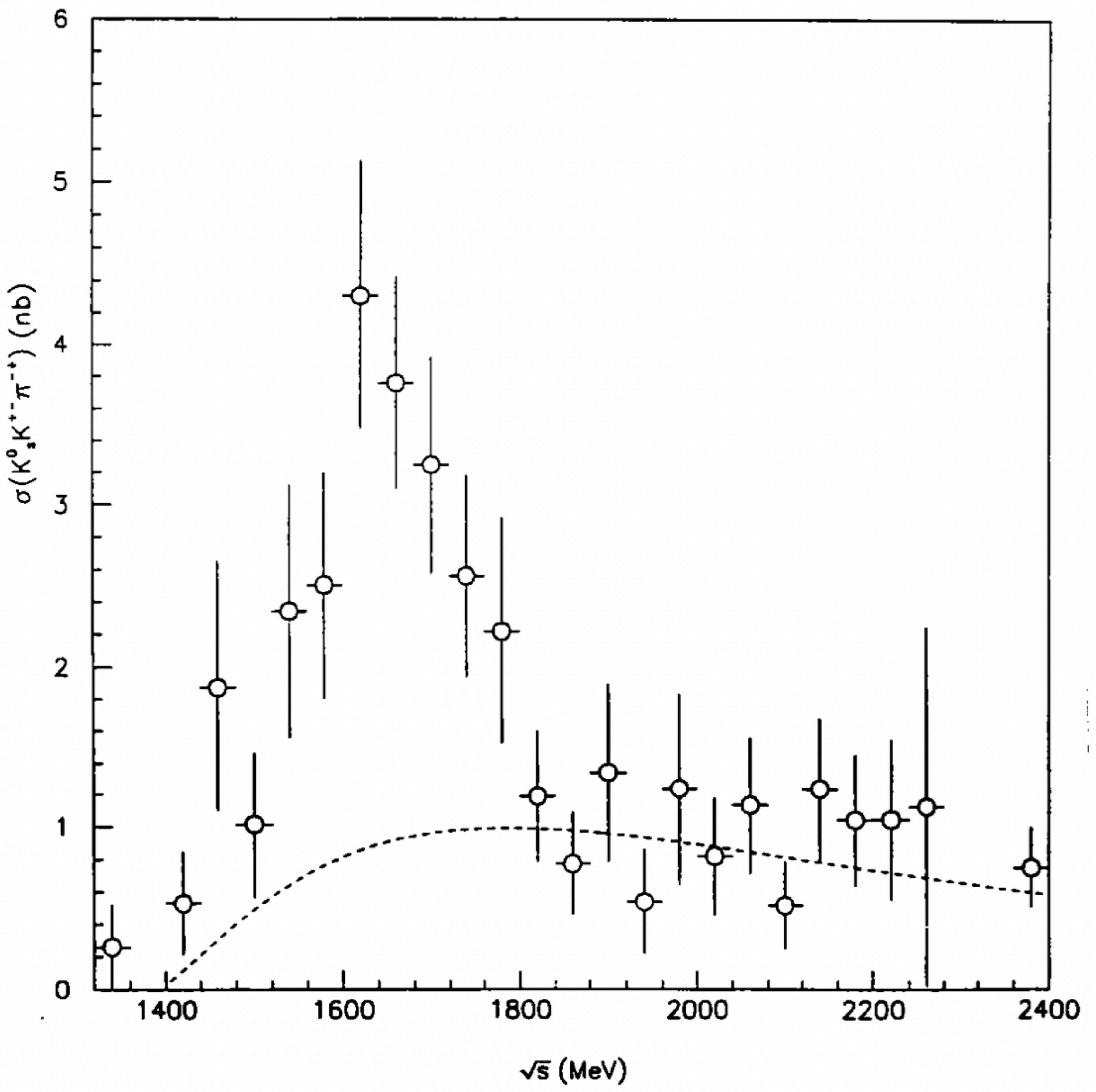


Fig. 2 $K_s^0 K^{\pm}\pi^{\mp}$ cross section. The dashed line shows the ρ, ω, ϕ tail contribution.

D. Bisello et al., Z. Phys. C 52, 227 (1991)

Lattice QCD

Optimized operator constructed from applying the eigenvectors extracted from applying the variational method $h^\dagger = \sum_i v_i O_i$

Finite volume spectrum $\Rightarrow C_{ij}(t) = \sum_\alpha \langle 0 | O_i | \alpha \rangle \langle \alpha | O_j | 0 \rangle e^{-E_\alpha t}$

Single meson operators: $\sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \bar{\psi} \overleftrightarrow{D} \overleftrightarrow{D} \dots \overleftrightarrow{D} \psi$

Momentum is quantized $\vec{p} = \frac{2\pi}{L}\vec{n}$

Meson-meson operators: $\sum_{\vec{p}_1 + \vec{p}_2 = \vec{P}} C(\vec{p}_1, \vec{p}_2; \vec{P}) h_1^\dagger(\vec{p}_1) h_2^\dagger(\vec{p}_2)$

No interactions

$$E = \sqrt{m_1^2 + \left(\frac{2\pi \vec{n}_1}{L}\right)^2} + \sqrt{m_2^2 + \left(\frac{2\pi \vec{n}_2}{L}\right)^2}$$

Variational Method

Diagonalize matrix of correlation functions to produce the finite volume spectrum:

$$C(t)v^{\alpha}(t) = \lambda^{\alpha}(t)C(t_0)v^{\alpha}(t)$$

$$\sim e^{-E^{\alpha}(t-t_0)}$$

$$\langle 0|O_i|\alpha\rangle = (V_i^{\alpha})^{-1}\sqrt{2E^{\alpha}}e^{E^{\alpha}t_0/2}$$

Use the orthonormality of the eigenvectors to distinguish states, and extract energies from the principle correlators $\lambda^{\alpha}(t)$.

Coupled channel with nonzero spin

Orbital and angular momentum couple $\boldsymbol{\ell} \otimes \boldsymbol{S} \to \boldsymbol{J}$

Can use K-matrix to handle this (ex. 0^{-+},1^{--} scattering in $J^P = 1^+$)

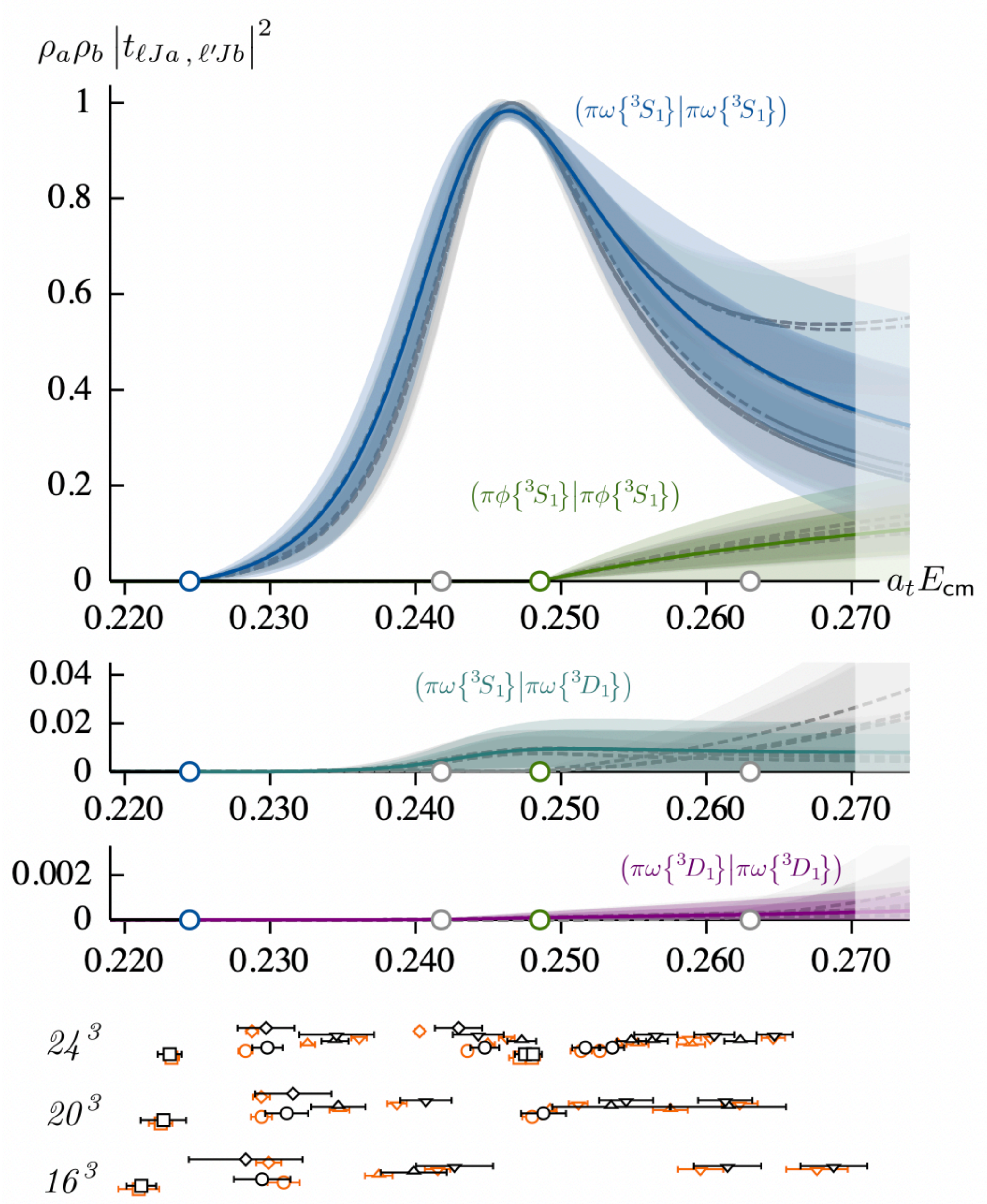
$$K_{1^+} = \begin{pmatrix} \{^3S_1|^3S_1\} & \{^3S_1|^3D_1\} \\ \{^3S_1|^3D_1\} & \{^3D_1|^3D_1\} \end{pmatrix}$$

ℓ	J^P
0	1^+
1	$(0,1,2)^-$
2	$(1,2,3)^+$
3	$(2,3,4)^-$
...	

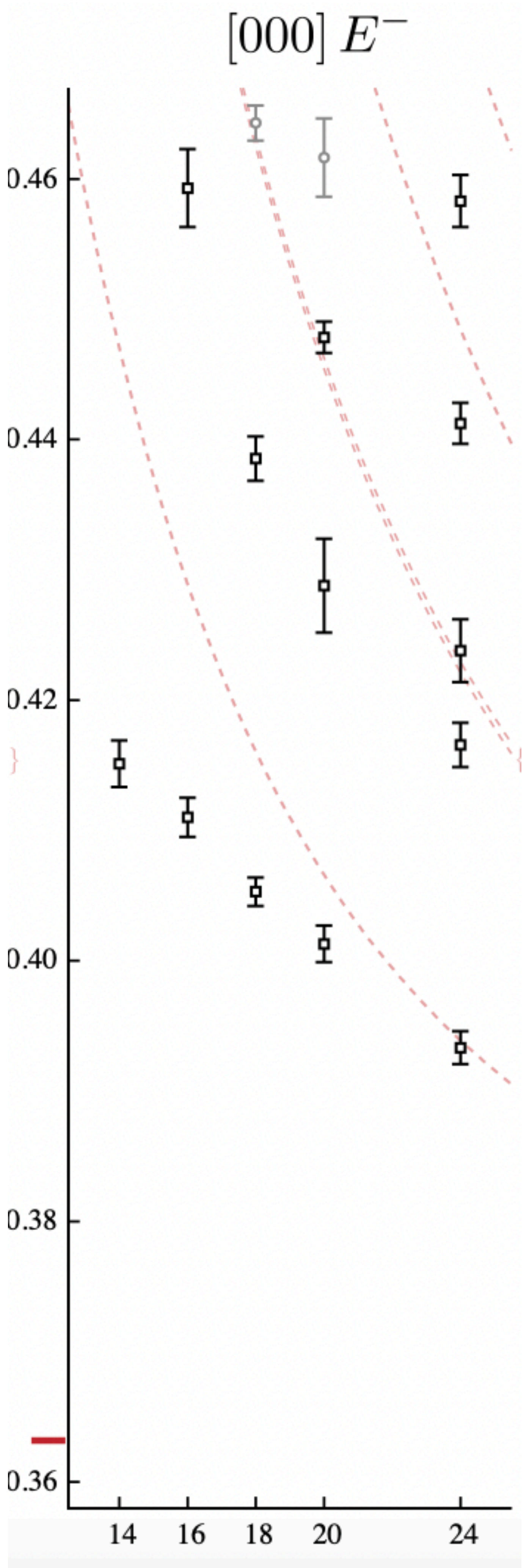
Done in both non-resonant and resonant systems:

"Dynamically-coupled partial-waves in $\rho\pi$ isospin-2 scattering from lattice QCD"- A. Woss, C. Thomas, J. Dudek

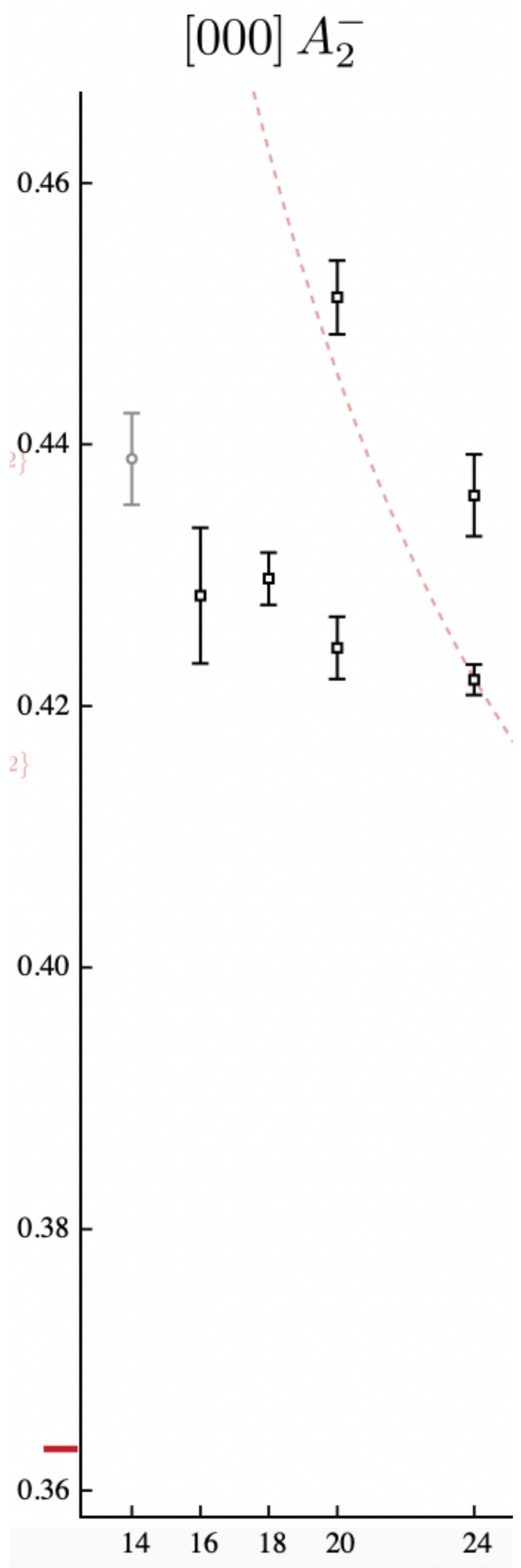
"The b_1 resonance in coupled $\pi\omega, \pi\phi$ scattering from lattice QCD"- A. Woss, C. Thomas, J. Dudek



How do we solve this?



