$K\pi$ scattering length at physical quark masses using all-to-all methods

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July 27, 2021

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$K\pi$ scattering length and pion-mass dependence

momentum	p
isospin	I
scattering pase shift	$\delta^I(p)$
scattering length	$a_0^I = -\lim_{p\to 0} \frac{\delta^I(p)}{p}$

- short selection of previous results

ref.	(#) m_π/MeV	$m_\pi a_0^{\frac{1}{2}}$	$m_\pi a_0^{\frac{3}{2}}$
Wilson2019	239	0.46(3)	
	284	0.79(13)	
Sasaki2014	extrap.	0.142(14)(27)	-0.0469(24)(20)
	166	0.158(36)	-0.108(12)
Janowski2014	phys	0.1562(50)	-0.0674(33)

- strong pion-mass dependence
- demands for evaluation at physical point

lattice setup

- C0 ensemble
  - Möbius domain wall fermions
  - geometry $= 48^3 \times 96$, $a^{-1} = 1.7295(38)\,\mathrm{GeV}$
  - $m_\pi = 139.17(35)\,\mathrm{MeV}$, $m_\pi L = 3.86$
- all-to-all propagators (Foley *et al.* 2005)
  - 2000 eigenmodes (low-modes) (light-quarks only)
  - time-diluted spin-color-diagonal Z_2 noises (high-modes)
- statistic
  - 23 configurations
  - all 96 source-time positions

all-to-all propagators

low modes

- eigenmodes of hermitian Dirac operator $\gamma_5 D$

$$(\gamma_5 D_{yx})^{-1} = \sum_{i=1}^{N_{low}} \frac{1}{\lambda_i} v_y^{(i)} v_x^{(i)\dagger}$$

- introduce notation

$$w_x^{(i)\dagger} = \frac{1}{\lambda_i} v_x^{(i)\dagger} \gamma_5$$

high modes

- stochastic noise $\eta^{(i)}$, $i = 1 \ldots N_{high}$

$$\langle \eta_x^{(i)} \rangle_i = 0 \,, \quad \langle \eta_x^{(i)} \eta_y^{(i)\dagger} \rangle_i = \delta_{x,y}$$

- invert Dirac matrix

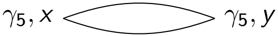
$$D\psi^{(i)} = \eta^{(i)} \,, \quad D_{xy}^{-1} = \langle \psi_x^{(i)} \eta_y^{(i)\dagger} \rangle_i$$

- introduce notation

$$w_x^{(i)^\dagger} = \eta_x^{(i)\dagger} \,, \quad v_x^{(i)} = \psi_x^{(i)}$$

- combine both methods (Foley *et al.* 2005), use exact low modes and correct for bias from cut-off N_{low} via high modes
- propagator

$$S_{yx} = \langle v_y w_x^\dagger \rangle_i \,, \quad i \in \{1 \ldots (N_{low} + N_{high})\}$$

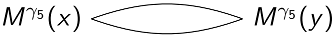


- correlation function

$$\begin{aligned} c(x,y) &= \mathrm{Tr}[S(y,x)\gamma_5 S(x,y)\gamma_5] \\ &= \mathrm{Tr}[v(y)w^\dagger(x)\gamma_5 v(x)w^\dagger(y)\gamma_5] \\ &= \mathrm{Tr}[w^\dagger(x)\gamma_5 v(x)w^\dagger(y)\gamma_5 v(y)] \\ &= \mathrm{Tr}[M^{\gamma_5}(x)M^{\gamma_5}(y)] \end{aligned}$$

- meson field

$$M^\Gamma(x) = w_i^\dagger(x)\Gamma v_j(x)$$



method to obtain $K\pi$ scattering length

- large-L expansion (Lüscher 1986)

$$E^I_{K\pi} = m_\pi + m_K - \frac{2\pi(m_\pi + m_K)}{m_\pi m_K L^3} a^I_0 (1 + c_1 \frac{a^I_0}{L} + c_2 \frac{(a^I_0)^2}{L^2}) + \mathcal{O}(L^{-6}) \,,$$
$$c_1 = -2.837297 \,,$$
$$c_2 = 6.375183$$

