First Results of the Hadron Spectrum from Stabilised Wilson Fermions

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Standard Wilson-Dirac Clover Action for LQCD

The standard $O(a)$-improved Wilson-Dirac operator can be split as:

$$D = D_W + C + m_0 = D'_W + C + 4 + m_0$$

Here D_W is the regular Wilson-Dirac operator, while D'_W is the off-diagonal part only. C is the clover term:

$$C = c_{SW} \frac{i}{4} \sigma_{\mu\nu} F_{\mu\nu}$$

This term is diagonal in spacetime.

A New Clover Action

An exponential variant has recently been proposed [1]

$$(4+m_0)+C \Rightarrow (4+m_0)\exp\left\{\frac{C}{(4+m_0)}\right\} = (4+m_0)\exp\left\{\frac{i}{4}\frac{c_{SW}}{(4+m_0)}\sigma_{\mu\nu}F_{\mu\nu}\right\}$$

The regular clover term can be understood as the first part of an expansion. For more details refer to the previous talk by A. Francis *Properties and ensembles of Stabilised Wilson Fermions*

[1] A. Francis, P. Fritzsch, M. Luscher, A. Rago, *Master-field simulations of O(a)-improved lattice QCD: Algorithms, stability and exactness*, Comput.Phys.Commun. 255 (2020) 107355

Numerical Implementation in CHROMA

- The exponential clover term has been implemented in CHROMA[2], on top of the regular QDP Clover term, to allow computation of more advanced observables.
- Publicly available for the LQCD community
- Tested against the existing openQCD implementation.

The key part of the implementation of the action is calculating $\exp[\sigma_{\mu\nu} F_{\mu\nu}]$, which is a 12×12 matrix, block diagonal (6×6). The exponential needs to be calculated as precisely as possible. This is only computed once before calling the solver, while creating the linear operator.
Using the Cayley-Hamilton theorem we can show that:

$$A^m = \sum_{i=0}^{n} c_i(m, A) A^i \;\rightarrow\; \exp(A) = \sum_{m=0}^{\infty} \frac{1}{m!} \sum_{i=0}^{n} c_i(m, A) A^i = \sum_{i=0}^{n} b_i(N, A) A^i$$

The coefficients $b_i(N, A)$ are found iteratively using Horner's scheme and the coefficients of the characteristic polynomial of A

[2] R. G. Edwards, B. Joó, *The Chroma Software System for Lattice QCD*, 2005, POS Lattice2004

Software Stack

To perform the spectroscopy analysis we used these tools:

- The `CHROMA` stack[3], including `QUDA`[4] and `QPHIX`
- `LALIBE`[5], a code built on top of `CHROMA` with many useful tools for spectroscopy
- `gvar` and `lsqfit`[6] for bayesian fitting

The computation was mostly done on the CORI machine from the National Energy Research Scientific Computing Center (NERSC), LBNL, USA

[3] R. G. Edwards, B. Joó, *The Chroma Software System for Lattice QCD*, 2005, POS Lattice2004

[4] M. A. Clark, R. Babich, K. Barros, R. Brower, and C. Rebbi, *Solving Lattice QCD systems of equations using mixed precision solvers on GPUs*, Comput. Phys. Commun. 181, 1517 (2010) [arXiv:0911.3191 [hep-lat]].

[5] `https://github.com/callat-qcd/lalibe`

[6] Created by G. Peter Lepage (Cornell University) 2008, copyright (c) 2008-2020 G. Peter Lepage

Ensembles' Properties

Ensemble	Size	β	N_{confs}	$N_{sources}$
a094m400mL6.2trMc	$32^3 \times 96$	3.8	234	3744
a094m300mL4.5trMc	$32^3 \times 96$	3.8	167	2758
a094m200mL3.3trMc	$32^3 \times 96$	3.8	210	3328
a064m400mL6.4trMc	$48^3 \times 96$	4.0	200	3200

Ensemble	t_0/a^2	a [fm]	m_π [MeV]	$m_\pi L$	L [fm]
a094m400mL6.2trMc	2.4420(36)	0.094	409.14(69)	6.2	3.0
a094m300mL4.5trMc	2.4540(27)	0.094	290.9(1.2)	4.5	3.0
a094m200mL3.3trMc	2.4645(35)	0.094	219.4(1.9)	3.3	3.0
a064m400mL6.4trMc	5.2470(57)	0.064	409.42(46)	6.4	3.1

Using Gaussian smearing, we compute both smeared-point (PS) and smeared-smeared (SS) correlators.

Smearing Parameters

We performed a scan for the Gaussian smearing [7] [8] parameters to use, we selected $N_s = 32$ iteration and $\sigma = 3.86$ for the smearing width.