$J^P = (2, \ldots)^-$



$J^P = (3, \ldots)^-$

Broken rotational symmetry of the lattice causes different resonances to be in the same representation.

Typical scattering calculations are able to isolate a SINGLE resonance.

All other irreps will feature a minimum of TWO resonances

$^3D_{1,2,3}$ states are expected to be nearly degenerate.

SU(3) Flavor

Two neutral members basis states $I = I_z = Y = 0$

$$|\mathbf{1}\rangle = \frac{1}{\sqrt{3}}\left(|\bar{u}u\rangle + |\bar{d}d\rangle + |\bar{s}s\rangle\right)$$

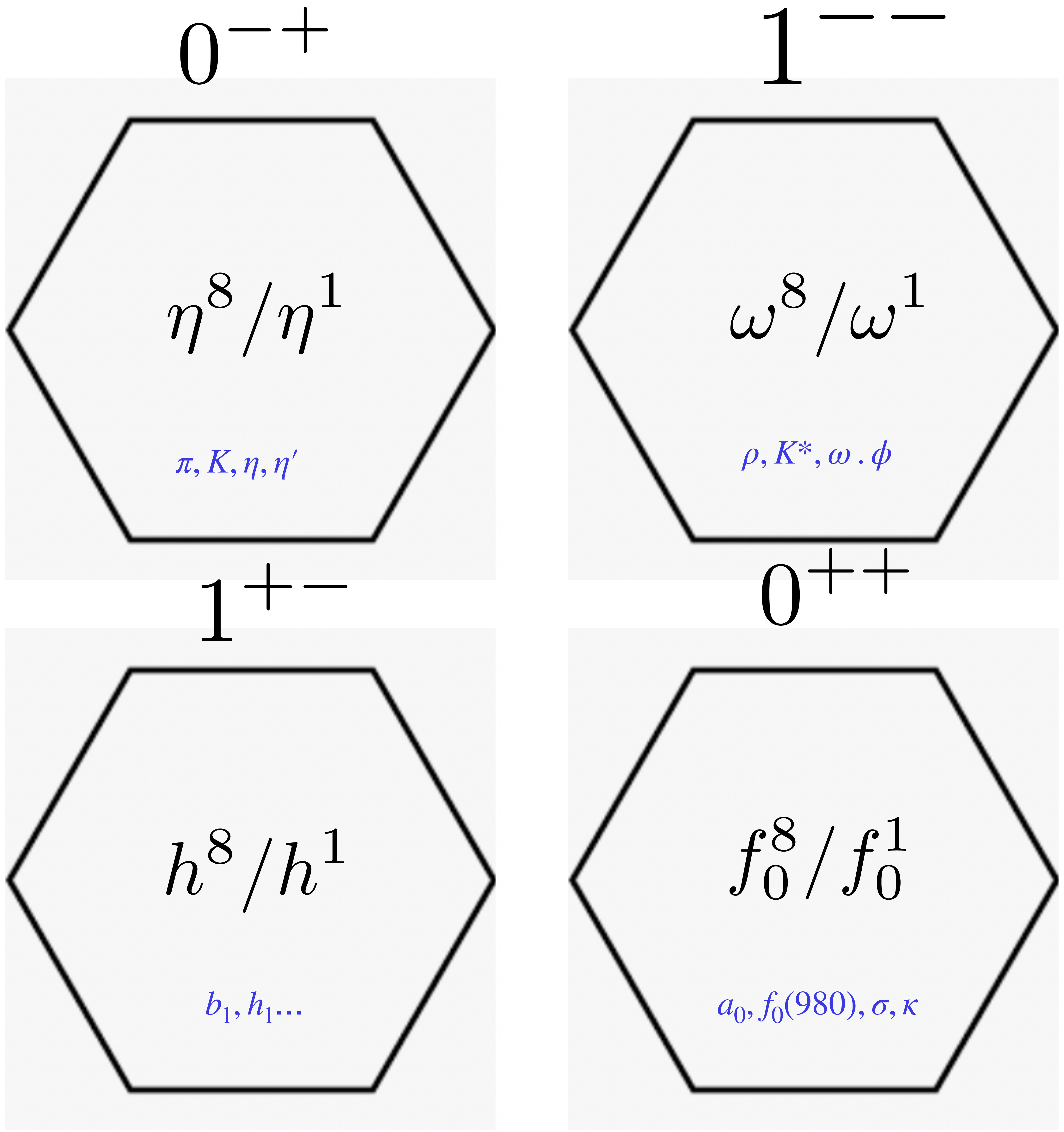
$$|\mathbf{8}\rangle = \frac{1}{\sqrt{6}}\left(|\bar{u}u\rangle + |\bar{d}d\rangle - 2|\bar{s}s\rangle\right)$$

Pseudoscalar have small mixing angle from SU(3) states $\sim -10°$

$$|\eta\rangle \sim |\eta^8\rangle \qquad |\eta'\rangle \sim |\eta^1\rangle$$

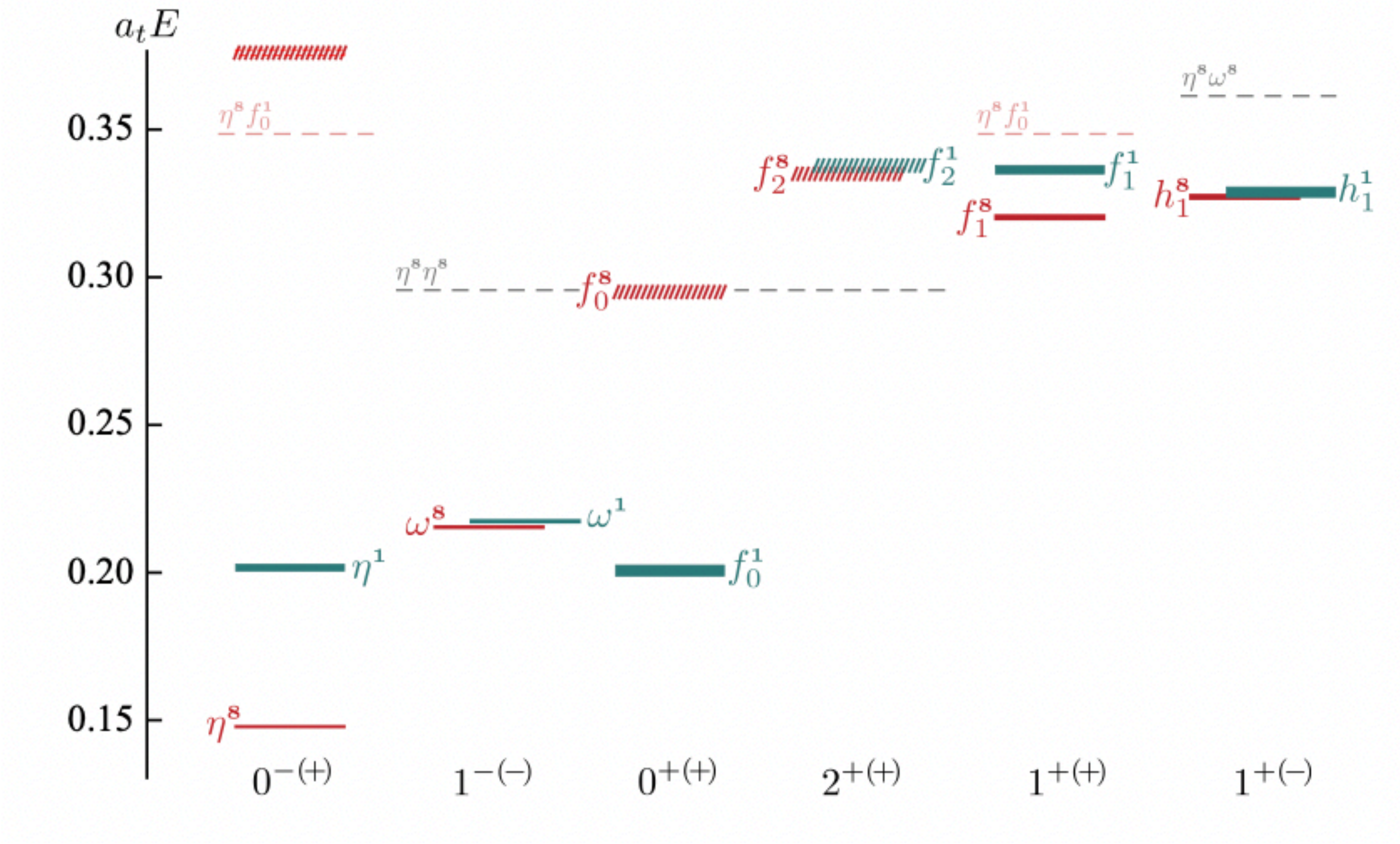
Mixing splits into light and strange quarks (OZI)

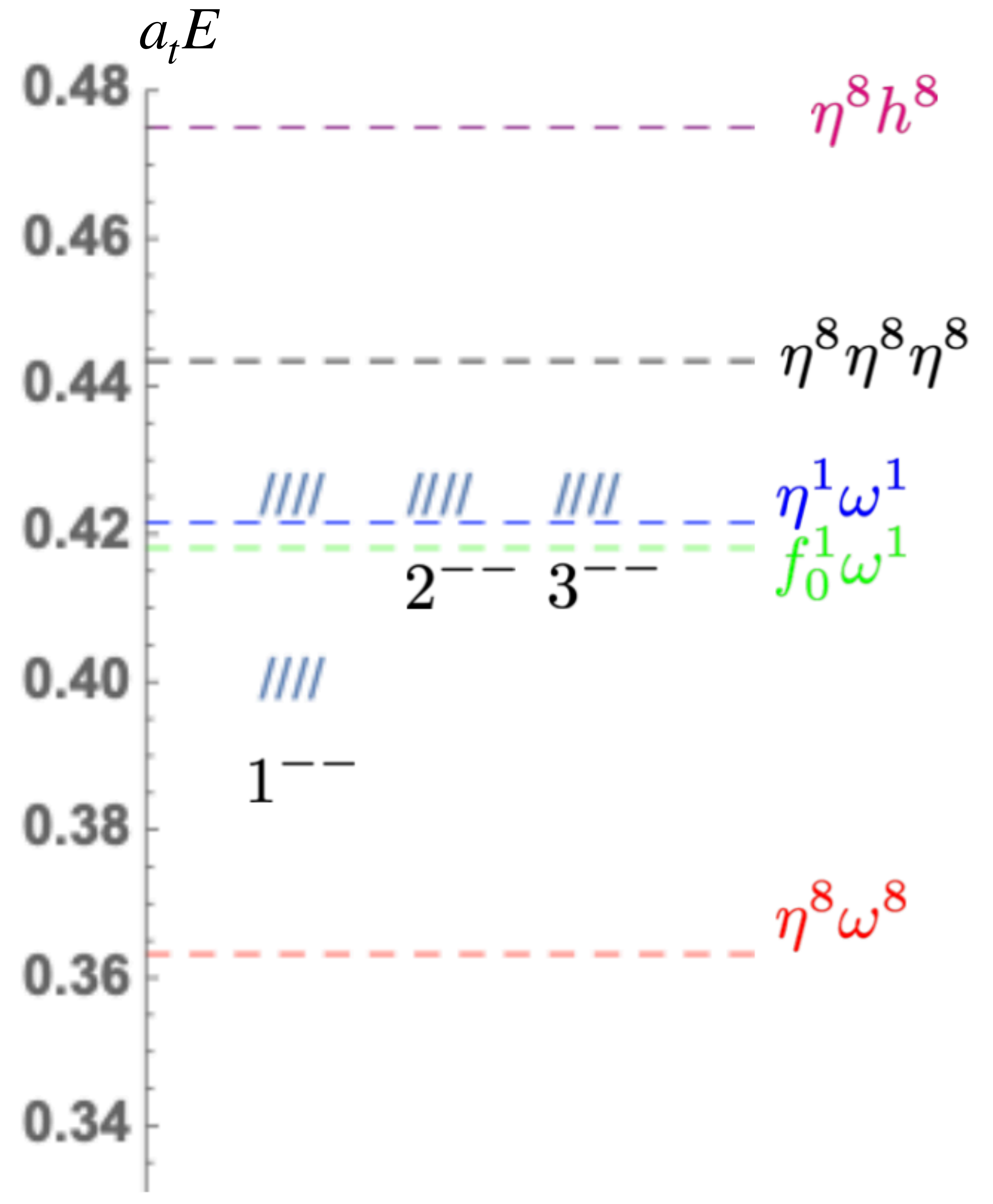
$$|\omega\rangle \sim \frac{1}{\sqrt{2}}\left(|\bar{u}u\rangle + |\bar{d}d\rangle\right) \qquad |\phi\rangle \sim |\bar{s}s\rangle$$



SU(3) Flavor

η^8	0.1478(1)	η^1	0.2017(11)
ω^8	0.2154(2)	ω^1	0.2174(3)
		f_0^1	0.2007(18)





Lattice QCD

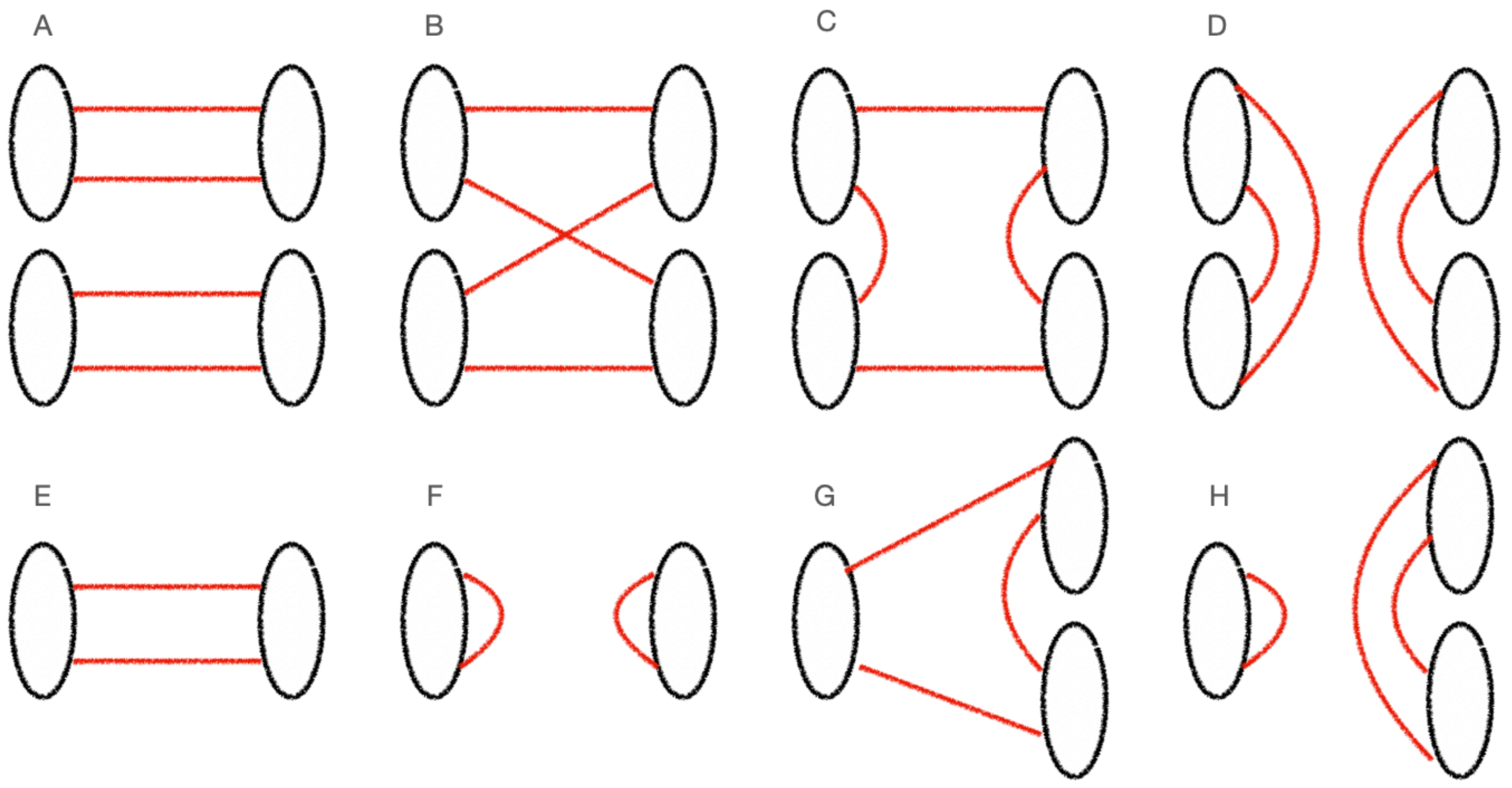
Finite volume spectrum $\Rightarrow C_{ij}(t) = \sum_{\alpha} \langle 0 | O_i | \alpha \rangle \langle \alpha | O_j | 0 \rangle e^{-E_\alpha t}$

