# Nuclear Parity Violation from 4-quark Interactions

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# Premise

- Study hadronic parity violation in $\Delta I = 1$ nucleon-nucleon interaction
- Aim to calculate the parity-odd pion-nucleon coupling $h_\pi^1$
- Comes from the weak Lagrangian

$$\mathcal{L}_{PV}^{w} = -\frac{h_\pi^1}{\sqrt{2}} \bar{N} \left(\vec{\tau} \times \vec{\pi}\right)_3 N + \ldots$$

- Experimental result by NPDGamma collaboration (Blyth *et al.*, Phys.Rev.Lett. 121, 2018)

$$h_\pi^1 = [2.6 \pm 1.4(stat) \pm 0.2(syst)] \times 10^{-7}$$

# Calculation on Lattice

- Only one study on the lattice (Wasem, Phys.Rev C85, 2012)

$$h_\pi^1 = \left(1.099 \pm 0.505^{+0.058}_{-0.064}\right) \times 10^{-7}$$

- However, with lots of systematics not accounted for
  - $N\pi$ represented by 3-quark interpolator
  - "quark-loop" contributions neglected
  - No renormalization performed
  - Single lattice at pion mass 389 MeV
- We try to benchmark a recently proposed technique for this calculation
  - Using PCAC relation, get rid of the soft-pion in the final state

$$F_\pi h_\pi^1 \approx -\frac{(\delta m_N)_{4q}}{\sqrt{2}} \quad \text{where} \ \ (\delta m_N)_{4q} \equiv (m_n - m_p)_{4q}$$

  - Use Feynman-Hellmann theorem to obtain the mass splitting

# The Parity-odd Coupling

- One obtains the coupling constant from the matrix element

$$h_\pi^1 = -\frac{i}{2m_N} \lim_{p_\pi \to 0} \langle n\pi^+ | \mathcal{L}_{PV}^w |p\rangle$$

- Using PCAC, construct a parity-even chiral partner (Feng, Guo, Seng, Phys.Rev.Let 120, 2018)

$$\lim_{p_\pi \to 0} \langle n\pi^+ | \mathcal{L}_{PV}^w |p\rangle \approx -\frac{\sqrt{2}i}{F_\pi} \langle p| \mathcal{L}_{PC}^w |p\rangle$$

- With the effective Lagrangian

$$\mathcal{L}_{PC}^w = -\frac{G_F}{\sqrt{2}} \frac{\sin^2\theta_W}{3} \sum_i (C_i\theta_i + S_i\theta_i^{(s)})$$

# 4-quark Operators

- The 4-quark operators thus derived are

$$\theta_1 = \bar{q}_a\gamma_\mu q_a \bar{q}_b\gamma_\mu\tau_3 q_b \qquad \theta_2 = \bar{q}_a\gamma_\mu q_b \bar{q}_b\gamma_\mu\tau_3 q_a$$
$$\theta_3 = \bar{q}_a\gamma_\mu\gamma_5 q_a \bar{q}_b\gamma_\mu\gamma_5\tau_3 q_b$$
$$\theta_1^{(s)} = \bar{s}_a\gamma_\mu s_a \bar{q}_b\gamma_\mu\tau_3 q_b \qquad \theta_2^{(s)} = \bar{s}_a\gamma_\mu s_b \bar{q}_b\gamma_\mu\tau_3 q_a$$
$$\theta_3^{(s)} = \bar{s}_a\gamma_\mu\gamma_5 s_a \bar{q}_b\gamma_\mu\gamma_5\tau_3 q_b \qquad \theta_4^{(s)} = \bar{s}_a\gamma_\mu\gamma_5 s_b \bar{q}_b\gamma_\mu\gamma_5\tau_3 q_a$$

- Using Fierz identities we can get rid of the colour-crossed operators. In the light quark sector we get

$$\mathcal{O}_S = \bar{q}\,\mathbb{1}\otimes\mathbb{1}\,q\;\bar{q}\,\mathbb{1}\otimes\tau_3\,q$$
$$\mathcal{O}_P = \bar{q}\,\gamma_5\otimes\mathbb{1}\,q\;\bar{q}\,\gamma_5\otimes\tau_3\,q$$
$$\mathcal{O}_V = \bar{q}\,\gamma_\mu\otimes\mathbb{1}\,q\;\bar{q}\,\gamma_\mu\otimes\tau_3\,q$$
$$\mathcal{O}_A = \bar{q}\,\gamma_\mu\gamma_5\otimes\mathbb{1}\,q\;\bar{q}\,\gamma_\mu\gamma_5\otimes\tau_3\,q$$

