Scattering from generalised ϕ^4

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- We want to investigate techniques to extract scattering amplitudes from Euclidean Lattice field theory
- As a tool we are using a variation of ϕ^4 theory with two fields with different masses
- ϕ^4 theory as been used many times to test method for scattering quantities calculation e.g. [F. Romero-López , A. Rusetsky, N. Schlage and C. Urbach (2021)]
- We test a proposal to extract the scattering length from lattice simulations [M. Bruno, M. T. Hansen (2021)] and Talk M. Bruno, "Variations on the Maiani-Testa approach and the inverse problem", Hadron Spectroscopy and Interactions, 27 Jul 2021, 06:45 local time.

BH method

One of the results derived in [M. Bruno, M. T. Hansen (2021)]

$$\langle \pi_0 | \tilde{N}_0(t) N^\dagger(t_i) | \pi_0 \rangle_c = \mathcal{N} e^{-m_N (t-t_i)} \Big[8\pi (m_N + m_\pi) a_{N\pi} (t - t_i) - 16 a_{N\pi}^2 \sqrt{2\pi (m_N + m_\pi) m_N m_\pi (t - t_i)} \Big] + O(t^0)$$

- $a_{N\pi}$ scattering length
- N is the interpolating field of the Nucleon and
- $\tilde{N}_0(t) = \frac{1}{V} \sum_{\mathbf{x}} N(t, \mathbf{x})$
- m_π,m_N masses of the particles
- $\langle \cdot \rangle_c$ is the connected part

The model

- Two real fields ϕ_0 and ϕ_1
- Different masses $m_0 < m_1$
- $Z_2 \otimes Z_2$ symmetry $\phi_0 \to -\phi_0 \otimes \phi_1 \to -\phi_1$

$$\mathcal{L} = \sum_{i=0,1} \left(\frac{1}{2} \partial_\mu \phi_i \partial_\mu \phi_i + \frac{1}{2} m_i \phi_i^2 + \lambda_i \phi_i^4 \right) + \mu \phi_0^2 \phi_1^2$$

- ϕ_0 mimic the π and ϕ_1 the N
- On the lattice $\partial_\mu = \phi(x+\mu) - \phi(x)$
- Metropolis-Hastings algorith to generate ensembles
- Implementation with Kokkos to have a performance portable implementation [H. C. Edwards, C. R. Trott, D. Sunderland (2014)]

- Fitting the BH formula reads

$$C_4^{\mathrm{BH}}(t_f,t,t_i) \equiv \frac{\langle\tilde{\phi}_0(t_f)\tilde{\phi}_1(t)\tilde{\phi}_1(t_i)\tilde{\phi}_0(0)\rangle}{\langle\tilde{\phi}_0(t_f)\tilde{\phi}_0(0)\rangle\langle\tilde{\phi}_1(t)\tilde{\phi}_1(t_i)\rangle} - 1$$

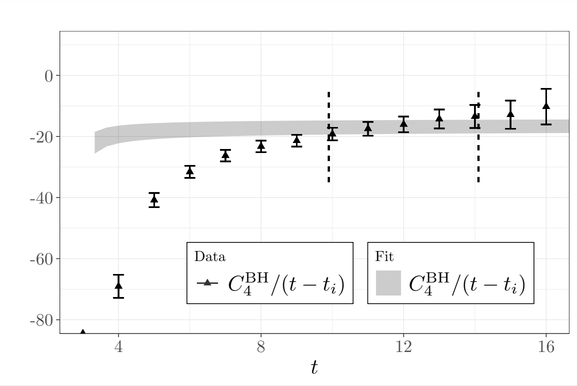
$$\xrightarrow[t\gg t_i\gg 0]{T\gg t_f\gg t} \frac{2}{L^3}\left[\pi\frac{a_0}{\mu_{01}}(t-t_i) - 2a_0^2\sqrt{\frac{2(t-t_i)}{\mu_{01}}} + O\left((t-t_i)^0\right)\right]$$

- $\tilde{\phi}_i = \sum_{\mathbf{x}} \phi(t,\mathbf{x})$
- $\mu_{01} = M_0 M_1/(M_0+M_1)$
- $2M_0 \ \sim M_1$ masses of the particle
- $m_0 = -4.952$, $m_1 = -4.85$, $\lambda_0 = \lambda_1 = \mu/2 = 2.5$