Single meson operators: $\sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \bar{\psi} \overleftrightarrow{D} \overleftrightarrow{D} \ldots \overleftrightarrow{D} \psi$

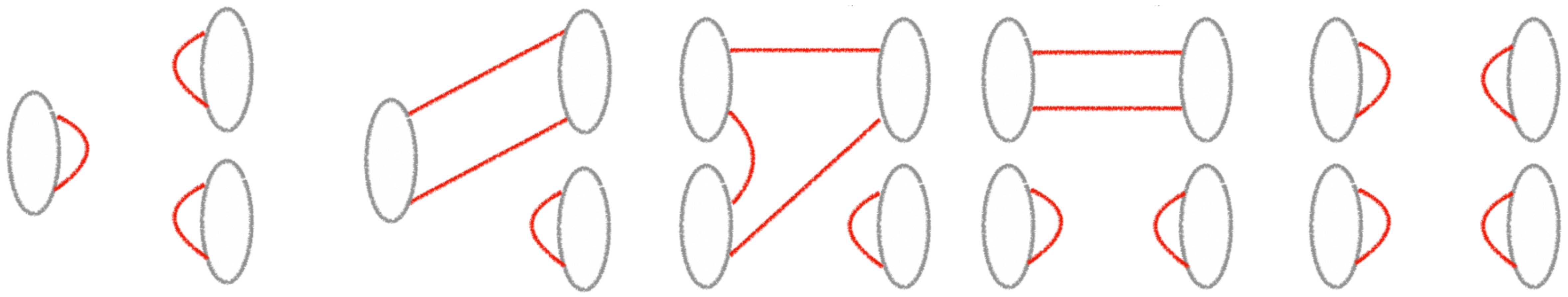
Meson-meson operators: $\sum_{\vec{p}_1+\vec{p}_2=\vec{P}} C(\vec{p}_1, \vec{p}_2; \vec{P}) h_1^\dagger(\vec{p}_1) h_2^\dagger(\vec{p}_2)$

Will include $\eta^8(\vec{p_1})\omega^8(\vec{p_2})$, $\eta^1(\vec{p_1})\omega^1(\vec{p_2})$, $f_0^1(\vec{p_1})\omega^8(\vec{p_2})$

$\eta^8 \omega^8$



$\eta^1 \omega^1 / f_0^1 \omega^1$



Channels in SU(3) Flavor

Conventional $\bar{q}q$ mesons live in either a singlet ($\bar{3} \otimes 3 \to 1$) or octet ($\bar{3} \otimes 3 \to 8$) representations.

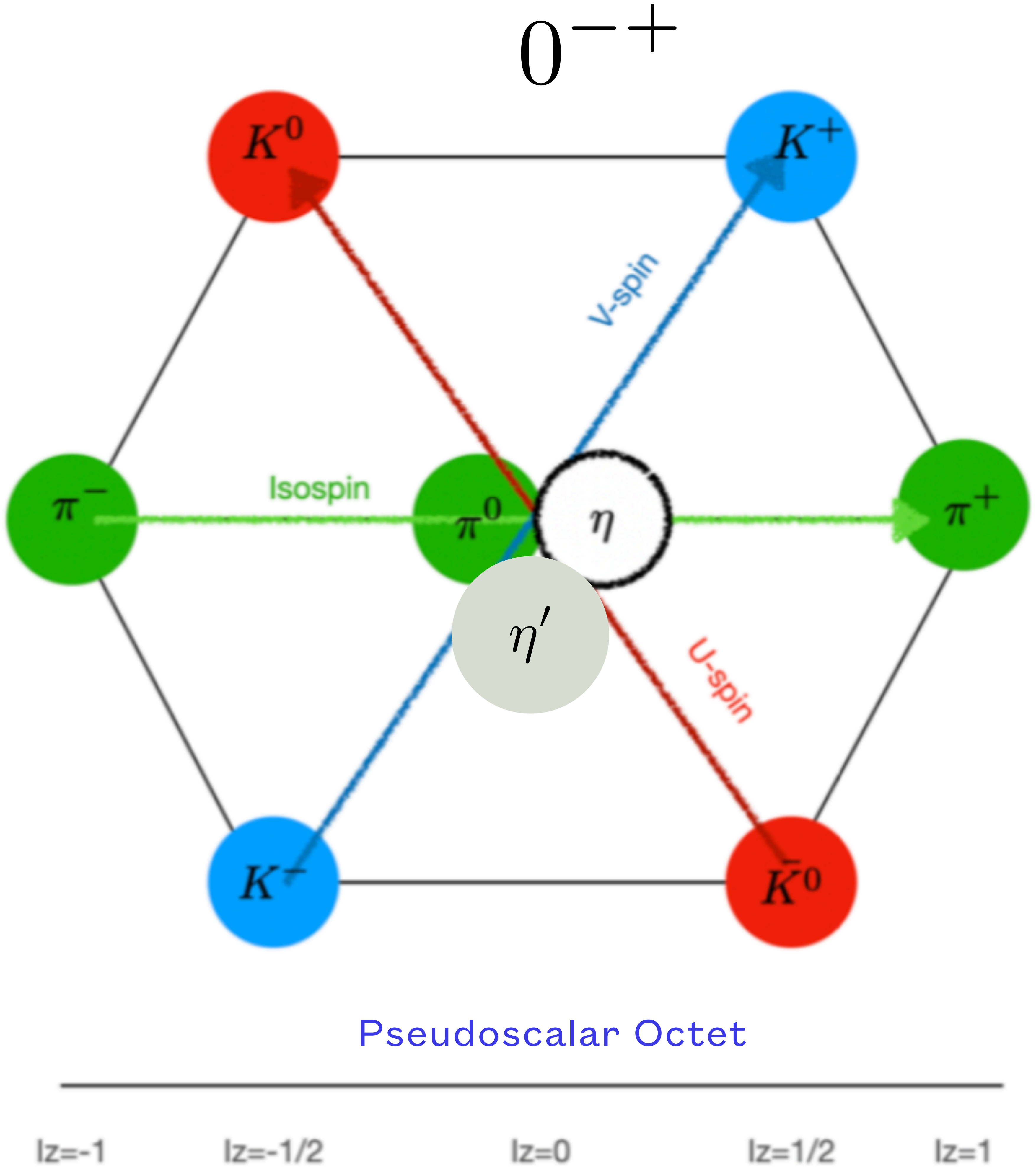
Two ways to project to flavor singlet $8 \otimes 8 \to 1$, and trivially $1 \otimes 1 \to 1$.

Charge conjugation in neutral member of the octet $|I = I_z = Y = 0\rangle$ for $8 \otimes 8 \to 1$:

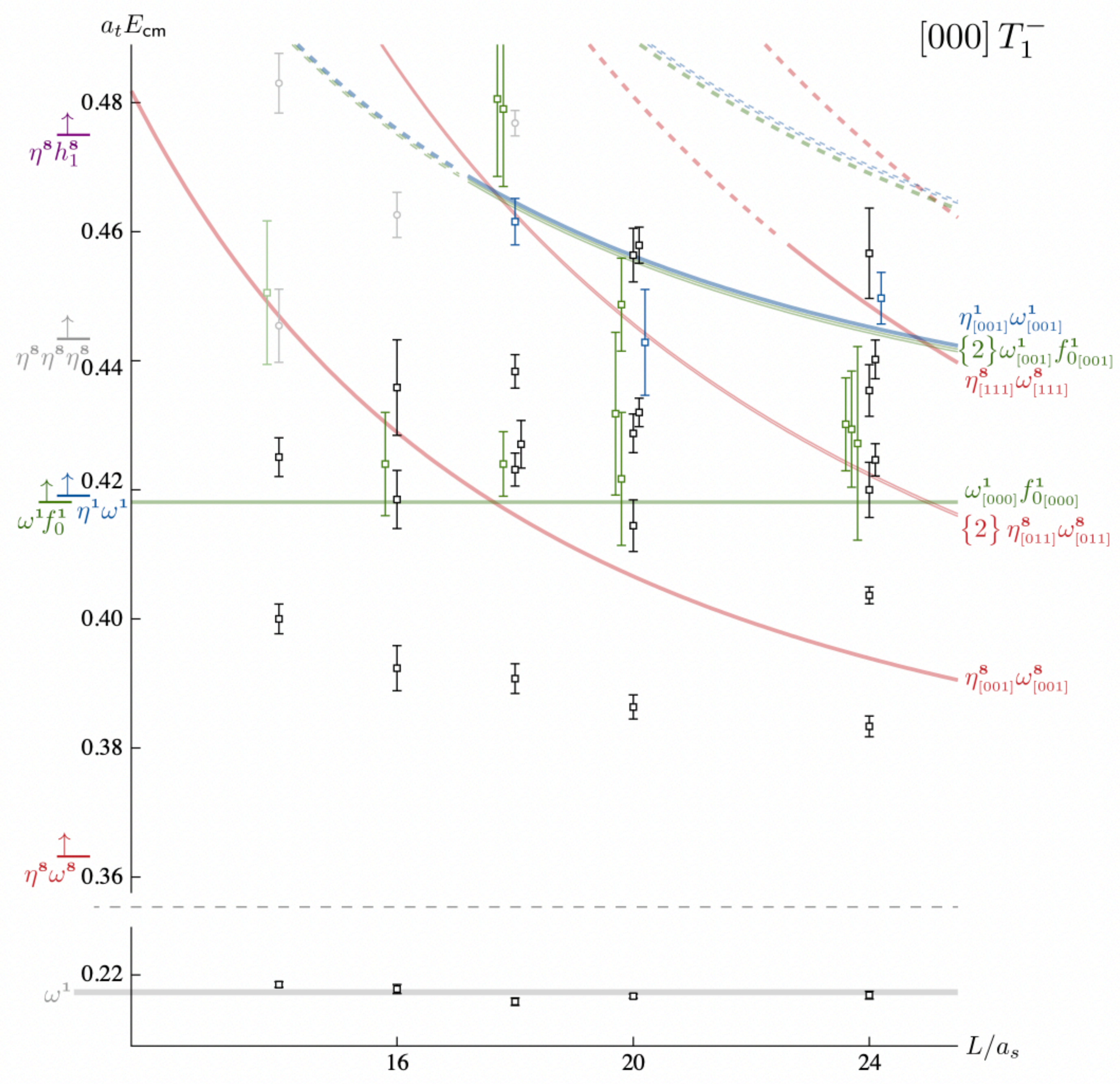
$$\hat{C}(|8_1, C_1\rangle \otimes |8_2, C_2\rangle) \to C_1 C_2(|8_1, C_1\rangle \otimes |8_2, C_2\rangle)$$

⇒ channels with C=- :
$\eta^8(0^{-+})\omega^8(1^{--}), f_0^1(0^{++})\omega^1(1^{--}), \eta^1(0^{-+})\omega^1(1^{--})$

⇒ can't have identical particles with C=-



Pseudoscalar Octet



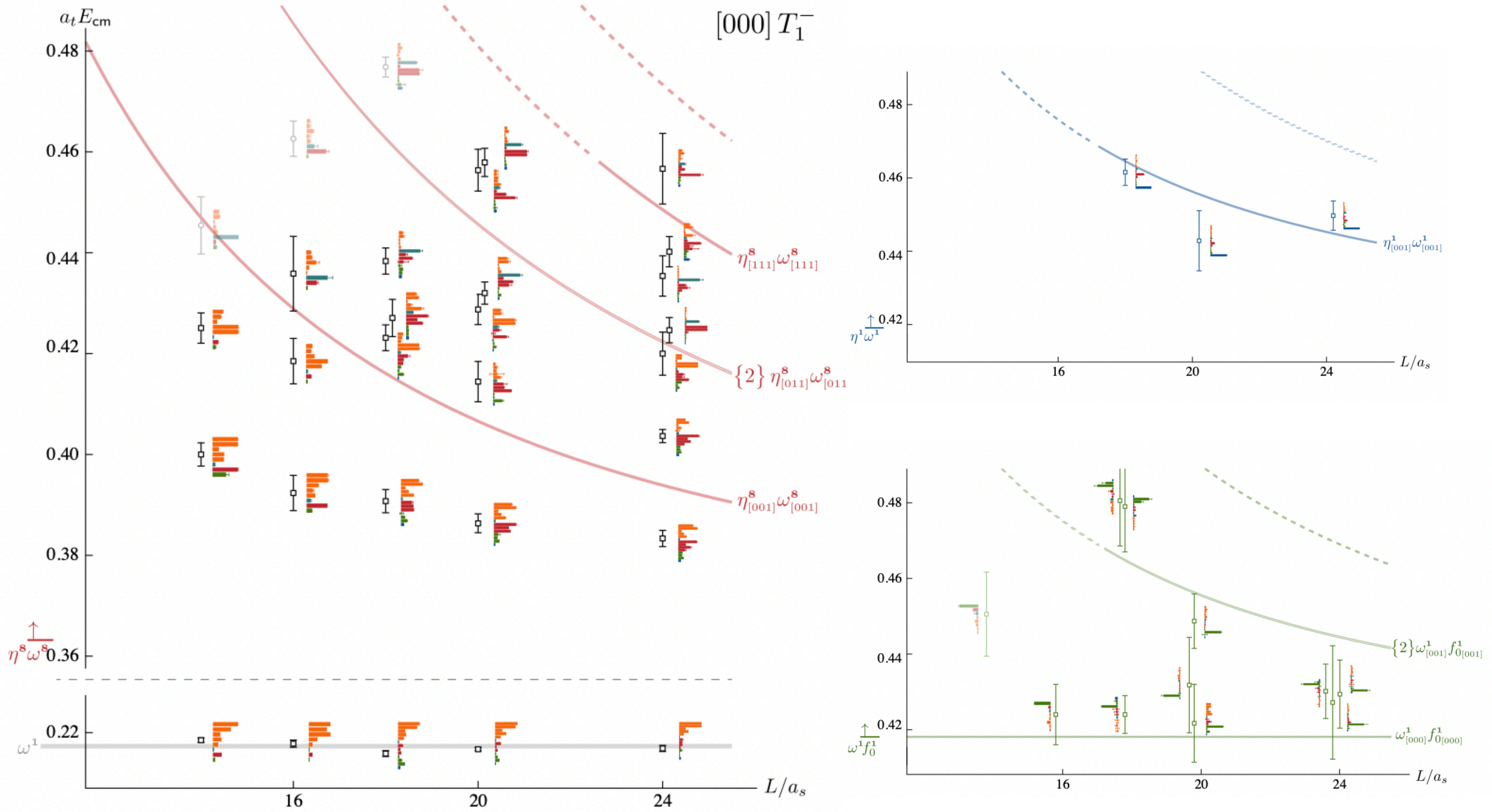
$J^P = (1,3,...)^-$

Three resonances in a single irrep.