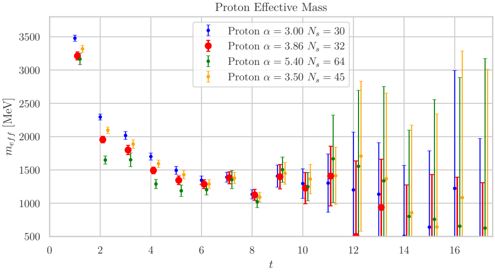


Figure 1: Nucleon mass on the a094m200mL3.3trMc ensemble with different Gaussian smearing parameters.

[7] T. A. DeGrand and R. D. Loft, Comput. Phys. Commun. 65, 84 (1991)

[8] Stephan Güsken, *A study of smearing techniques for hadron correlation functions*, Nuclear Physics B, (1990)

Bayesian Fitting of Correlation Function

For the analysis we used a Bayesian framework with constraints[9]. The n-state function used as ansatz reads:

$$C(t, Z_{P,n}, Z_{S,n}, E_n) = \sum_{n=0}^{n=N} Z_{S,n} Z_{S/P,n} e^{-E_n t}$$

Where $Z_{P,n}$ and $Z_{S,n}$ are the amplitudes for the point and smeared sources/sinks. We fit both the PS and SS correlators at once.

[9] G.P. Lepage, B. Clark, C.T.H. Davies, K. Hornbostel, P.B. Mackenzie, C. Morningstar, and H. Trottier, *Con- strained curve fitting*, Nucl. Phys. B Proc. Suppl. 106, 1220 (2002

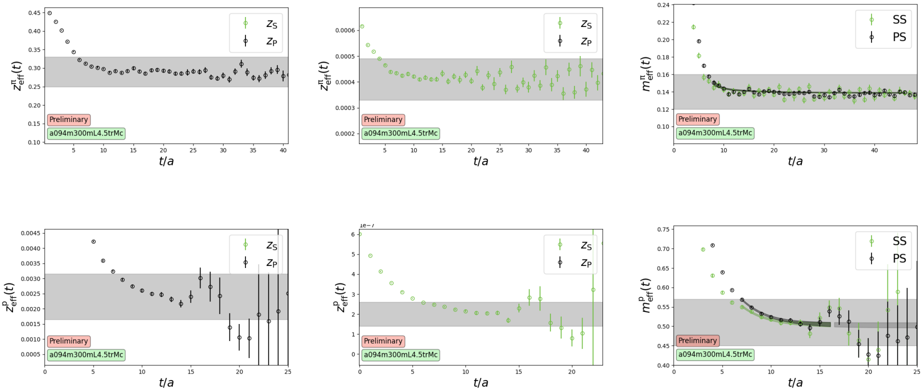
Bayesian Fitting of Correlation Function

To constrain the fit we used a set of priors for the energies and the amplitudes. These enter the fit through an addition to the χ^2 function to be minimized:

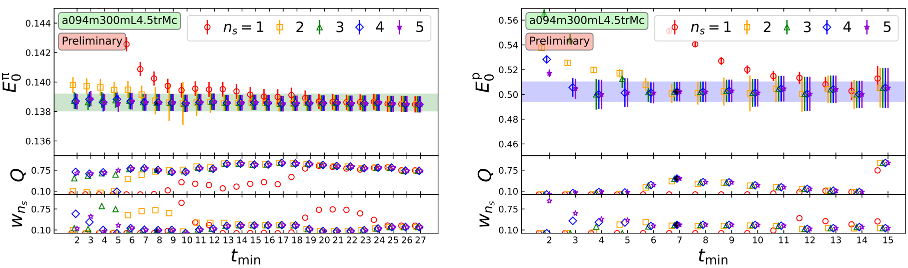
$$\chi^2_{prior} = \sum_{n=0}^{N} \frac{(Z_{P,n} - \tilde{Z}_{P,n})^2}{\tilde{\sigma^2}_{Z_{P,n}}} + \sum_{n=0}^{N} \frac{(Z_{S,n} - \tilde{Z}_{S,n})^2}{\tilde{\sigma^2}_{Z_{S,n}}} + \sum_{n=0}^{N} \frac{(E_n - \tilde{E}_n)^2}{\tilde{\sigma^2}_{E_n}}$$

The priors for $E_0, Z_{P,n}, Z_{S,n}$ are chosen as normally distributed, while the excited-state energy priors are set to be log-normal. The excited-state energy splittings are set to the value of $2m_\pi$ with a width allowing for fluctuations down to one pion mass within one standard deviation.