# Feynman-Hellmann Theorem

- Considering a small perturbation $H \to H + \lambda H_\lambda$
- According to Feynman-Hellmann theorem

$$\frac{\partial m_N}{\partial \lambda} = \langle N | H_\lambda | N \rangle$$

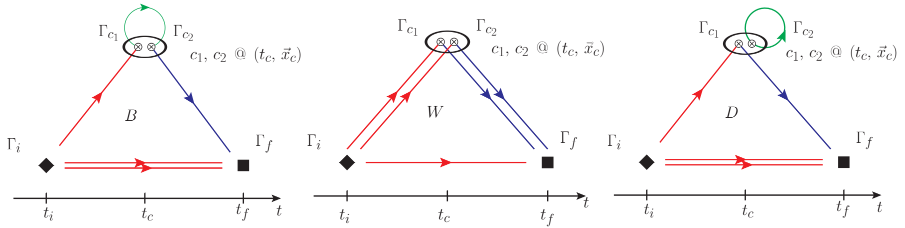
- We can calculate the energy shift as

$$\left| \frac{\partial m_N}{\partial \lambda} \right|_{\lambda=0} (t, \tau) = \frac{1}{\tau} \left[ \frac{\partial_\lambda C_\lambda(t)}{C(t)} - \frac{\partial_\lambda C_\lambda(t+\tau)}{C(t+\tau)} \right]_{\lambda=0}$$

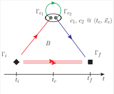
- Here $C_\lambda$ is the 2 point correlator calculated in the presence of the source $\lambda$ and $C$ is the unperturbed correlator

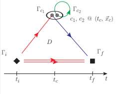
# Diagrams

- We obtain $\partial_\lambda C_\lambda$ by calculating $\sum_x \langle p|\mathcal{L}^w_{PC}(x)|p\rangle$
- The nucleon interpolators at source and sink are of the form $(u^T C\gamma_5 d)u$
- Inserting the 4-quark operators gives the 3-pt correlator with 3 sets of diagrams



# Method for B/D




- Calculate x-dependent loops for a number of time-diluted sources

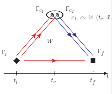
$$L_f^{(k)}(x_c)_{\alpha\beta}^{ab} = \phi^{(k)}(x_c)_\alpha^a \, \xi^{(k)}(x_c)_\beta^{b*}$$

- With $E[L_f^{(k)}(x_c)_{\alpha\beta}^{ab}] \equiv S^{(f)}(x_c; x_c)_{\alpha\beta}^{ab}$
- Average over stochastic sources to get $\bar{L}(x_c)$
- Build sequential source

$$B : \Phi(x_c; x_i) = \Gamma_c \, \bar{L}(x_c) \, \Gamma_c \, S(x_c; x_i)$$
$$D : \Phi(x_c; x_i) = Tr(\Gamma_c \, \bar{L}(x_c)) \, \Gamma_c \, S(x_c; x_i)$$

- Invert on $\Phi$ to get the sequential propagator

# Method for W




- Define a set of scalar noise sources $\eta^{(k)}(x)$ such that

$$E[\eta^{(k)}(x)\eta^{(l)}(y)] = \delta_{kl}\delta_{x,y}$$

- Build W type sequential propagator at the source

$$\tilde{W}^{(i)}(\Gamma_c) = \sum_{x_c} S(x_f; x_c)\Gamma_c \eta^{(i)}(x_c) S(x_c; x_i)$$

- Calculate expectation of a product of two such propagators

$$E[\tilde{W}^{(i)}(\Gamma_c)\tilde{W}^{(k)}(\Gamma_c)] = \sum_{x_c} \tilde{W}(x_f; x_c, \Gamma_c; x_i)\tilde{W}(x_f; x_c, \Gamma_c; x_i)$$