$\Rightarrow \rho\{^3 2S_1\}, \rho\{^3 D_1\}, \rho\{^3 D_3\}$

Very dense in energy levels.

Appears to be a decoupling within the heavier channels $f_0^1\eta^1, \eta^1\omega^1$



$a_t E_{\mathsf{cm}}$
$[000]\ T_1^-$
$\eta^{8}_{[111]}\omega^{8}_{[111]}$
$\{2\}\ \eta^{8}_{[011]}\omega^{8}_{[011]}$
$\eta^{8}_{[001]}\omega^{8}_{[001]}$
$\eta^{8}\omega^{8}$
ω^{1}
L/a_s
$\eta^{1}_{[001]}\omega^{1}_{[001]}$
$\eta^{1}\omega^{1}$
$\{2\}\omega^{1}_{[001]}f^{1}_{0[001]}$
$\omega^{1}_{[000]}f^{1}_{0[000]}$
$\omega^{1}f^{1}_{0}$

Parameterizations, J=2,3

J=2 dynamically coupled in P- and F-waves

$$K_{ij} \to (2k_i)^{\ell} K_{ij}^{\ell\ell'} (2k_j)^{\ell'}$$

Can handle this with the K-matrix $\boldsymbol{t}^{-1} = \boldsymbol{K}^{-1} + \boldsymbol{I}$

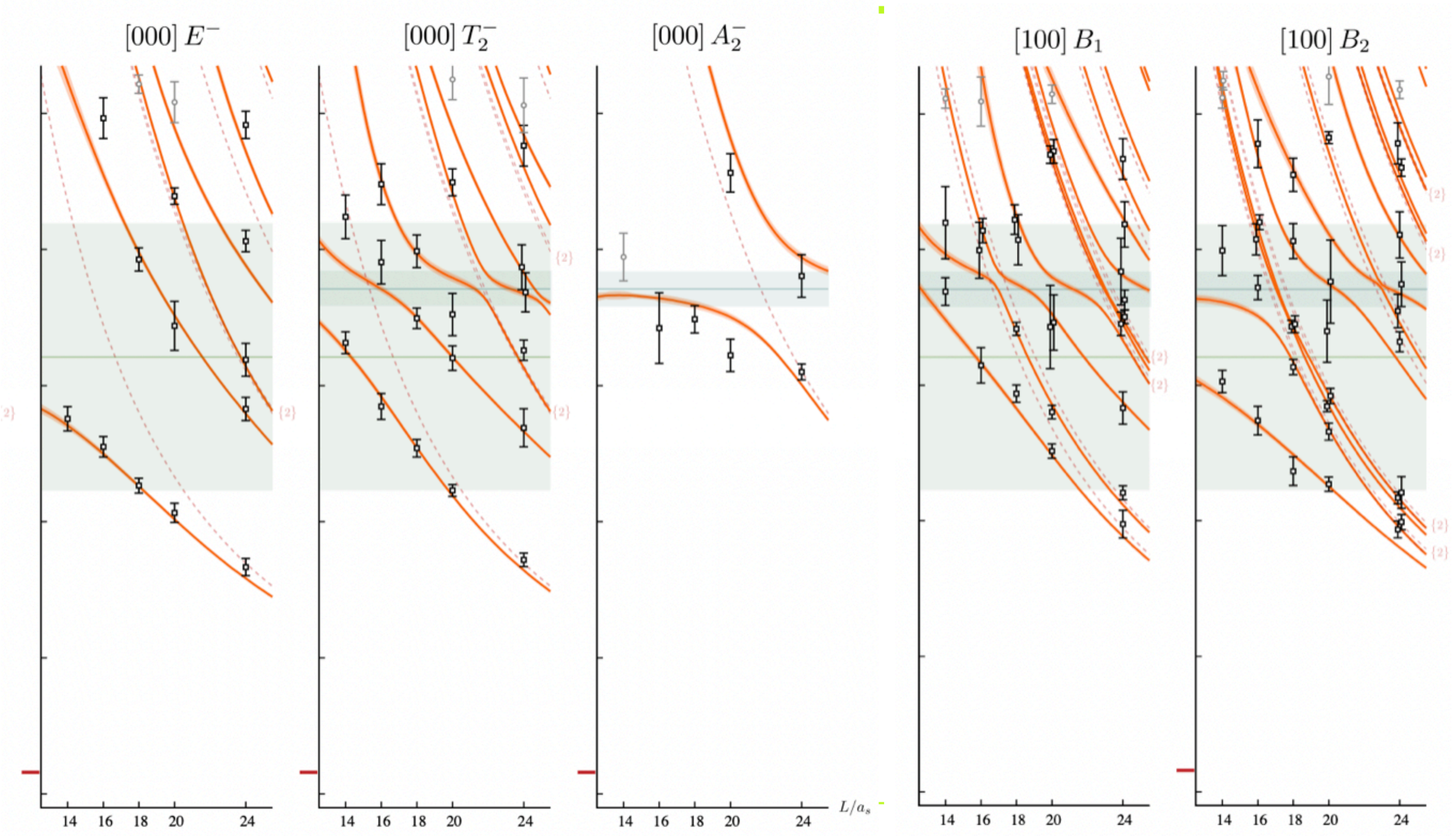
$$K_{J=2} = \begin{bmatrix} ({}^3P_2|{}^3P_2) & ({}^3P_2|{}^3F_2) \\ ({}^3P_2|{}^3F_2) & ({}^3F_2|{}^3F_2) \end{bmatrix}$$

J=3 Breit-Wigner parameterization

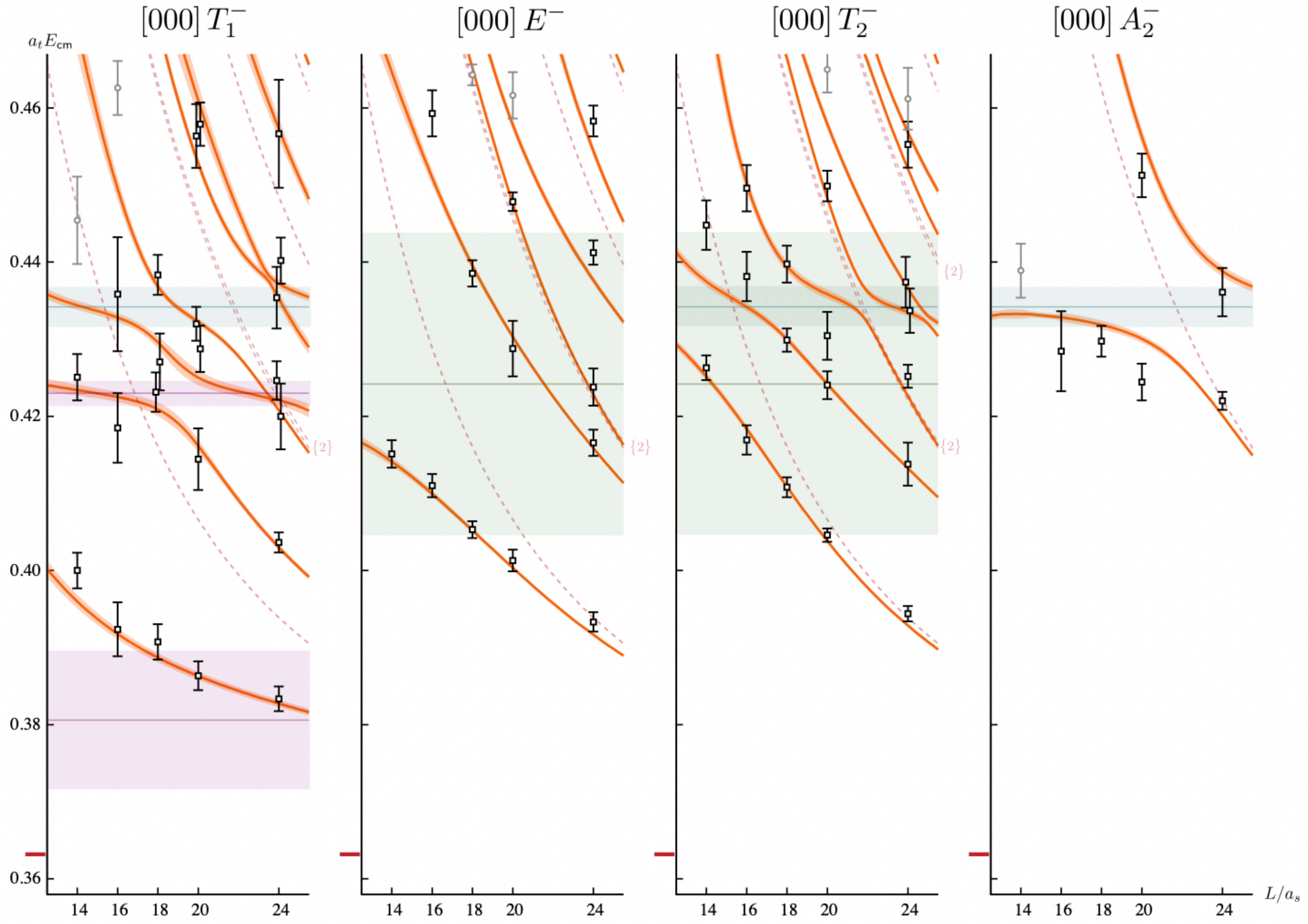
	J^P
$\ell = 0$	1^+
$\ell = 1$	$(0, 1, 2)^-$
$\ell = 2$	$(1, 2, 3)^+$
$\ell = 3$	$(2, 3, 4)^-$
...	...

$$I(s) = I(s_0) - \frac{s - s_0}{\pi} \int_{s_{thr}}^{\infty} \frac{\rho(s')}{(s' - s_0)(s' - s - i\epsilon)} ds'$$

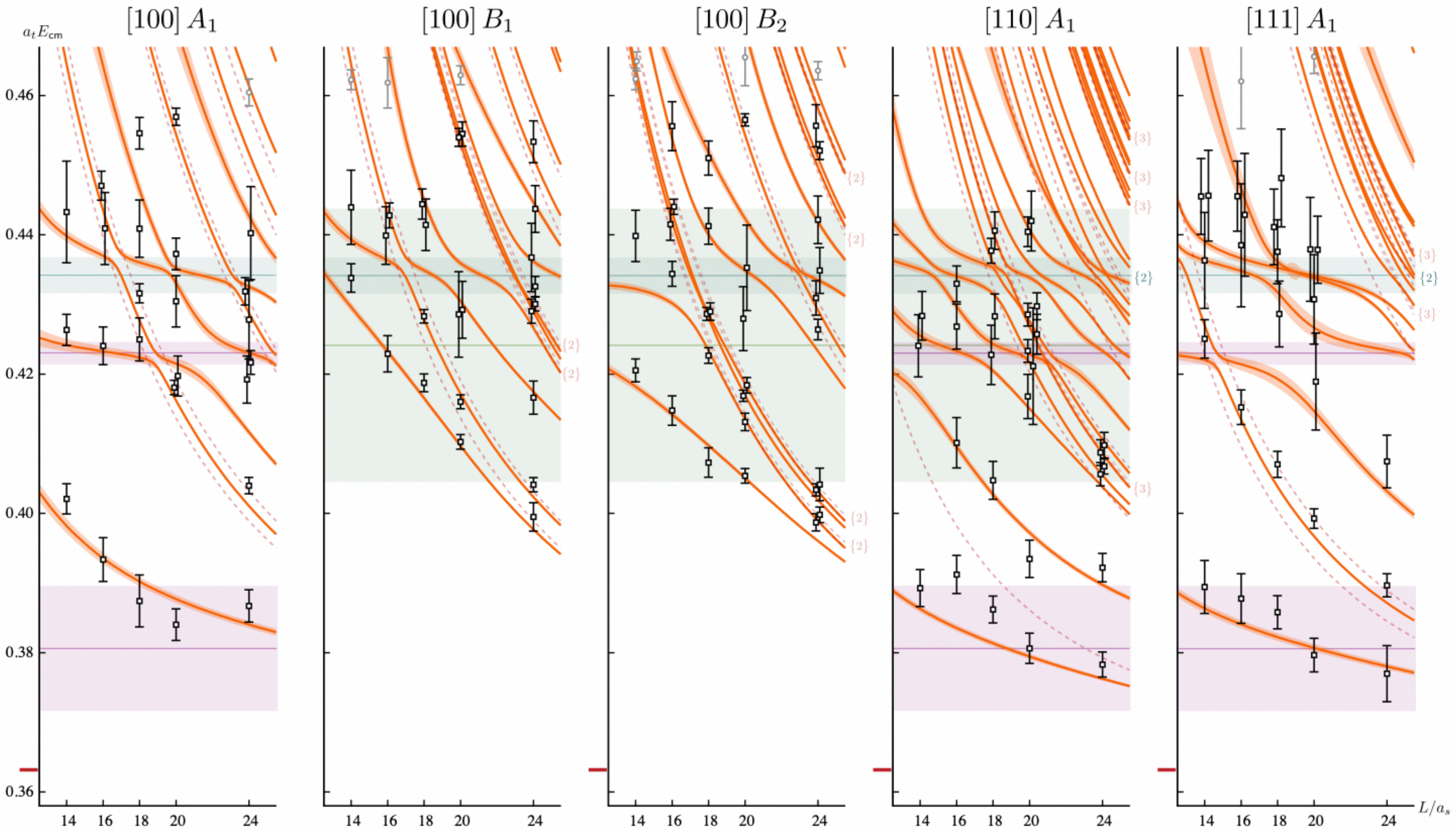
Im$\boldsymbol{I} = -\boldsymbol{\rho}$



[000] E^-
[000] T_2^-
[000] A_2^-
[100] B_1
[100] B_2
14
16
18
20
24
L/a_s



[000] T_1^-
[000] E^-
[000] T_2^-
[000] A_2^-
$a_t E_{\mathsf{cm}}$
0.46
0.44
0.42
0.40
0.38
0.36
14
16
18
20
24
L/a_s
{2}



$[100]\ A_1$
$[100]\ B_1$
$[100]\ B_2$
$[110]\ A_1$
$[111]\ A_1$
$a_t E_{\rm cm}$
0.46
0.44
0.42
0.40
0.38
0.36
14
16
18
20
24
L/a_s
{2}
{3}

