

String breaking in $N_f=2+1$ QCD

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String breaking

- String breaking describes the flattening of the potential $V(r)$ between a static quark anti-quark pair at large distance $r = |\mathbf{r}|$
- It is due to the formation of two static-light mesons in the ground state
- It has been observed as a mixing of “string-like” and two-meson operators [Drummond, 9805012; Philipsen and Wittig, 9807020; Knechtli and Sommer, 9807022, 0005021; Bali et al., 0505012, Bulava et al., 1902.04006]

$$C(\mathbf{r}, t) = \begin{pmatrix} \square & \sqrt{2} \times \text{wavy} & \text{wavy} \\ \sqrt{2} \times \text{wavy} & 2 \times \text{wavy} + \text{wavy} \text{wavy} & \sqrt{2} \times \text{wavy} \\ \text{wavy} & \sqrt{2} \times \text{wavy} & \text{wavy} + \text{wavy} \text{wavy} \end{pmatrix}$$

[Bulava et al., 1902.04006]

CLS ensembles

Set of $N_f = 2 + 1$ ensembles from CLS (Coordinated Lattice Simulations) [M. Bruno et al., 1411.3982]

id	N_{conf}	N_{conf}^W	t_0/a^2	N_s	N_t	m_π [MeV]	m_K [MeV]	$m_\pi L$
N202	244	1802	5.154(19)	48	128	420	420	6.5
N203	94	752	5.1433(74)	48	128	340	440	5.4
N200	104	1664	5.1590(76)	48	128	280	460	4.4
D200	209	1117	5.1802(78)	64	128	200	480	4.2

- same lattice spacing $a = 0.06426(76)$ fm [M. Bruno, T. Korzec, S. Schaefer, 1608.08900]
- quark masses $m_{ud} \equiv m_u = m_d$, m_s vary along a trajectory
 $\sum_{f=u,d,s} m_{bare,f} = \text{const.}$

Smearing

Temporal gauge links: HYP2 static action [Hasenfratz and Knechtli, 0103029; Della Morte, Shindler, Sommer, 0506008; Donnellan et al, 1012.3037]

Quarks: Distillation [Peardon et al, 0905.2160], stochastic evaluation of quark diagrams [Morningstar et al, 1104.3870]

Generalized Eigenvalue Problem (GEVP)

For fixed inter-quark separation \mathbf{r} solve

$$C(t) v_n(t, t_0) = \lambda_n(t, t_0) C(t_0) v_n(t, t_0), \quad n = 0, 1, \dots, N-1 (= 3), \quad t \geq t_0$$

$$\lambda_n(t, t_0) \simeq \exp\{-E_n(t - t_0)\}$$

Effective masses $E_n^{\text{eff}} = \ln[\lambda_n(t, t_0)/\lambda_n(t + a, t_0)]$: theory [Blossier et al, 0902.1265] predicts convergence for $t_0 \leq t/2$

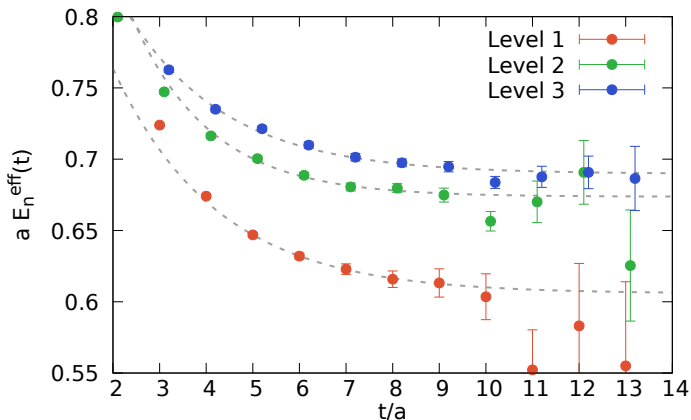
$$E_n^{\text{eff}} = E_n + \epsilon(t, t_0), \quad \epsilon(t, t_0) = \mathcal{O}(\exp\{-(E_{N+1} - E_n)t\})$$