Example of Effective Mass Fit Results

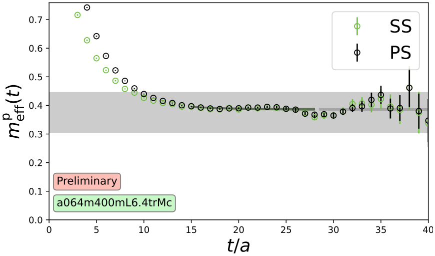


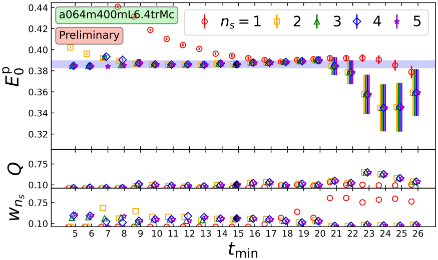
Fit Stability Considerations



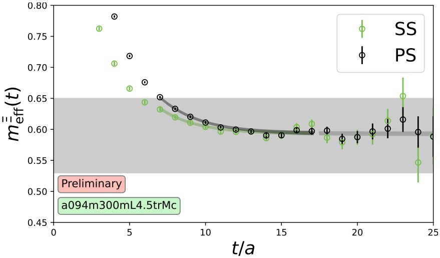
Stability for the ground state as the t_{min} of the fitting window is changed for fits with different number of states. Here Q is the probability that the augmented χ^2 from the fit could have been larger, by chance, assuming the best-fit model is correct and W_{n_s} is the relative weight one the fit over the others, at fixed t_{min}

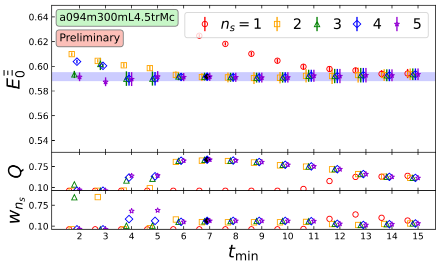
Fit Stability Considerations

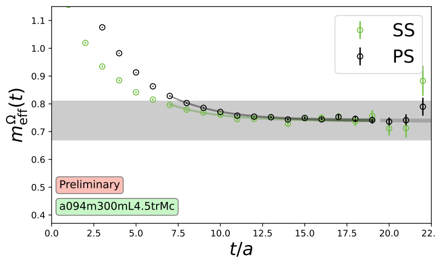


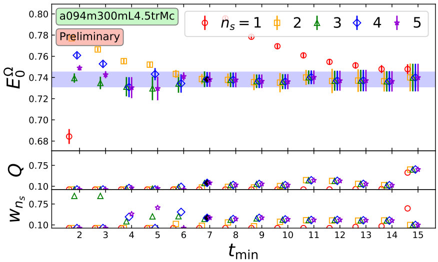


Other Hadrons...

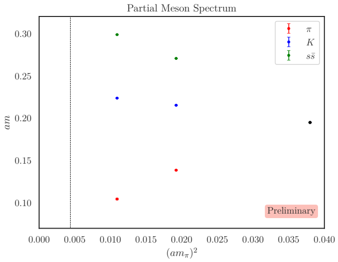


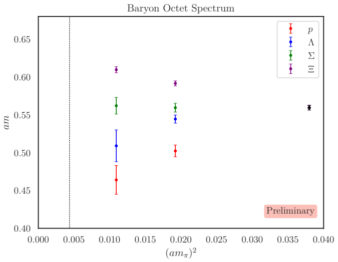


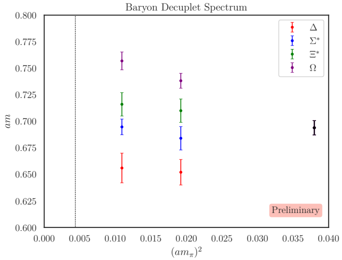




Hadron Spectrum







Preliminary results for the hadron spectrum at $\beta = 3.8$ with SWF. The black point is the $SU(3)$ flavor symmetric point.

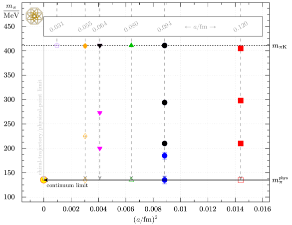
Moving Forward

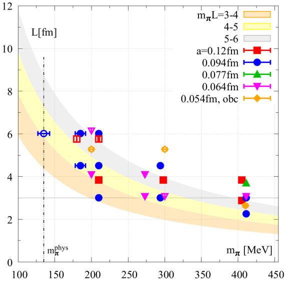
Summary:

- Implemented SWF for spectrum measurements in CHROMA
- Performed measurements on available configurations
- Bayesian method applied to determine ground-state masses

Going Forward:

- Repeat analysis once the 500 configurations per ensemble target is reached
- Refine the analysis and perform infinite volume and chiral extrapolations
- Establish robust results as future benchmarks for the openLAT initiative





Thank You

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