Resonance interpretation

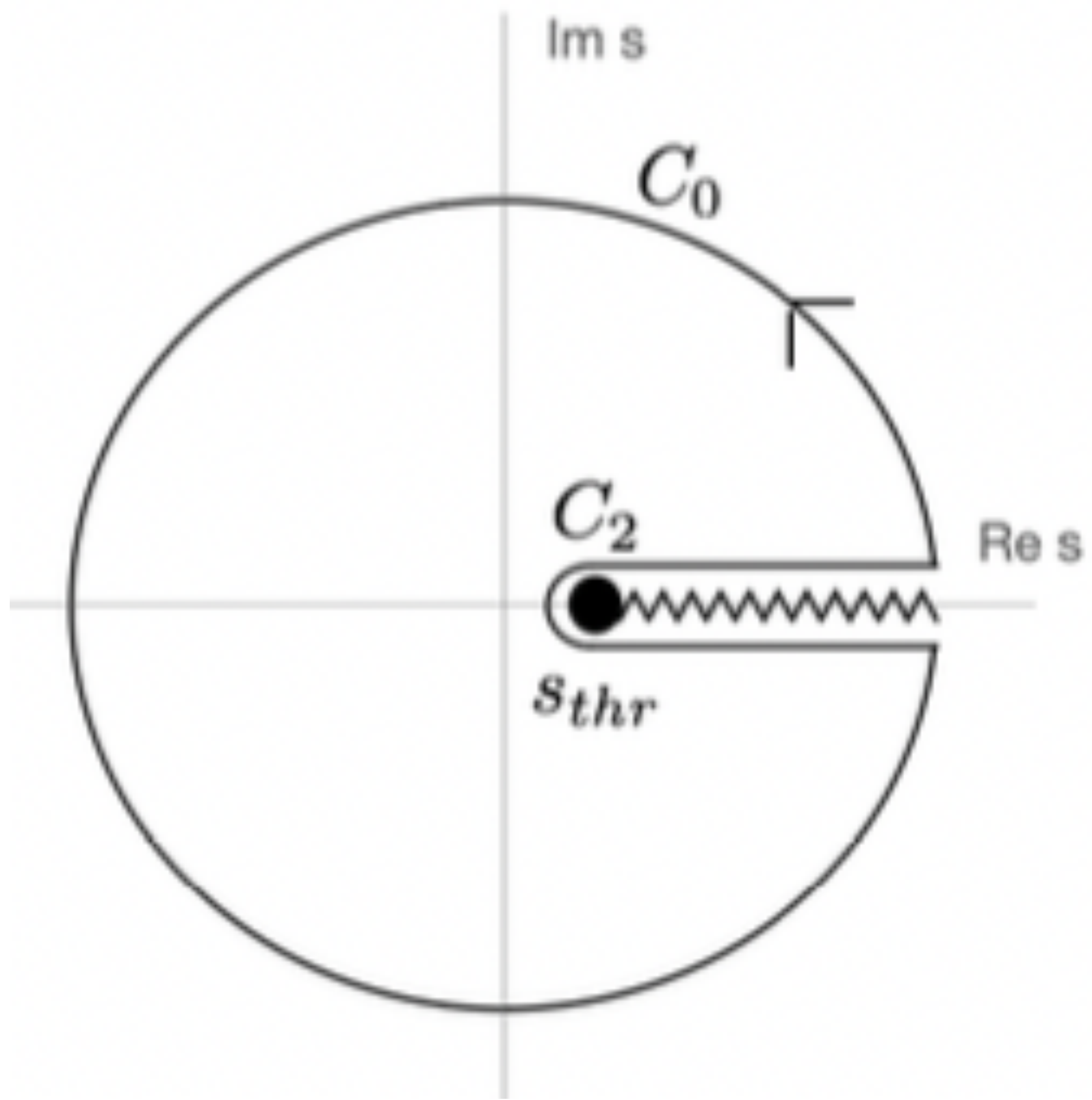
$$t(s) = \frac{N(s)}{D(s)}$$

Write dispersively $\frac{1}{2\pi i}\oint \frac{D(s')}{s'-s} = D(s_0) - \frac{s-s_0}{\pi}\int_{s_{thr}}^{\infty} \frac{N(s')\rho(s')}{(s'-s)(s'-s_0)} ds'$

⇒ can add poles to D(s) that feature as zeros in t(s)

⇒ create nearby poles in t(s)

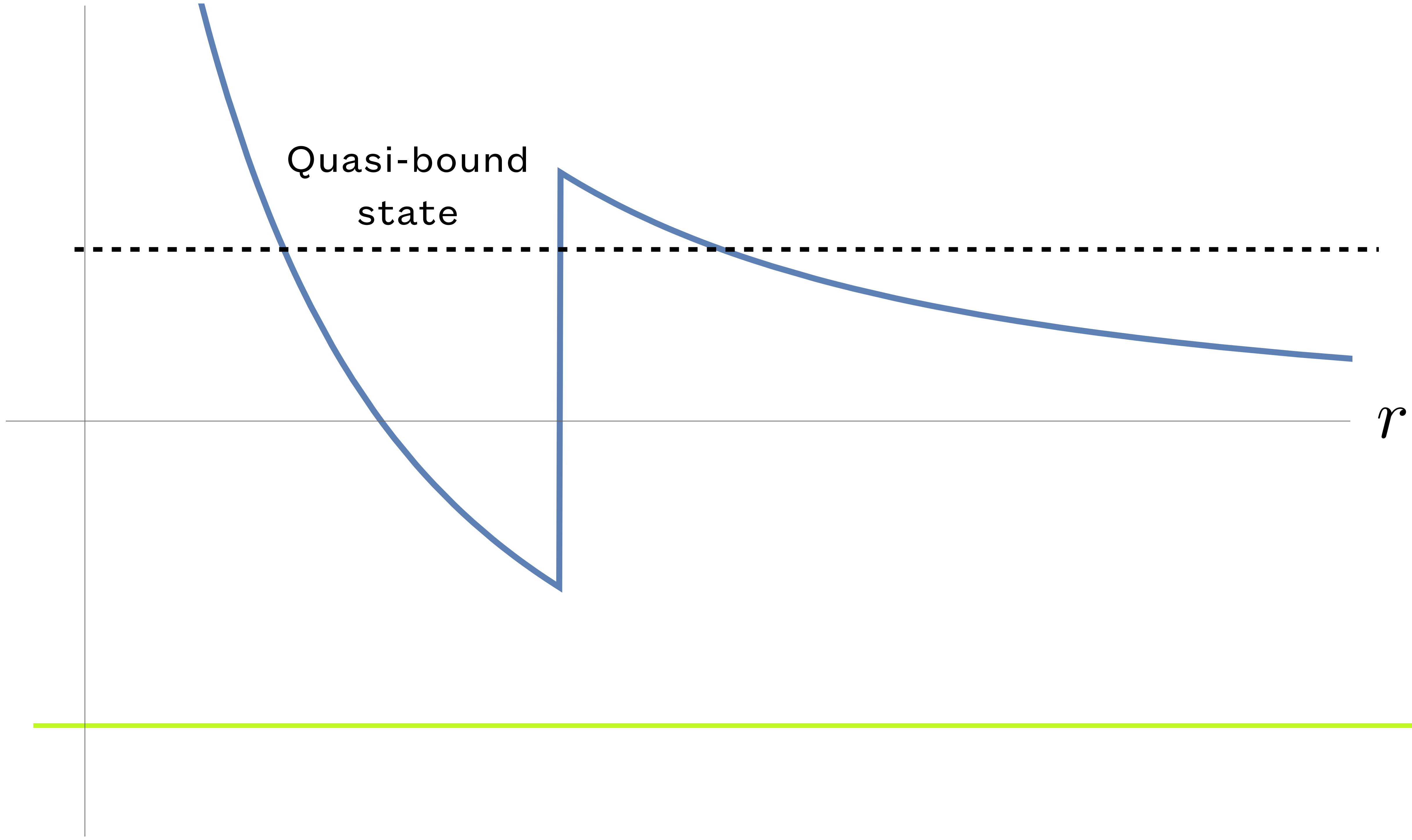
⇒ these “CDD” poles have an interpretation that they would be stable particles if there were not lighter mesons for which it to decay

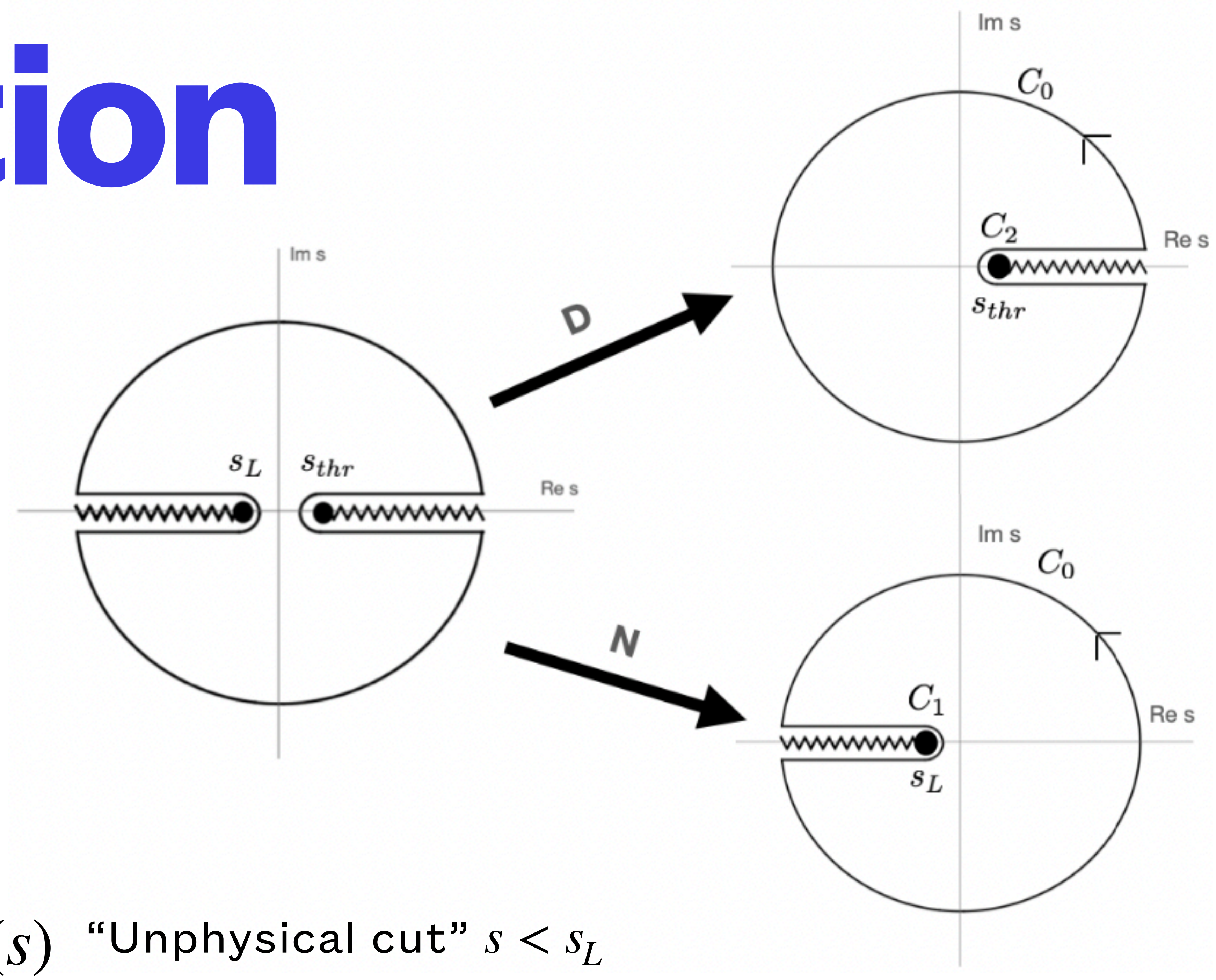


Resonance interpretation

In N.R. scattering, the scattering amplitude is completely determined by the potential.

$$V_{eff}(r) = V(r) + \frac{\ell(\ell+1)}{r^2}$$





$$t(s) = \frac{N(s)}{D(s)}$$

“Unphysical cut” $s < s_L$

“Physical cut” $s > s_{thr}$

$$D(s) = D(s_0) - \frac{s - s_0}{\pi}\int_{s_{thr}}^{\infty} \frac{N(s')\rho(s')}{(s'-s)(s'-s_0)} ds' + \sum_\alpha \frac{g_\alpha^2}{m_\alpha^2 - s}$$

Can add poles to D(s) that produce zeros in t(s)

Comparing to the ω_J^*, ϕ_J^*

Vector states are mixtures of the singlet and octet states

$$\omega = \sqrt{\frac{2}{3}}\omega_1 + \sqrt{\frac{1}{3}}\omega_8\,;\, \phi = \sqrt{\frac{1}{3}}\omega_1 - \sqrt{\frac{2}{3}}\omega_8$$

Pseudoscalar states have little mixing from SU(3) eigenstates $\eta \sim \eta_8, \eta' \sim \eta_1$

If we assume excited J^{--} have the same quark content as the vector states, we need to know the result of the octet couplings to find the partial width of the isoscalar resonances to pseudoscalar-vector final states.

We can still guess what the result of the octet calculation would be by assuming an exact OZI symmetry.

Comparing to the ω_J^*, ϕ_J^*

We first re-write the couplings in the basis of familiar meson states:

$$|\eta^8 \otimes \omega^8 \to \mathbf{1}\rangle = \frac{1}{2\sqrt{2}}\left(K^+\bar{K}^{*-} + K^-\bar{K}^* - K^0\bar{K}^{*0} - \bar{K}^0K^{*0} + \pi^+\rho^- + \pi^-\rho^+ - \pi^0\rho^0 - \eta_8\omega_8\right) : g^1$$

$$|\eta^8 \otimes \omega^8 \to \mathbf{8}\rangle = \sqrt{\frac{1}{20}}\left(K^+K^{*-} + K^-\bar{K}^* - K^0\bar{K}^{*0} - \bar{K}^0K^{*0}\right) - \sqrt{\frac{1}{5}}\left(\pi^+\rho^- + \pi^-\rho^+ - \pi^0\rho^0 - \eta_8\omega_8\right) : g^8$$

$$|\eta^8 \otimes \omega^1 \to \mathbf{8}\rangle = \eta_8\omega_1 = \sqrt{\frac{2}{3}}\eta\omega + \sqrt{\frac{1}{3}}\eta\phi : h^8$$

OZI disallowed decays:

$$\phi^* \to \rho\pi \sim \sqrt{\frac{1}{3}}\frac{1}{2\sqrt{2}}g^1 + \left(-\sqrt{\frac{2}{3}}\right)\left(-\sqrt{\frac{1}{5}}\right)g^8$$

$$\phi^* \to \eta\omega \sim \sqrt{\frac{1}{3}}\left(-\frac{1}{2\sqrt{2}}\right)\sqrt{\frac{1}{3}}g^1 + \left(-\sqrt{\frac{2}{3}}\right)\left(-\sqrt{\frac{1}{5}}\right)\sqrt{\frac{1}{3}}g^8 + \left(-\sqrt{\frac{2}{3}}\right)\sqrt{\frac{2}{3}}h^8$$