- In ϕ^4 model the BH formula reads

$$C_4^{\rm BH}(t_f,t,t_i) \equiv \frac{\langle\tilde{\phi}_0(t_f)\tilde{\phi}_1(t)\tilde{\phi}_1(t_i)\tilde{\phi}_0(0)\rangle}{\langle\tilde{\phi}_0(t_f)\tilde{\phi}_0(0)\rangle\langle\tilde{\phi}_1(t)\tilde{\phi}_1(t_i)\rangle} - 1$$

$$\xrightarrow[t\gg t_i\gg 0]{T\gg t_f\gg t} \frac{2}{L^3}\left[\pi\frac{a_0}{\mu_{01}}(t-t_i) - 2a_0^2\sqrt{\frac{2(t-t_i)}{\mu_{01}}} + O\left((t-t_i)^0\right)\right]$$

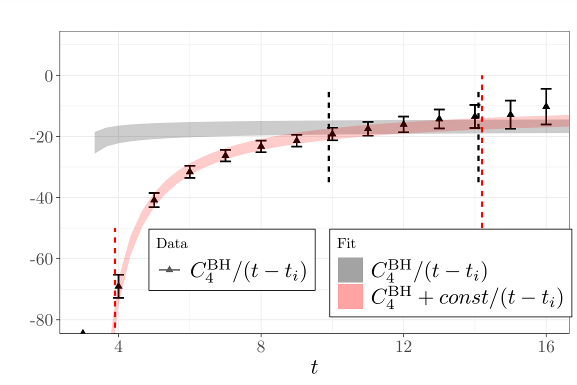


- Example plot for $L22T96$, $t_f = 16$, $t_i = 3$
- fit [10,14] $\chi^2/d.o.f \sim 0.7$
- fit [6,14] $\chi^2/d.o.f \sim 5$

- Fitting the constant term

$$C_4^{\mathrm{BH}}(t_f,t,t_i) \equiv \frac{\langle\tilde{\phi}_0(t_f)\tilde{\phi}_1(t)\tilde{\phi}_1(t_i)\tilde{\phi}_0(0)\rangle}{\langle\tilde{\phi}_0(t_f)\tilde{\phi}_0(0)\rangle\langle\tilde{\phi}_1(t)\tilde{\phi}_1(t_i)\rangle} - 1$$

$$\xrightarrow[t\gg t_i\gg 0]{T\gg t_f\gg t} \frac{2}{L^3}\left[\pi\frac{a_0}{\mu_{01}}(t-t_i) - 2a_0^2\sqrt{\frac{2(t-t_i)}{\mu_{01}}} + \color{red}{const}\right]$$

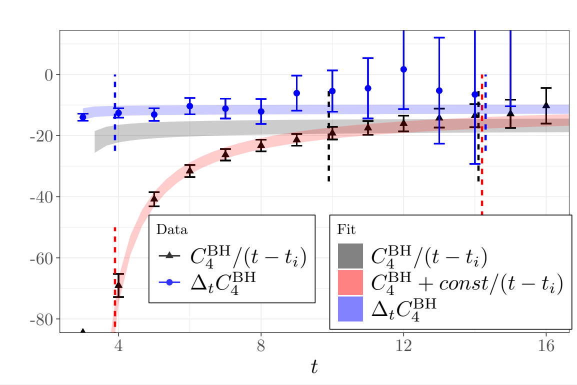


- $\color{red}{C_4^{\mathrm{BH}} + const}$ fit [4,14]
  $\chi^2/d.o.f \sim 0.2$
  $a_0 = -0.31(6)$

- We can cancel the constant term using a shifted correlator

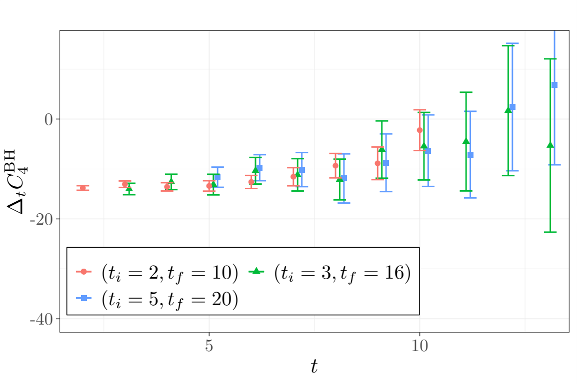
$$\Delta_t C_4^{\text{BH}}(t_f,t,t_i) = C_4^{\text{BH}}(t_f,t+1,t_i) - C_4^{\text{BH}}(t_f,t,t_i)$$

$$\Delta_t C_4^{\text{BH}}(t_f,t,t_i) \approx \frac{2}{L^3}\left[\pi\frac{a_0}{\mu_{01}} - 2a_0^2\sqrt{\frac{2}{\mu_{01}}}\left(\sqrt{t+1-t_i}-\sqrt{t-t_i}\right)\right].$$