- need energy of $K\pi$ state in isospin channel I

$$C^I_{K\pi}(t) = \langle O^I_{K\pi}(t) O^{I\dagger}_{K\pi}(0) \rangle$$

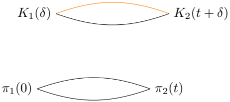
isospin I $K\pi$ contributions

- $K\pi$ correlation function $\langle O^I_{K\pi} O^{I\dagger}_{K\pi} \rangle$

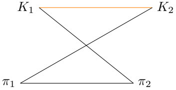
$$O^{I=\frac{3}{2}}_{K\pi} = (\bar{s}\gamma_5 u)\,(\bar{d}\gamma_5 u)$$
$$O^{I=\frac{1}{2}}_{K\pi} = (\bar{s}\gamma_5 d)(\bar{d}\gamma_5 u) - \frac{1}{2}(\bar{s}\gamma_5 u)(\bar{d}\gamma_5 d) + \frac{1}{2}(\bar{s}\gamma_5 u)(\bar{u}\gamma_5 u)$$

- $K\pi$ diagrams (Nagata *et al.* 2009)

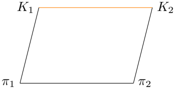
D



C



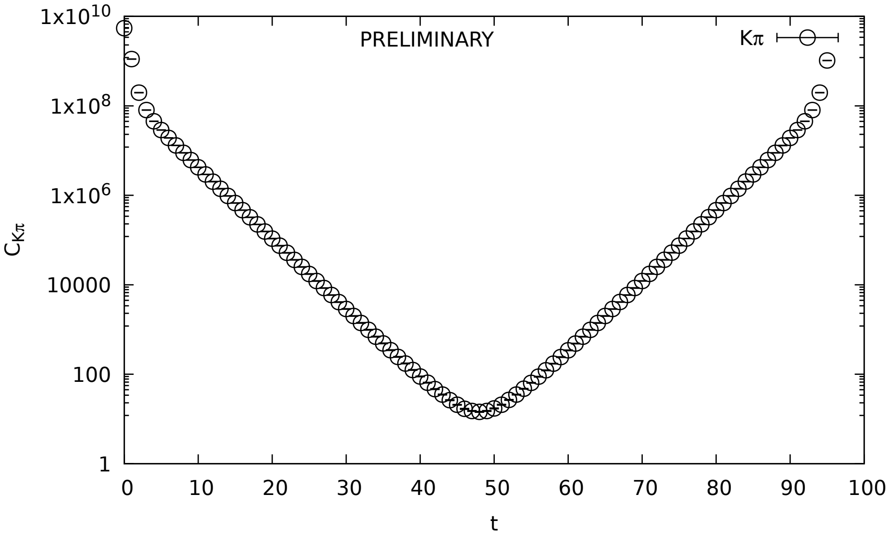
R



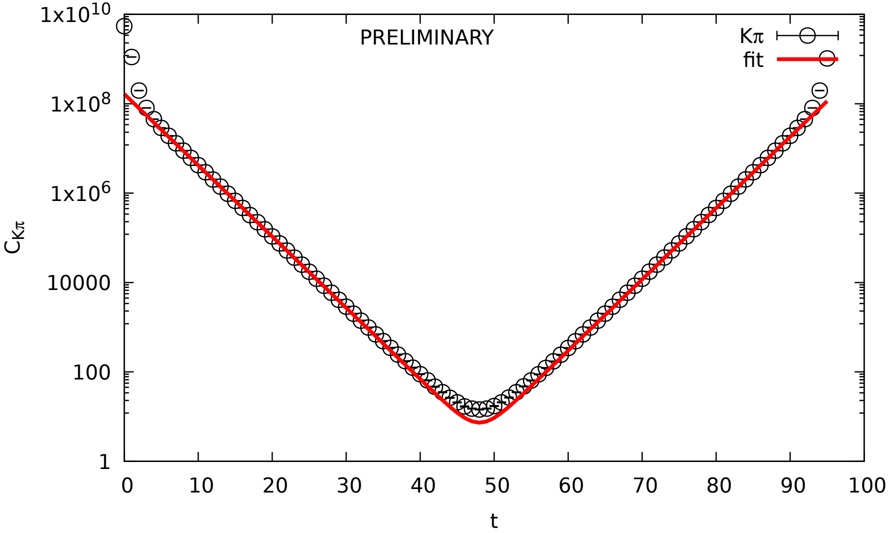
- Fierz re-arrangement
  - could use different noises (expensive)
  - offset in time direction (Kuramashi *et al.* 1993) (we will use this)
- Isospin contributions

$$C^{I=\frac{1}{2}}_{K\pi} = D + \frac{1}{2}C - \frac{3}{2}R$$
$$C^{I=\frac{3}{2}}_{K\pi} = D - C$$