Leads to the constraints:

$$g^8 = -\frac{\sqrt{5}}{4}g^1; h^8 = -\frac{1}{2\sqrt{2}}g^1$$

Comparing to the ω_J^*, ϕ_J^*

We write the partial widths as $\Gamma = g^2 \frac{\rho}{M}$

OZI relations together with a sum over the charged states give us the following partial widths:

$$\Gamma(\omega^\star \to \pi\rho) = 3\,\tfrac{\rho}{M}\,\tfrac{3}{16}\left(g^1\right)^2$$
$$\Gamma(\omega^\star \to K\overline{K}^*) = 4\,\tfrac{\rho}{M}\,\tfrac{3}{64}\left(g^1\right)^2$$
$$\Gamma(\omega^\star \to \eta\omega) = 1\,\tfrac{\rho}{M}\,\tfrac{1}{16}\left(g^1\right)^2$$

$$\Gamma(\phi^\star \to K\overline{K}^*) = 4\,\tfrac{\rho}{M}\,\tfrac{3}{32}\left(g^1\right)^2$$
$$\Gamma(\phi^\star \to \eta\phi) = 1\,\tfrac{\rho}{M}\,\tfrac{1}{4}\left(g^1\right)^2$$

$$\Gamma(\rho^\star \to \pi\omega) = 1\,\tfrac{\rho}{M}\,\tfrac{3}{16}\left(g^1\right)^2$$
$$\Gamma(\rho^\star \to K\overline{K}^*) = 2\,\tfrac{\rho}{M}\,\tfrac{3}{32}\left(g^1\right)^2,$$

We attempt to rescale the angular momentum barrier factors:

$$g^1 = \left|\frac{k^{phys}(M^{phys})}{k(M)}\right|^{\ell} |c_{\eta^8\omega^8}|$$

Comparing to the ω_J^*, ϕ_J^*

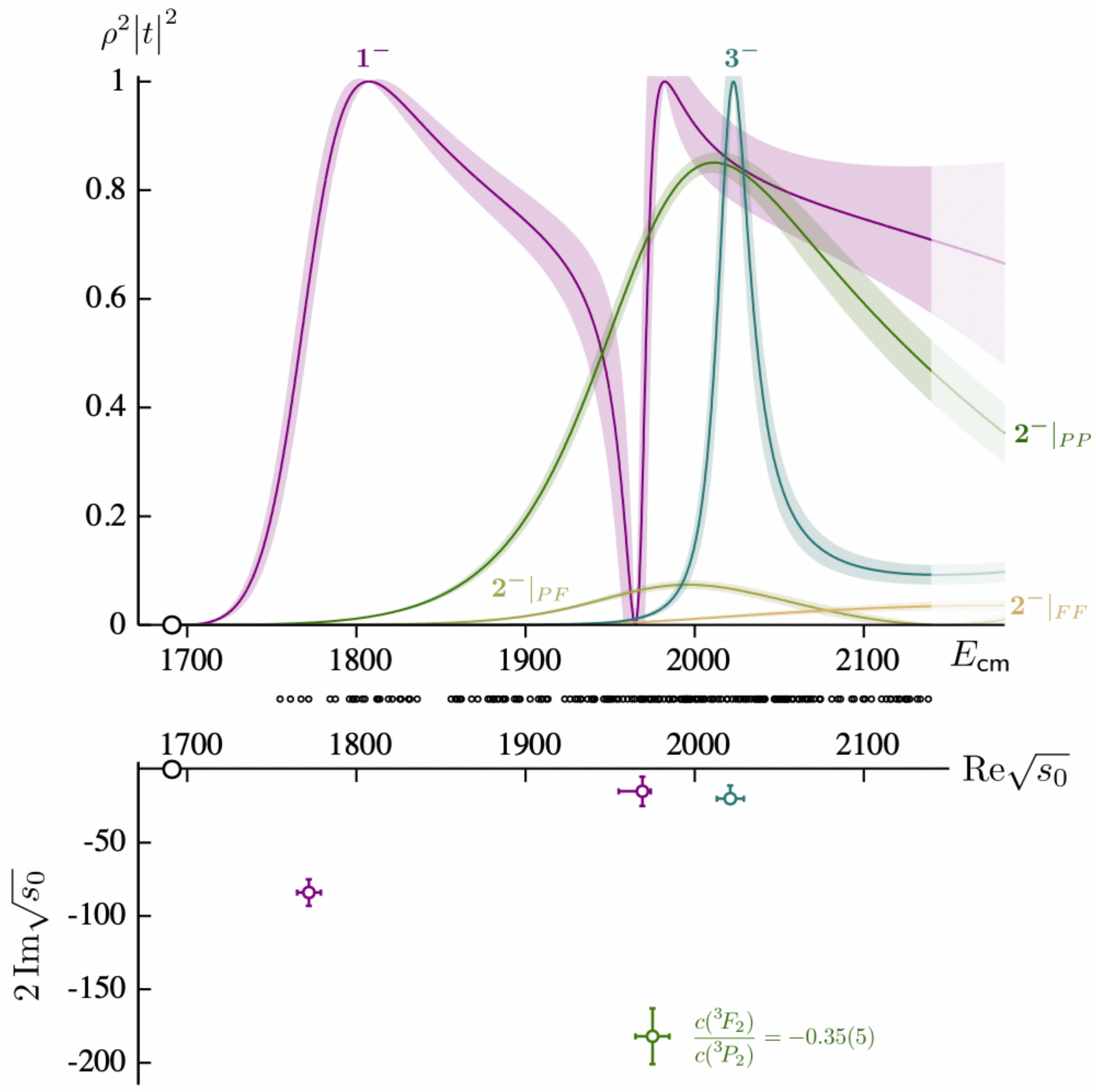
Prediction | **Experiment**

Prediction	Experiment
$\Gamma(\omega_3 \to \pi\rho) = 62$ MeV	$\Gamma^{tot}_{\omega_3(1670)} \sim 168(10)$ MeV
$\Gamma(\omega_3 \to K\bar{K}^*) = 2$ MeV	
$\Gamma(\omega_3 \to \eta\omega) = 1$ MeV	
$\Gamma(\phi_3 \to K\bar{K}^*) = 20$ MeV	$\Gamma^{tot}_{\phi_3(1850)} \sim 87(25)$ MeV
$\Gamma(\phi_3 \to \eta\phi) = 3$ MeV	
$\Gamma(\rho_3 \to \pi\omega) = 22$ MeV	$\Gamma^{\pi\omega}_{\rho_3} \sim 30(10)$ MeV
$\Gamma(\rho_3 \to K\bar{K}^*) = 2$ MeV	$\Gamma^{K\bar{K}\pi}_{\rho_3} \sim 7$ MeV

$$\Gamma(\rho_2 \to \pi\omega, K\overline{K}^*) = 125, 36\,\text{MeV}$$
$$\Gamma(\omega_2 \to \pi\rho, K\overline{K}^*, \eta\omega) = 365, 36, 17\,\text{MeV}$$
$$\Gamma(\phi_2 \to K\overline{K}^*, \eta\phi) = 148, 44\,\text{MeV}\,,$$

Comparing to the ω_J^*, ϕ_J^*

Prediction	Experiment	Prediction	Experiment
$\Gamma(\omega_b \to \pi\rho) = 25$ MeV	$\Gamma^{tot}_{\omega(1650)} \sim 315(35)$ MeV	$\Gamma(\omega_a \to \pi\rho) = 384$ MeV	
$\Gamma(\omega_b \to K\bar{K}^*) = 3$ MeV	$\Gamma^{\pi\rho}_{\omega(1650)} \sim 84$ MeV	$\Gamma(\omega_a \to K\bar{K}^*) = 4$ MeV	$\Gamma^{\pi\rho}_{\omega(1420)} \sim 240$ MeV
$\Gamma(\omega_b \to \eta\omega) = 1$ MeV		$\Gamma(\omega_a \to \eta\omega) = 5$ MeV	$\Gamma^{tot}_{\omega(1420)} \sim 290(120)$ MeV
$\Gamma(\phi_b \to K\bar{K}^*) = 13$ MeV		$\Gamma(\phi_a \to K\bar{K}^*) = 154$ MeV	$\Gamma^{tot}_{\phi(1680)} \sim 150(50)$ MeV
$\Gamma(\phi_b \to \eta\phi) = 5$ MeV		$\Gamma(\phi_a \to \eta\omega) = 25$ MeV	
$\Gamma(\rho_b \to \pi\omega) = 9$ MeV	$\Gamma^{\pi\omega}_{\rho(1700)} \sim 0$ MeV	$\Gamma(\rho_a \to \pi\omega) = 133$ MeV	$\Gamma^{tot}_{\rho(1450)} \sim 400(60)$ MeV
$\Gamma(\rho_b \to K\bar{K}^*) = 3$ MeV	$\Gamma^{tot}_{\rho(1700)} \sim 250(100)$ MeV	$\Gamma(\rho_a \to K\bar{K}^*) = 9$ MeV	$\Gamma^{\pi\omega}_{\rho(1450)} \sim 52 - 78$ MeV



Add the $[011]A_1$ irreps and fit all simultaneously

Very good constraint $N_{\text{dof}} = 180$

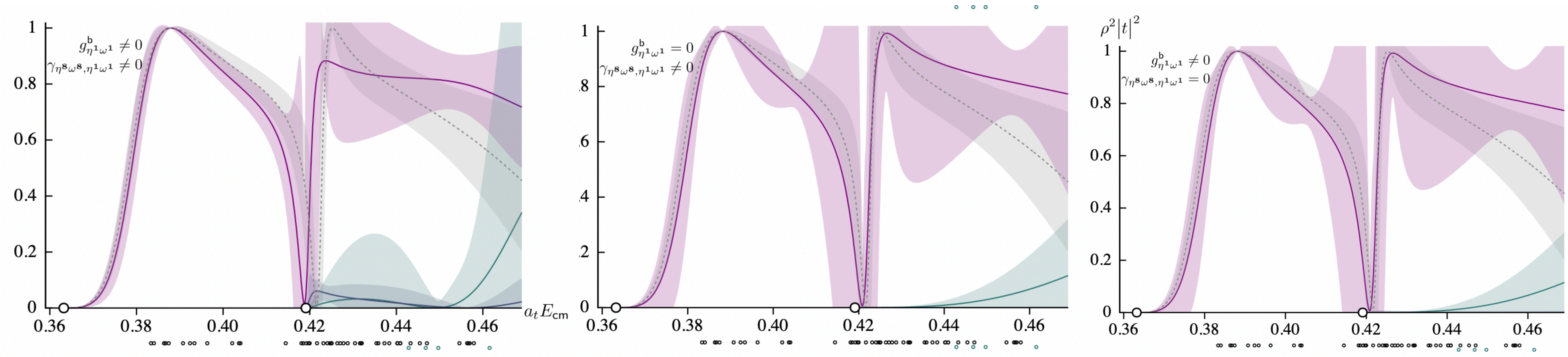
$$K_{J=1} = \frac{g_{\text{a}}^2}{m_{\text{a}}^2 - s} + \frac{g_{\text{b}}^2}{m_{\text{b}}^2 - s} + \gamma$$

$$K_{J=2} = \frac{1}{m_R^2 - s}\begin{bmatrix} g_P^2 & g_P g_F \\ g_P g_F & g_F^2 \end{bmatrix} + \begin{bmatrix} \gamma_{PP} & \gamma_{PF} \\ \gamma_{PF} & 0 \end{bmatrix}$$

$$K_{J=3} = \frac{g_F^2}{m_R^2 - s}$$

$$\chi^2/N_{\text{dof}} = 258.3/(192 - 12) = 1.43$$

Coupled-Channel $\eta^8\omega^8 - \eta^1\omega^1$



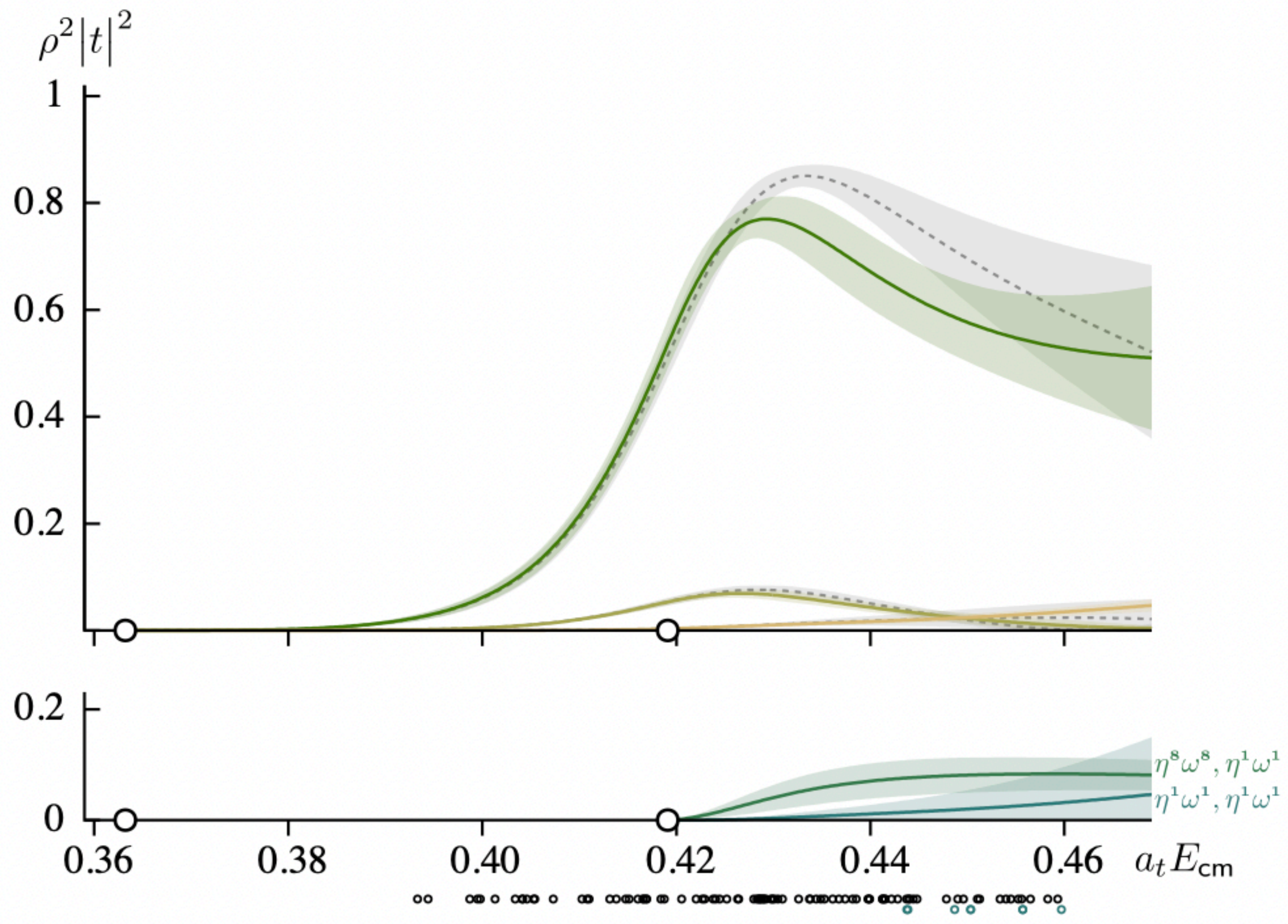
Only 4 levels with large $\eta^1\omega^1$ overlap.

Only real difference in fit-1 which features two $\eta^1\omega^1$ parameters.

Potentially a small coupling $c_{\eta^1\omega^1} \lesssim 0.04$ does not change overall width.

Statistical uncertainties on $f_0^1\omega^1$ energy levels prevent a proper C.C. analysis with this channel.

C.C. $2^{--}, \eta^8\omega^8 - \eta^1\omega^1$



Mild changes in the amplitude.

$a_t|c_{\eta^1\omega^1}| \sim 0.07(2)$ is small and comparable to F-wave coupling.