- $C_4^{\text{BH}} + const$ fit [4,14]
  $\chi^2/d.o.f \sim 0.2$
  $a_0 = -0.31(6)$
- $\Delta_t C_4^{\text{BH}}$ fit [4,14]
  $\chi^2/d.o.f \sim 0.2$
  $a_0 = -0.34(4)$

- We check the dependence on t_f and t_i in $\Delta_t C_4^{\rm BH}(t_f, t, t_i)$



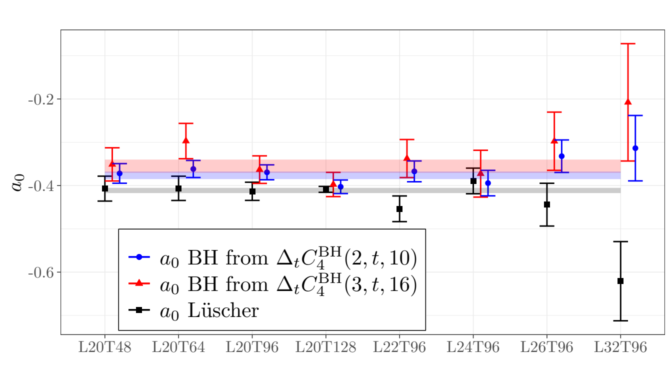
- Consistent result for different t_f and t_i
- Smaller error for smaller t_f and t_i

- We compare the result of BH method with the Lüscher threshold expansion [M. Lüscher (1986)]

$$\Delta E_2 = -\frac{2\pi a_0}{\mu_{01} L^3}\left[1 + c_1 \frac{a_0}{L} + c_2 \left(\frac{a_0}{L}\right)^2\right] + O\left(L^{-6}\right),$$

- $\Delta E_2 = E_2 - M_0 - M_1$
- E_2 the interacting two-particle energy
- $\mu_{01} = M_0 M_1/(M_0 + M_1)$
- $c_1 = -2.837297$, $c_2 = 6.375183$

$$\langle\tilde{\phi}_1(t)\tilde{\phi}_0(t)\tilde{\phi}_1(0)\tilde{\phi}_0(0)\rangle \xrightarrow[T-t\gg 0]{t\gg 0} A_2 e^{-E_2\frac{T}{2}}\cosh\left(E_2(t-\frac{T}{2})\right) + B_2 e^{-(M_0+M_1)\frac{T}{2}}\cosh\left((M_1-M_0)(t-\frac{T}{2})\right).$$



- In each ensemble both method are consistent
- Systematic difference between the average, possibly due to lattice artefacts
- Statistical error and scaling with L similar
- BH method is a promising alternative that will be interesting to try it in QCD where the continuum limit can be studied

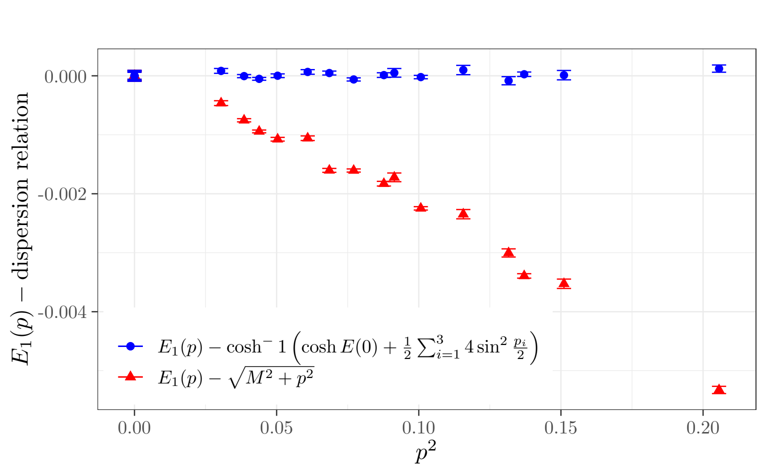
- We also investigate the computation of the scattering quantities at non zero momentum
- We studied s-wave scattering amplitude for two particles with Lüscher method [M. Lüscher (1991)]
- We compute the spectrum of our ϕ^4 model at $p \neq 0$ for the lighter particle.
- $3M_0 \sim M_1$ masses of the particle
- $m_0 = -4.9$, $m_1 = -4.65$, $\lambda_0 = \lambda_1 = \mu/2 = 2.5$

- One particle spectrum from the correlator

$$\langle \tilde{\phi}_0(t,p)\tilde{\phi}_0(0,-p)\rangle \approx |A_1| \left(e^{-E_1(p)t} + e^{-E_1(p)(T-t)}\right)$$