$K\pi$ correlation function $I = 1/2$

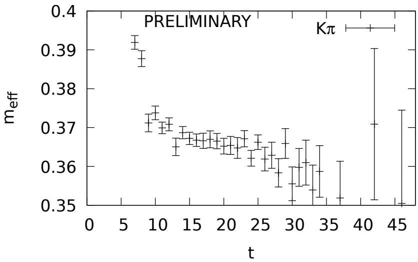


$K\pi$ correlation function $I = 1/2$

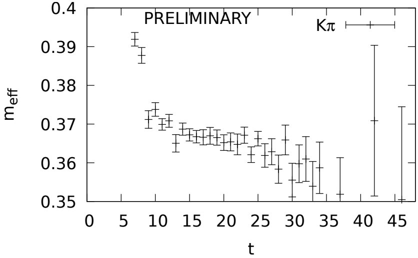


$K\pi$ correlation function affected by round-the-world effects

round-the-world contributions

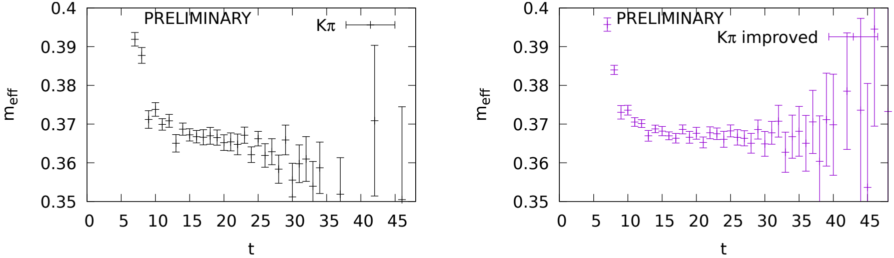


round-the-world contributions



$$\langle K(t+\delta)\pi(t)K(\delta)\pi(0)\rangle = \langle 0|K(\delta)\pi(0)|K\pi\rangle\,\langle K\pi|K(\delta)\pi(0)|0\rangle\, e^{-E_{K\pi}t}$$
$$+ \langle 0|K(\delta)\pi(0)|K\pi\rangle\,\langle K\pi|K(\delta)\pi(0)|0\rangle\, e^{-E_{K\pi}(T-t)}$$
$$+ \langle 0|\pi(0)|\pi\rangle\,\langle \pi|K(\delta)\pi(0)|K\rangle\,\langle K|K(0)|0\rangle\, e^{-m_\pi(T-t)}e^{-m_K(t-\delta)}$$
$$+ \langle 0|K(0)|K\rangle\,\langle K|K(\delta)\pi(0)|\pi\rangle\,\langle \pi|\pi(0)|0\rangle\, e^{-m_\pi t}e^{-m_K(T+\delta-t)}$$