Additional singularities

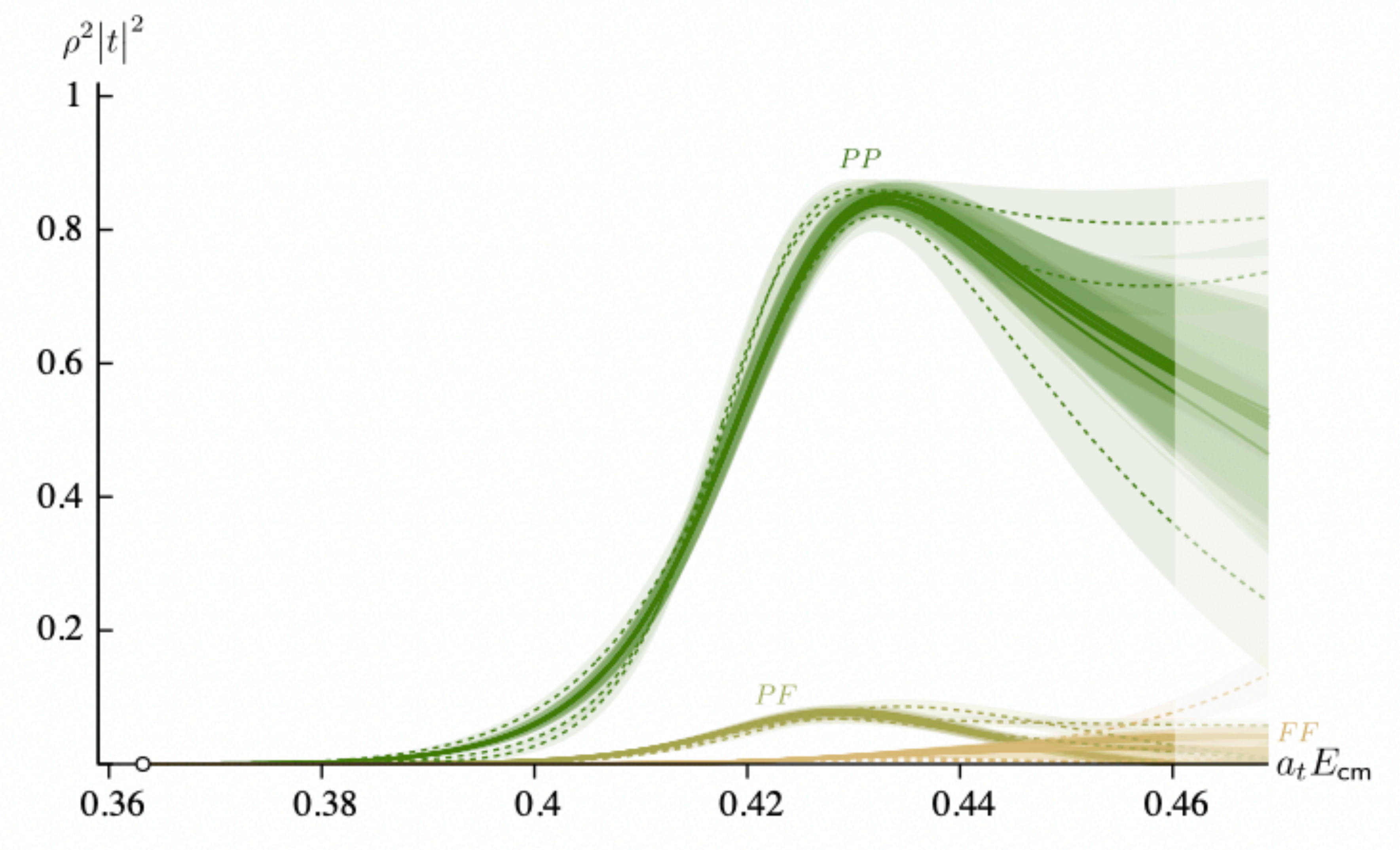
Unphysical sheet real axis pole $a_t\sqrt{s} \sim 0.23$ on many parameterizations

⇒ wanders a bit and remains far from physical scattering

Additional real axis pole $a_t\sqrt{s} \sim 0.24$ for simple phase space parameterization

⇒ not surprising this parameterization has poorer analytic properties

⇒ residue is real, a true p-wave bound state has imaginary coupling



Amplitude analytic structure

The full scattering amplitude T(s,t) relates all scattering channels s,t,u- through an analytic continuation.

s-channel unitarity constrains the “right hand cut” to form $2^{N_{chan}}$ Riemann sheets

⇒ built into our parameterizations

Analyticity requires poles off axis real valued poles be on unphysical sheets.

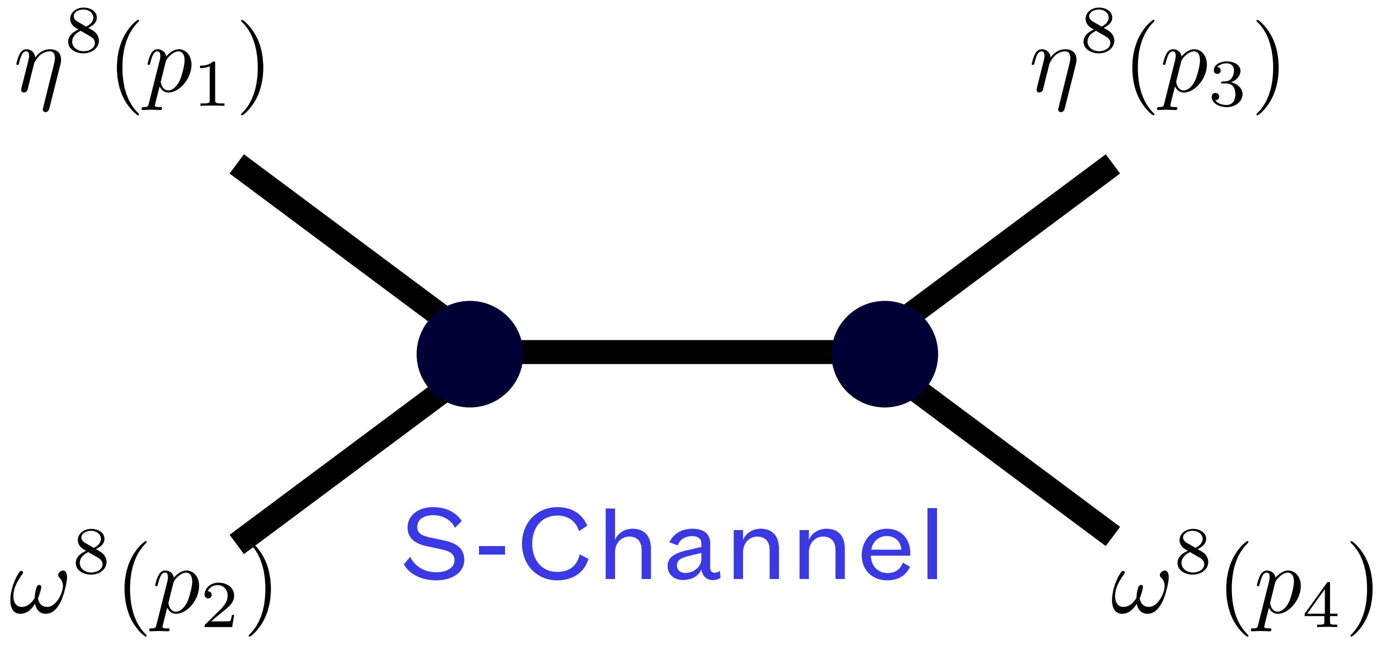
⇒ reject parameterizations that have these

t,u-channel unitarity manifests themselves in the form of a “left hand cut”

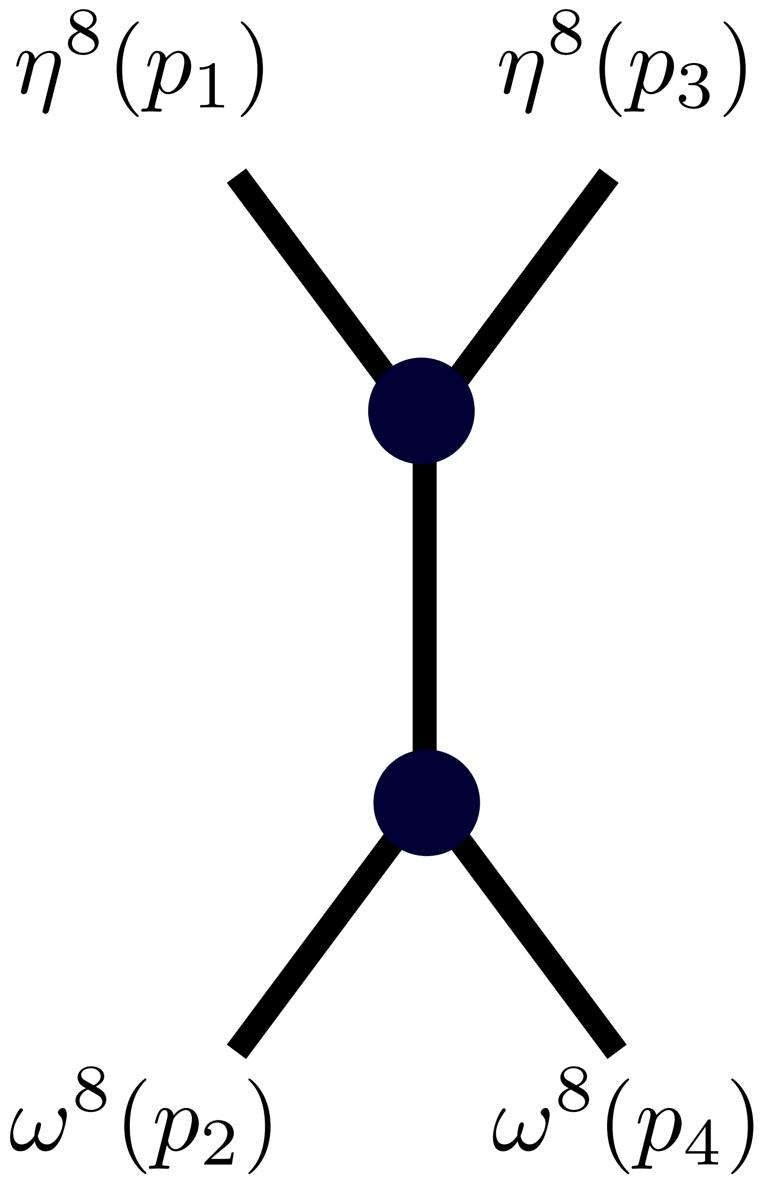
⇒ not described but we know where they are