We use a fixed GEVP: diagonalise at time $t = t_d$, define

$$\hat{C}_{i,j} = (v_i(t_0, t_d), C(t) v_j(t_0, t_d)), \quad R_n(t) = \frac{\hat{C}_{nn}(t)}{C_B^2(t)}, \quad n = 0, 1, 2$$

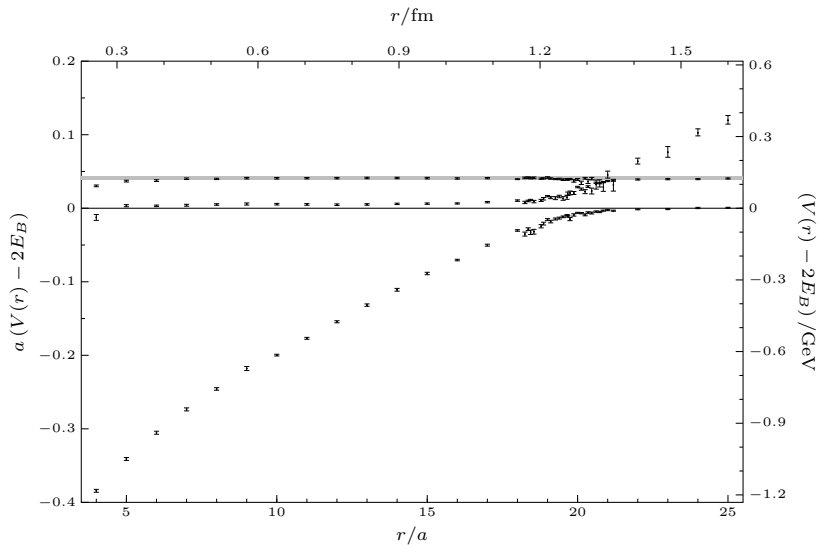
(C_B is the correlator of a single static-light meson, energy E_B), yields energy differences $E_n - 2E_B$

GEVP analysis



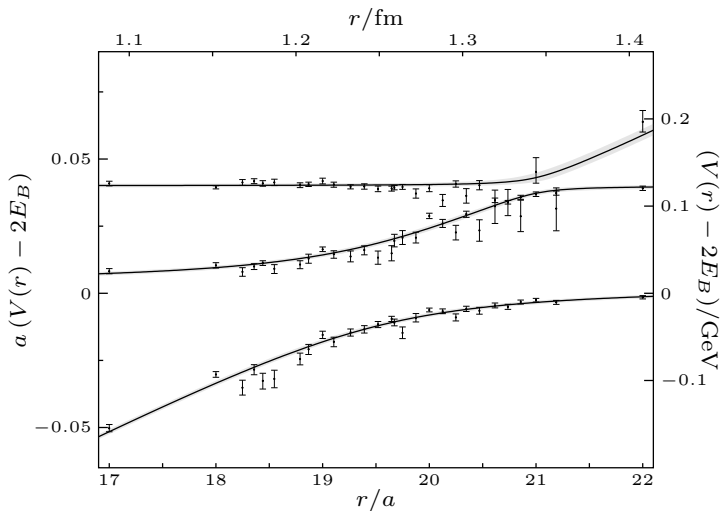
- Effective masses for N200, $r/a = 17$, fixed GEVP with $t_0/a = 5$, $t_d/a = 10$
- Fits $b \exp(-\Delta t) + c$ to exponential fall-off indicate exponential corrections $\Delta \approx 0.5/a \approx 1500 \text{ MeV}$ much larger than the gaps between the levels

Potential



Lowest three levels of the static potential for D200 ($m_\pi = 200$ MeV)

Model fits



Fits to model in the range $r/a \in [12, 25]$ for D200 ($m_\pi = 200 \text{ MeV}$)