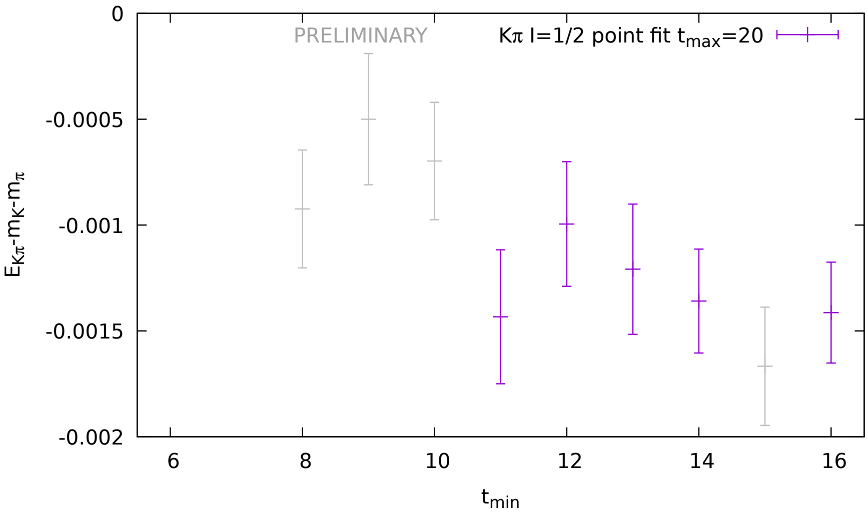
round-the-world contributions



$$\langle K(t+\delta)\pi(t)K(\delta)\pi(0)\rangle = \langle 0|K(\delta)\pi(0)|K\pi\rangle\,\langle K\pi|K(\delta)\pi(0)|0\rangle\, e^{-E_{K\pi}t}$$
$$+ \langle 0|K(\delta)\pi(0)|K\pi\rangle\,\langle K\pi|K(\delta)\pi(0)|0\rangle\, e^{-E_{K\pi}(T-t)}$$
$$+ \langle 0|\pi(0)|\pi\rangle\,\langle \pi|K(\delta)\pi(0)|K\rangle\,\langle K|K(0)|0\rangle\, e^{-m_\pi(T-t)}e^{-m_K(t-\delta)}$$
$$+ \langle 0|K(0)|K\rangle\,\langle K|K(\delta)\pi(0)|\pi\rangle\,\langle \pi|\pi(0)|0\rangle\, e^{-m_\pi t}e^{-m_K(T+\delta-t)}$$

- can compute and remove the leading round-the-world contributions
- obtained $\langle \pi|K(\delta)\pi(0)|K\rangle$ and $\langle K|K(\delta)\pi(0)|\pi\rangle$ from separate computations

$K\pi$ fit scan $I = 1/2$



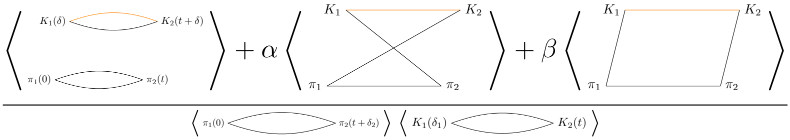
- gray points excluded by p-value ($p < 0.05$ or $p > 0.95$)
- Fits to the $K\pi$ two-point function have large errors

- cancel noise by using the following ratio (Kuramashi 1993)

$$r^I_{K\pi}(t) = \frac{C^I_{K\pi}(t)}{C_\pi(t)\ C_K(t)}\,.$$

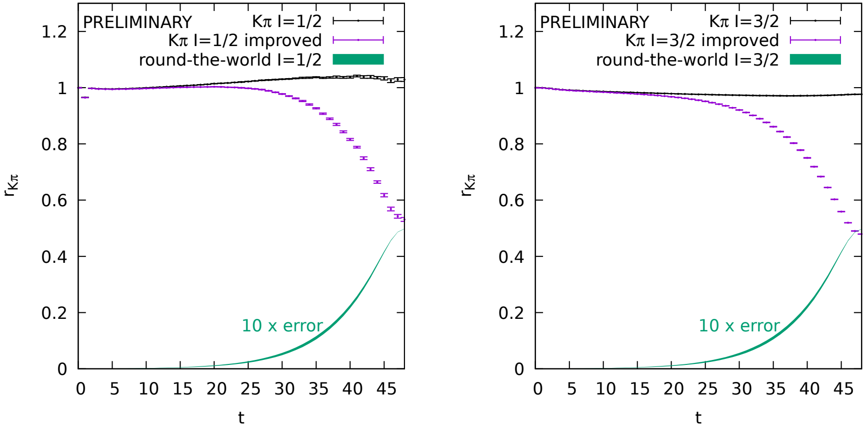
- as diagrams

$r^I_{K\pi} =$



$$\text{with} \begin{cases} \alpha = \frac{1}{2}, \quad \beta = -\frac{3}{2} & \text{for } I = \frac{1}{2} \\ \alpha = -1, \quad \beta = 0 & \text{for } I = \frac{3}{2} \end{cases}$$

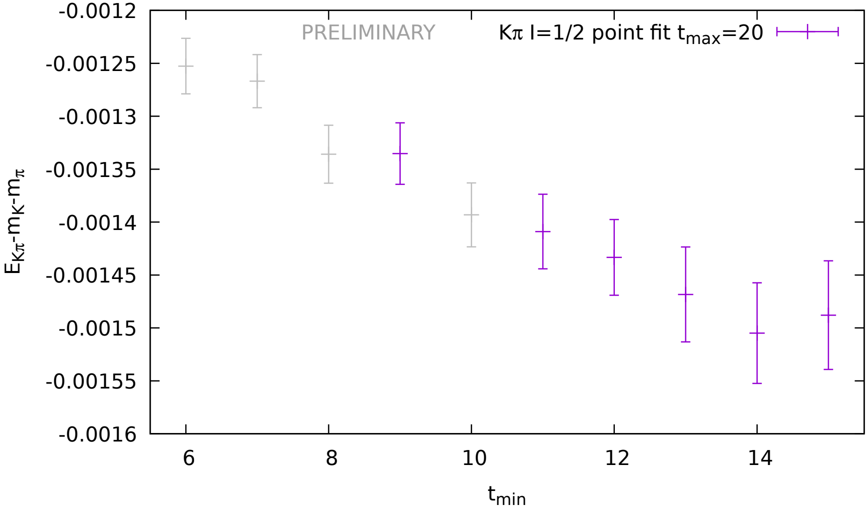
$K\pi$ contributions (ratio $r_{K\pi}$)