⇒ hope is we remain far enough away

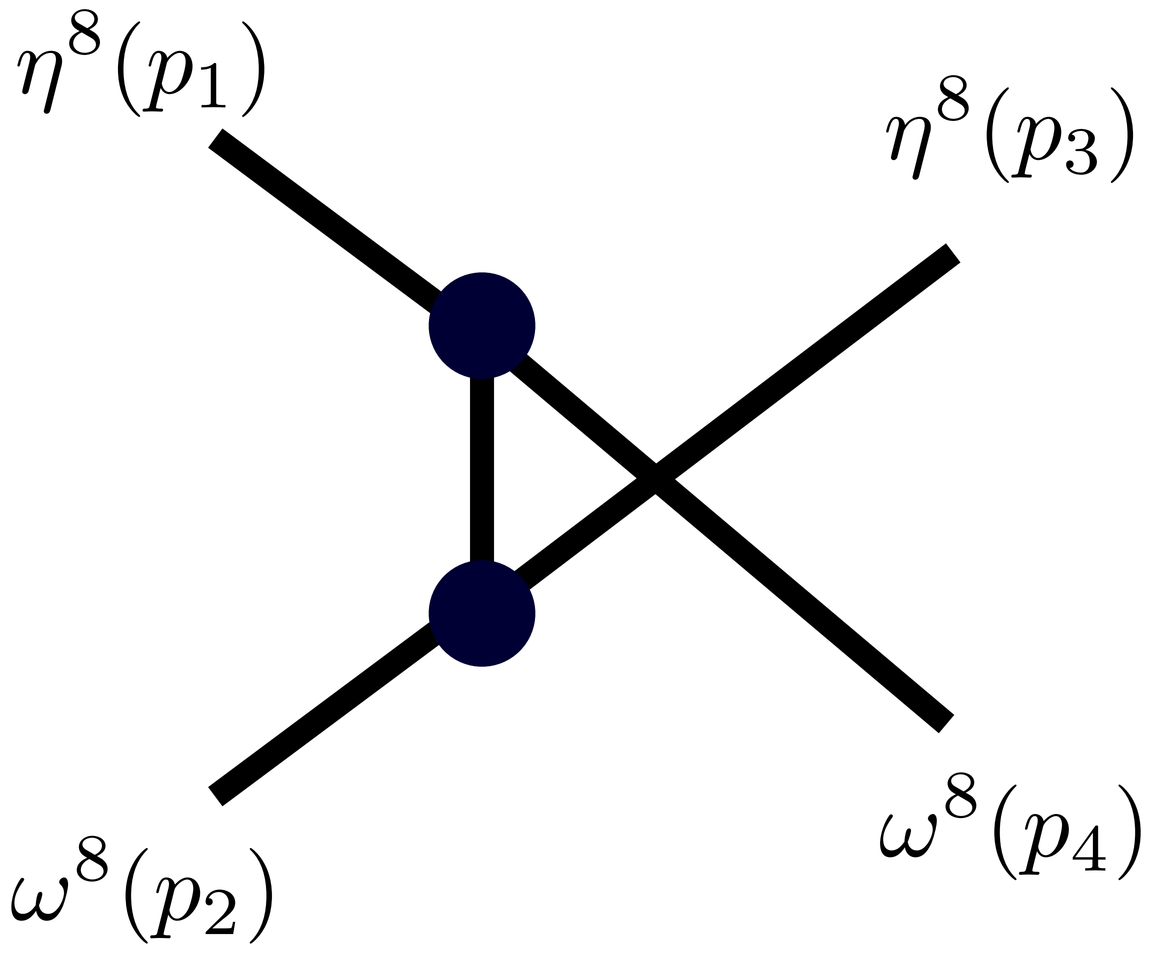
Cross Channels



S-Channel

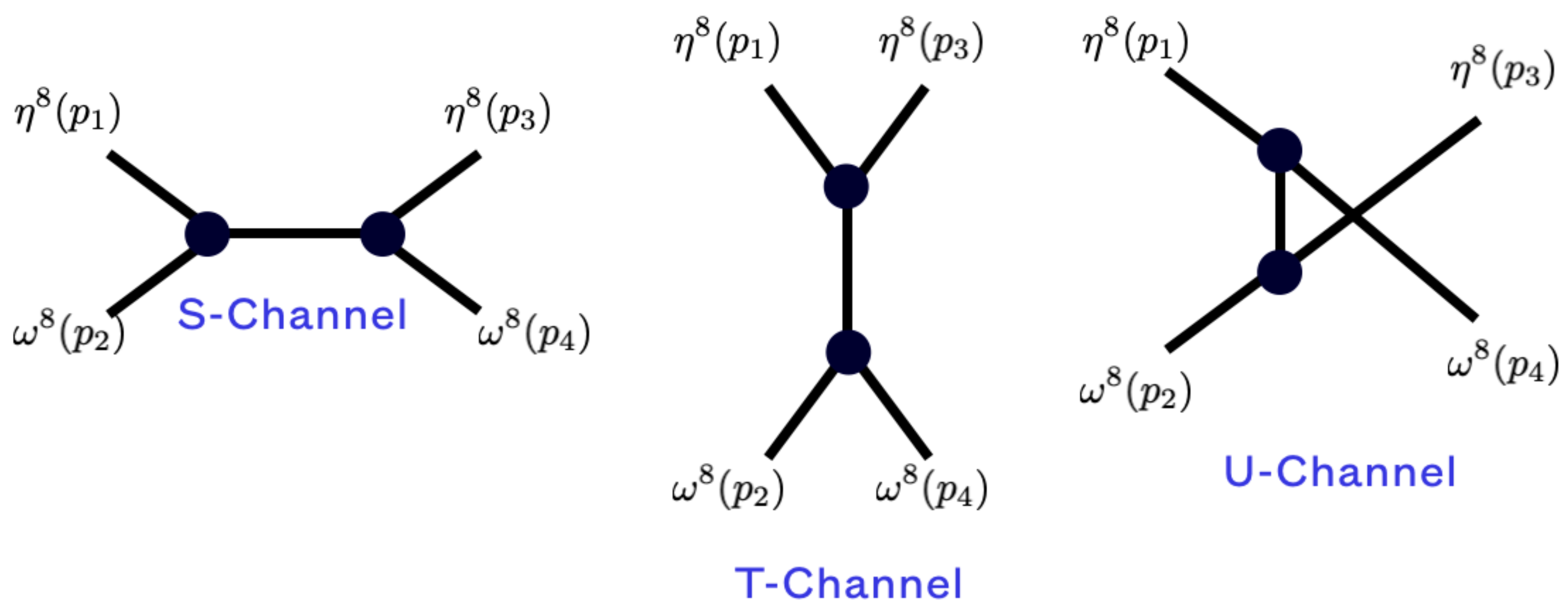


T-Channel

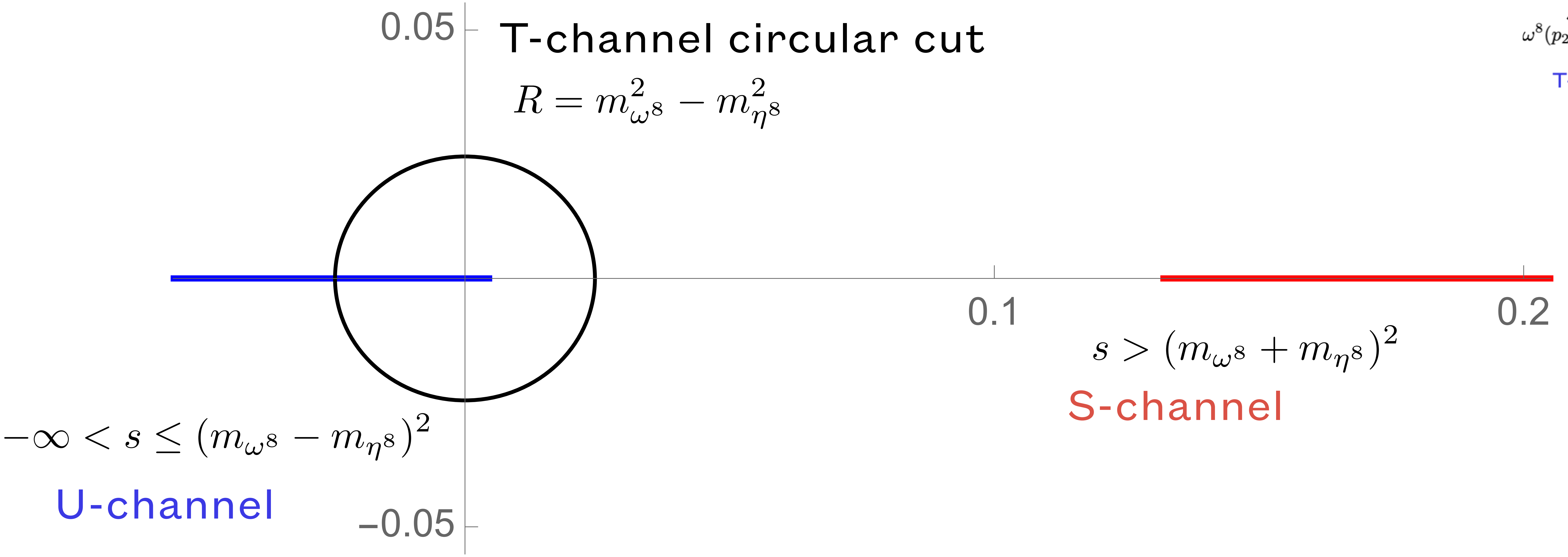


U-Channel

Cuts



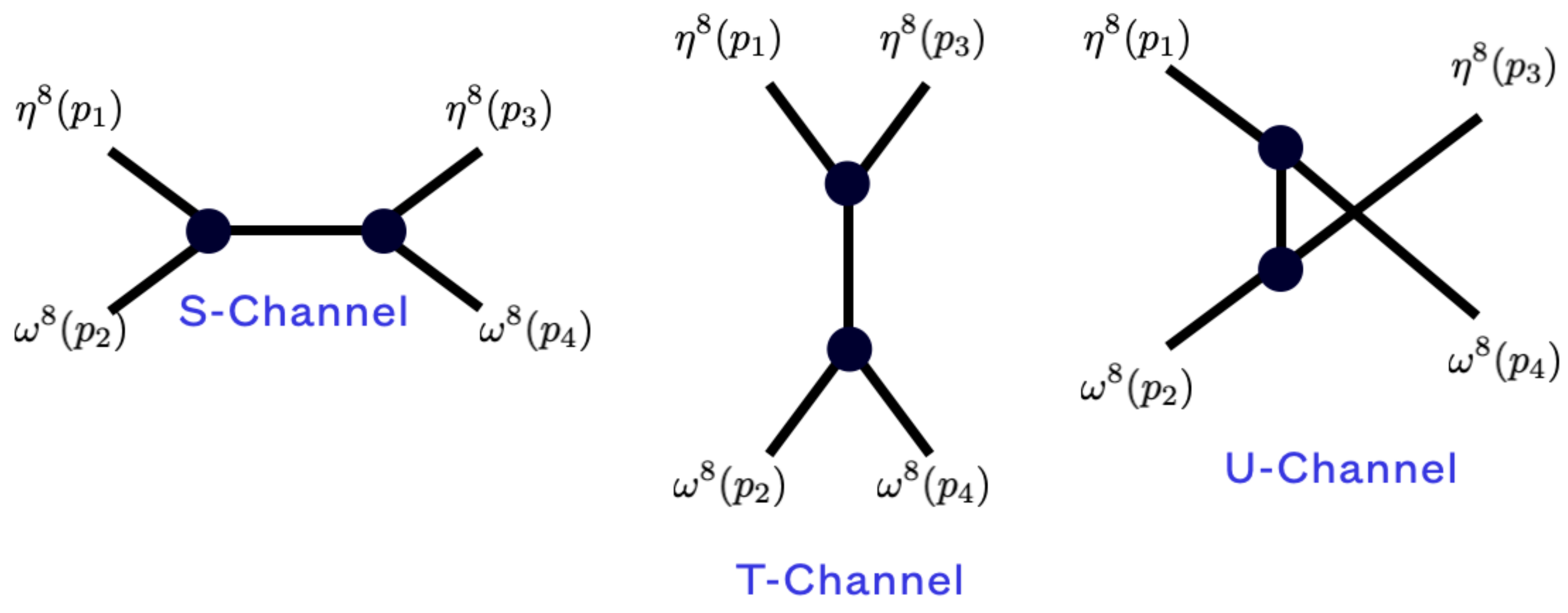
$\eta^8(p_1)$
$\eta^8(p_3)$
$\omega^8(p_2)$
$\omega^8(p_4)$
S-Channel
T-Channel
U-Channel

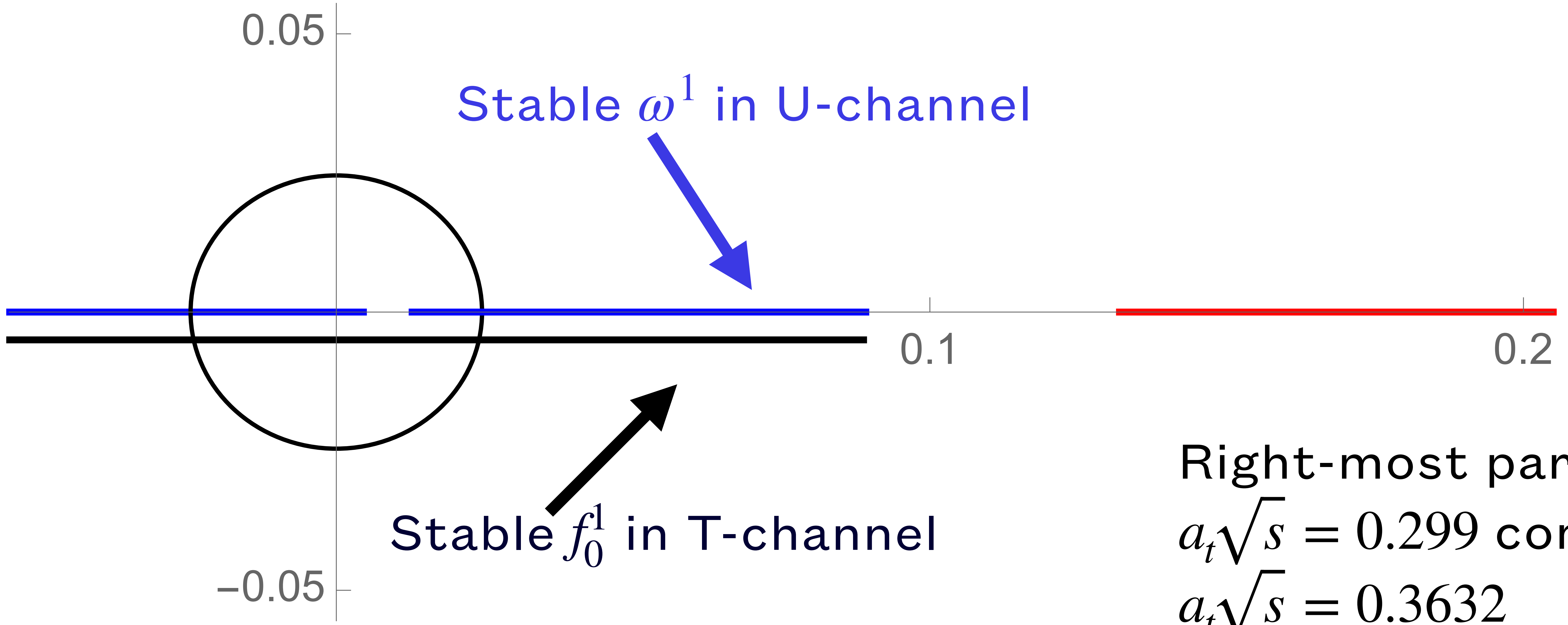


0.05
T-channel circular cut
$R = m_{\omega^8}^2 - m_{\eta^8}^2$
0.1
0.2
$s > (m_{\omega^8} + m_{\eta^8})^2$
S-channel
$-\infty < s \leq (m_{\omega^8} - m_{\eta^8})^2$
U-channel
−0.05

Cuts

Stable particles in cross-channels add additional singularities



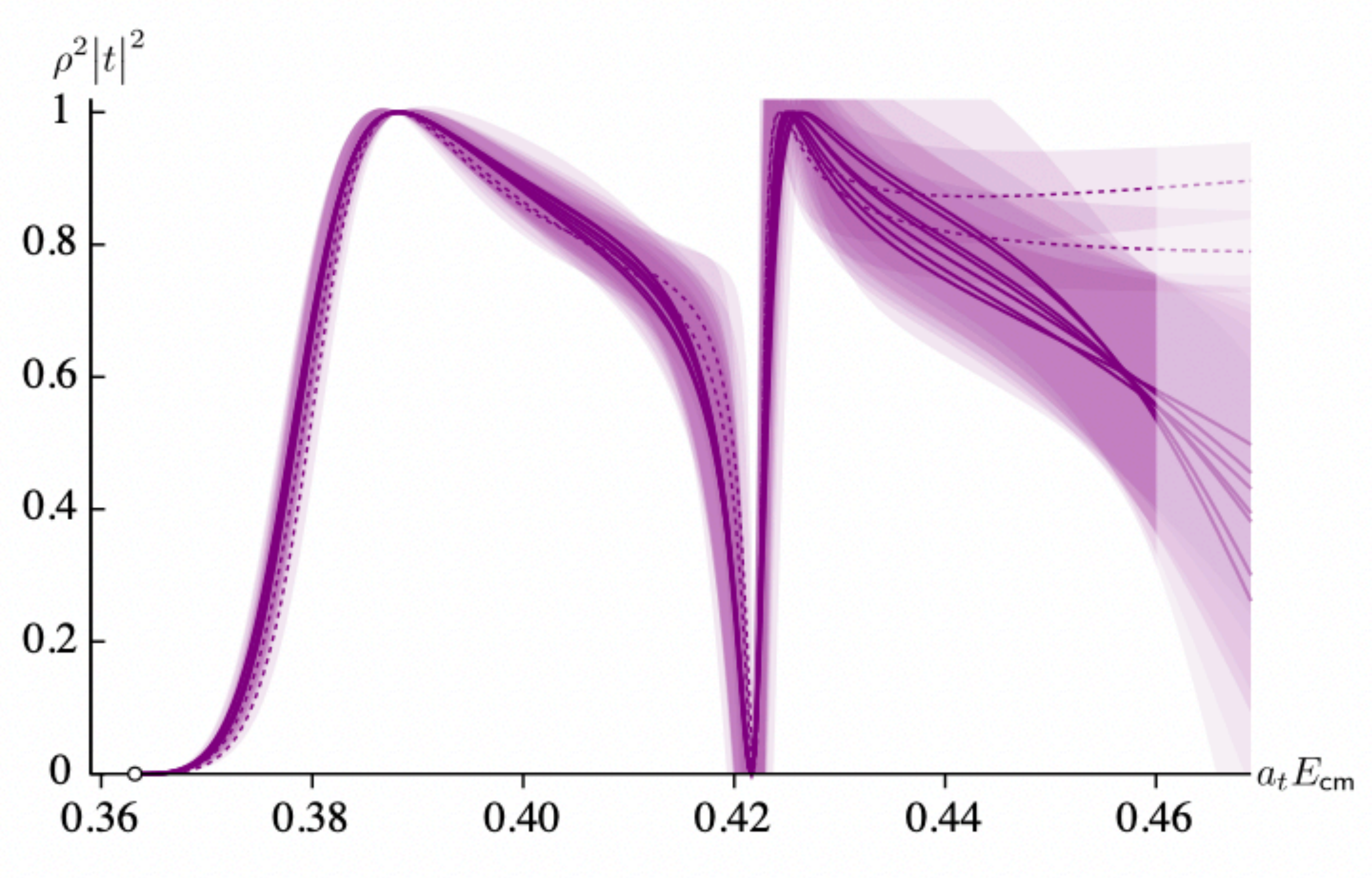


Right-most part of additional cuts at $a_t\sqrt{s} = 0.299$ compared to threshold of $a_t\sqrt{s} = 0.3632$

Additional Singularities

Physical sheet pole at $a_t\sqrt{s} = 0.278(26)$ wrong residue.

$\Rightarrow$ asses this as a “ghost” occurring from improper treatment of the LHC



Noisy third unphysical sheet pole lies beyond region of constraint $a_t E \sim 0.46$.

$\Rightarrow$ artifact not present in all parameterizations

$\Rightarrow$ could be feeling presence of a hybrid 1^{--} meson we expect in that region