Model Hamiltonian

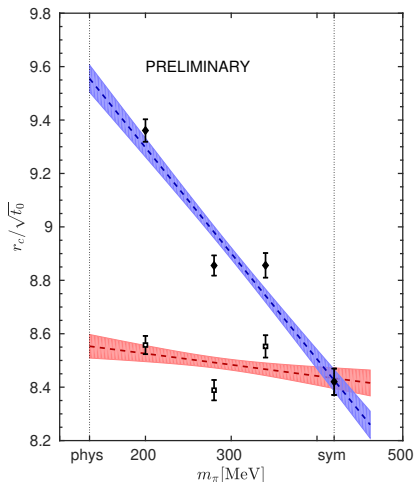
$$H(r) = \begin{pmatrix} \hat{V}(r) & g_1 & g_2 \\ g_1 & \hat{E}_1 & 0 \\ g_2 & 0 & \hat{E}_2 \end{pmatrix}, \quad \hat{V}(r) = \hat{V}_0 + \sigma r$$

- uncorrelated fit to spectrum with 6 parameters: $a\hat{E}_1, a\hat{E}_2, ag_1, ag_2, a^2\sigma, a\hat{V}_0$
- $\hat{V}(r), \hat{E}_1, \hat{E}_2$ are the asymptotic energy levels for $r \rightarrow \infty$ up to $O(r^{-1})$
- Notice: **string tension** σ is defined including the effects of *string breaking* (ground state potential is nowhere linear)
- Preliminary analysis, work is in progress to assess the systematic error on the model parameters

Two string breaking distances r_c and r_{c_s} defined by the crossings

$$\hat{V}(r_c) = \hat{E}_1 \text{ (static-light)} \quad \text{and} \quad \hat{V}(r_{c_s}) = \hat{E}_2 \text{ (static-strange)}$$

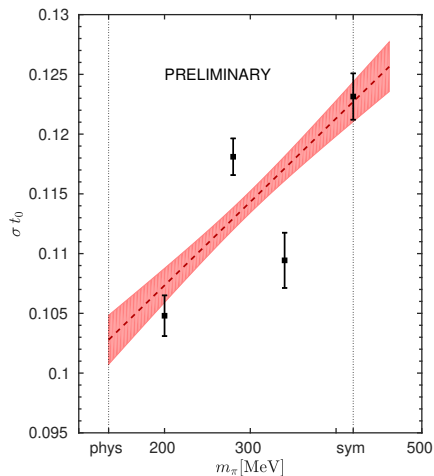
String breaking distances



At the physical point using $t_0 = 0.415(6)$ fm [M. Bruno, T. Korzec, S. Schaefer, 1608.08900]:

$$r_c = 1.255(19) \text{ fm (static-light)}, \quad r_{c_s} = 1.402(22) \text{ fm (static-strange)}$$

String tension



At the physical point using $t_0 = 0.415(6)$ fm [M. Bruno, T. Korzec, S. Schaefer, 1608.08900]:

$$\sqrt{\sigma} = 430(8) \text{ MeV}$$

Conclusions

- GEVP works as expected for string breaking
- We can measure the **three lowest levels of the static potential up to a separation $r = 1.6$ fm**
- We have results for multiple light quark masses
- **Robust definition of the string breaking distances at the physical point**

$$r_c/\sqrt{t_0} = 8.553(44), \quad r_c = 1.255(19) \text{ fm} \quad (\text{static-light})$$

$$r_{c_s}/\sqrt{t_0} = 9.555(51), \quad r_{c_s} = 1.402(22) \text{ fm} \quad (\text{static-strange})$$

Preliminary analysis, systematic error needs to be assessed

- **Robust definition of the string tension at the physical point**

$$\sigma t_0 = 0.1028(21), \quad \sqrt{\sigma} = 430(8) \text{ MeV} \quad (\text{preliminary})$$

For comparison quenched values translated from T_c/σ using $T_c r_0$ from [Necco, 0309017] and $r_0/\sqrt{t_0}$ from [Knechtli et al, 1706.04982] are $\sigma t_0 \in [0.143, 0.159]$