(increased error of green curve by factor of 10 to make it visible)

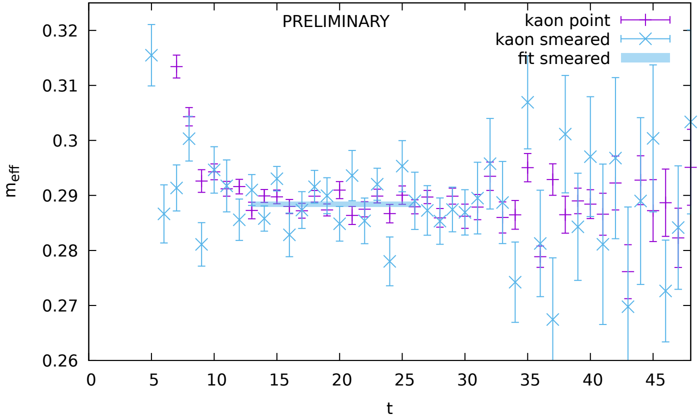
- upwards (downwards) trend of the $K\pi$ function reflects the positive (negative) sign of a_0
- round-the-world effects have a significant impact starting from early times

$K\pi$ ratio fit scan $I = 1/2$ improved



- fits to the ratio $r_{K\pi}$ have smaller errors than fits to $C_{K\pi}$
- plateau starts very late

smeared kaon

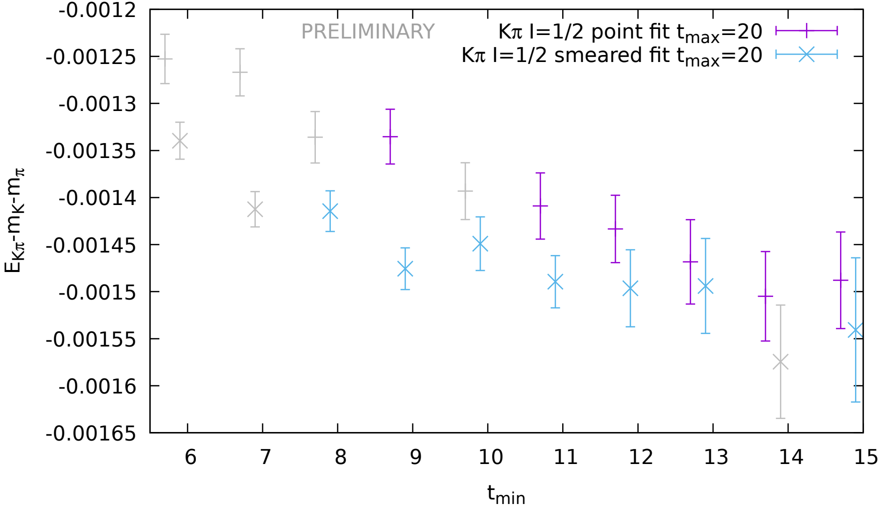


- smeared meson field

$$M^{\Gamma}(\rho, t) = \int \frac{d^3q}{(2\pi)^3}\rho(\vec{q})w^{\dagger}(t,\vec{q})\Gamma v(t,\vec{q})$$

- smeared kaon noisier, but less affected by excited states

$K\pi$ scattering energy $I = 1/2$



- fit to smeared $r_{K\pi}$ (blue) has a longer plateau region
- fits have competitive errors

conclusions

- physical point result important
- computed $K\pi$ correlation function
  - at the physical point
  - removed round-the-world contributions
  - computed ratio to cancel noise
  - smearing to get longer plateau
- $K\pi$ scattering length at physical point with competitive precision
- outlook: finalize the fits and systematic error budget

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