# Gauge Ensemble

- $N_f = 2 + 1 + 1$ twisted mass ensemble $cA211.30.32$

| $L^3 \times T$ | $a[fm]$ | $a\mu_l$ | $am_\pi$ | $m_\pi L$ | $m_\pi[MeV]$ |
|---|---|---|---|---|---|
| $32^3 \times 64$ | $0.097$ | $0.00300$ | $0.12530\,(16)$ | $4.01$ | $261.1\,(1.1)$ |

- 1262 configurations

# First Run on Test Ensemble

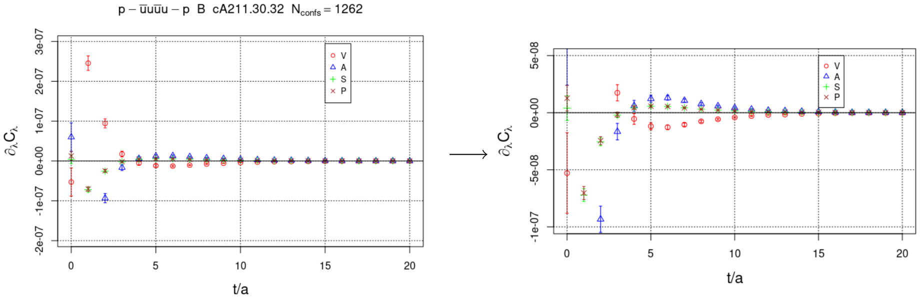
- Working with only light quark flavors

| | Noise Samples | Source Coordinates |
|---|---|---|
| B/D | 1 | 8 |
| W | 8 | 2 |

- The propagators are source and sink smeared
- all insertions with 4 quark operators at source and sink

# Results (Diagrams)

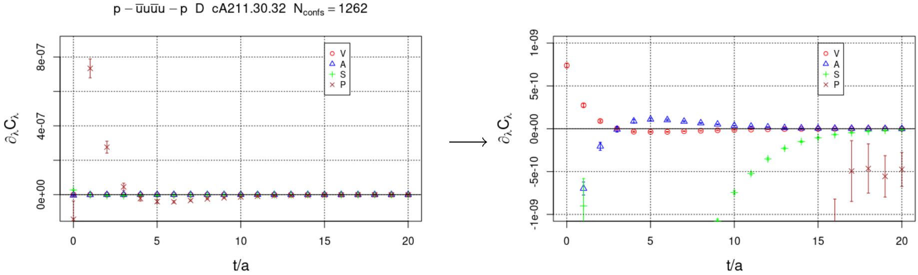
B type diagram for $\bar{u}\Gamma u\bar{u}\Gamma u$ insertion




- $V$ and $A$ seem to be symmetric about the zero-line
- $S$ and $P$ have almost identical signal

# Results (Diagrams)

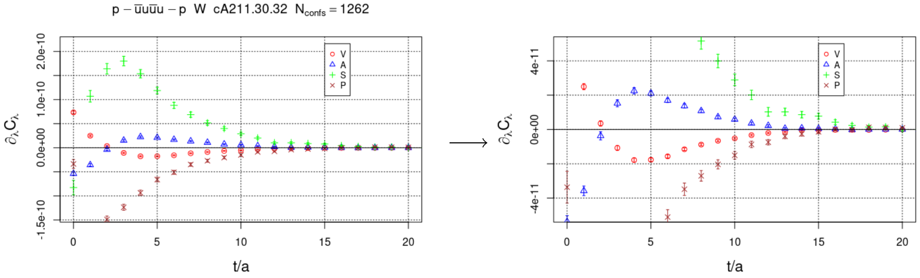
$D$ type diagram for $\bar{u}\Gamma u \bar{u}\Gamma u$ insertion




- The Signal from $\mathcal{O}_P$ is significantly more dominant
- $V$ and $A$ are about 2 orders of magnitude weaker compared to $B$-type

# Results (Diagrams)

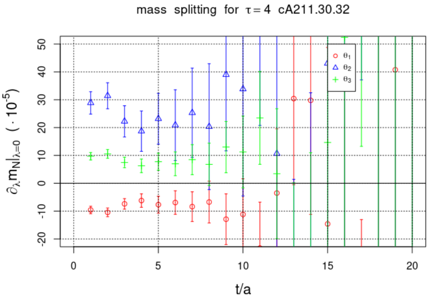
W type diagram for $\bar{u}\Gamma u \bar{u}\Gamma u$ insertion

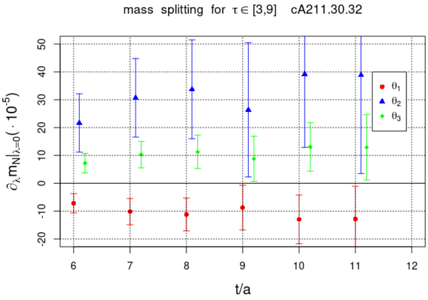



- Symmetry between $V$ and $A$ seen here as well
- Again signal is much weaker compared to $B$-type, which seems to be the major contributor to the final operators

# Preliminary Results

The energy shift for the 4-quark operators, calculated with the FHT analysis




- $\theta_1$ and $\theta_3$ look symmteric, which comes directly from the symmetry observed in the individual diagrams
- colour-crossed $\theta_2$ has more noise, could be due to disproportionate contribution from $\mathcal{O}_P$ in $D$

# Concluding Remarks

- Non-zero signal observed for all operators
- Need to have better control for $\theta_2$
- Next step is renormalization
- Also add strange quark
- Investigate other techniques and compare cost and signal quality
- Then calculate with the physical pion mass ensembles
- And finally estimate $h_\pi^1$

# The PCAC Relation

- The PCAC relation for an operator $\hat{O}$ goes as

$$\lim_{p_\pi \to 0} \langle a\pi^i | \hat{O} | b \rangle = \frac{i}{F_\pi} \langle a | \left[ \hat{O}, \hat{Q}^i_A \right] | b \rangle$$

- $\hat{Q}^i_A$ is the axial charge operator, $a$ and $b$ are hadrons, $F_\pi$ is the pion decay constant
- This relates $\mathcal{L}^w_{PV}$ to $\mathcal{L}^w_{PC}$ upto leading order in chiral EFT (Guo & Seng, Eur.Phys.J. C, 2019)

$$\lim_{p_\pi \to 0} \left\langle n\pi^+ \right| \mathcal{L}^w_{PV} \left| p \right\rangle \approx -\frac{\sqrt{2}i}{F_\pi} \left\langle p \right| \mathcal{L}^w_{PC} \left| p \right\rangle = \frac{\sqrt{2}i}{F_\pi} \left\langle n \right| \mathcal{L}^w_{PC} \left| n \right\rangle$$

# Mass Splitting and $h_\pi^1$

- One can write the matrix element in terms of the neutron-pion mass splitting $(\delta m_N)_{4q} \equiv (m_n - m_p)_{4q}$ induced by $\mathcal{L}_{PC}^w$

$$(\delta m_N)_{4q} = \frac{1}{m_N} \langle p| \mathcal{L}_{PC}^w |p\rangle = -\frac{1}{m_N} \langle n| \mathcal{L}_{PC}^w |n\rangle$$

- Then $h_\pi^1$ is obtained as

$$F_\pi h_\pi^1 \approx -\frac{(\delta m_N)_{4q}}{\sqrt{2}}$$

- This $(\delta m_N)_{4q}$ is equivalent to the energy shift $\partial_\lambda m_N|_{\lambda=0}$, as we are calculating using FHT

# FHT Method

- The 2-pt correlator in the presence of the external source $\lambda$

$$C_\lambda(t) = \langle\lambda| N(t)N^\dagger(0) |\lambda\rangle = \frac{1}{Z_\lambda}\int \mathcal{D}\Phi N(t)N^\dagger(0)e^{-S-S_\lambda}$$

- where $S_\lambda = \lambda \int d^4x\ \mathcal{L}^w_{PC}(x)$
- The partial derivative w.r.t. $\lambda$ relates $C_\lambda$ to the matrix element we want

$$-\frac{\partial C_\lambda(t)}{\partial\lambda}\bigg|_{\lambda=0} = -\,C(t)\int d^4x\ \langle\Omega| \mathcal{L}^w_{PC}(x) |\Omega\rangle + \int d^4x\ \langle\Omega| \mathcal{T}\left\{N(t)\mathcal{L}^w_{PC}(x)N^\dagger(0)\right\} |\Omega\rangle$$

- The first term vanishes due to lattice symmetries

# Very Naive Estimate of $h_\pi^1$

- If we naively add the operators with the known Wilson coefficients