$p = 2\pi n/L$ with $n = (0,0,0), (1,0,0), (1,1,0), (1,1,1)$

- The data are described well by the lattice bosonic dispersion relation



- Two particle spectrum from the correlator

$$\langle \hat{O}_2(t,p)\hat{O}_2(0,-p)\rangle \xrightarrow[T-t\gg 0]{t\gg 0} A_2 e^{-E_2(p)\frac{T}{2}} \cosh\left(E_2(p)(t-\frac{T}{2})\right)$$
$$+A_1 e^{-(E_1(p)+M_0)\frac{T}{2}} \cosh\left((E_1(p)-M_0)(t-\frac{T}{2})\right)$$

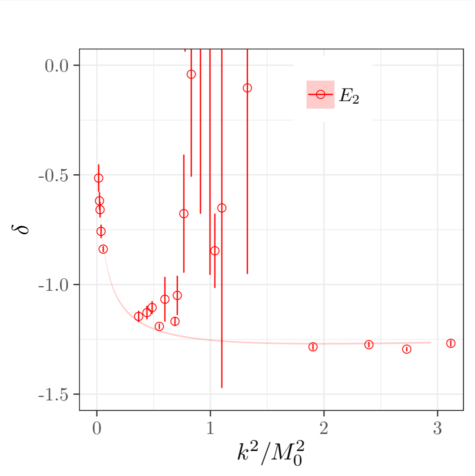
- $\hat{O}_2(t,p) = \tilde{\phi}_0(t,p)\tilde{\phi}_0(t,0)$ in the A1 irrep
- We also compute $\hat{O}_2(t,0) = \sum_{i=x,y,z} \tilde{\phi}_0(t,p_i)\tilde{\phi}_0(t,-p_i)$ in the A1 irrep
- Calculate the S-wave phase shift as [M. Lüscher (1991)]

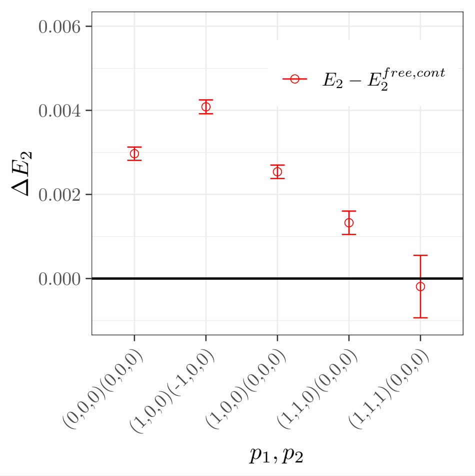
$$\cot\delta = \frac{Z_{0,0}(1,q^2)}{\pi^{3/2}\gamma q}$$

- $Z_{0,0}$ Lüscher zeta function
- $\gamma = E_2/E_{CM}$ with $E_{CM} = E_2 - p^2$
- $q = kL/2\pi$ with $k = \frac{E_{CM}}{4} - M_0^2$ the scattering momentum

- $k \cot \delta = \frac{1}{a_0} + \frac{r_0 k^2}{2}$
- $k = \frac{E_{CM}}{4} - M_0^2$

- $\hat{O}_2(t) = \tilde{\phi}(t, p_1)\phi(t, p_2)$
- $L = 32\ T = 32$

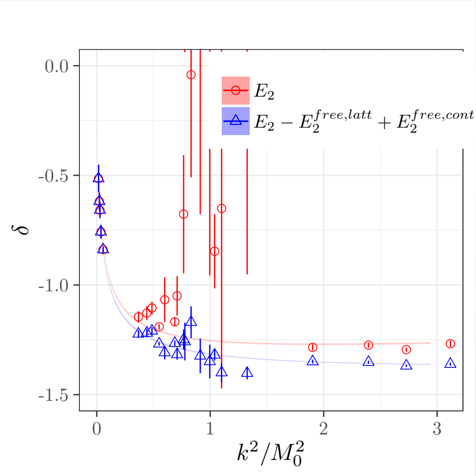


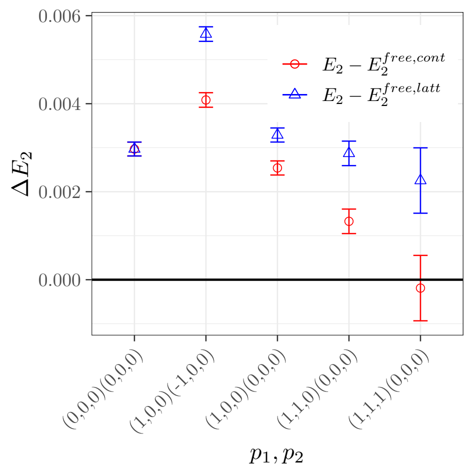


- Add a lattice artefact to the energy

$$E_2 \to E_2 - E_2^{free,latt} + E_2^{free,cont}$$

- Similar treatment in [K. Rummukainen and S. A. Gottlieb (1995)]





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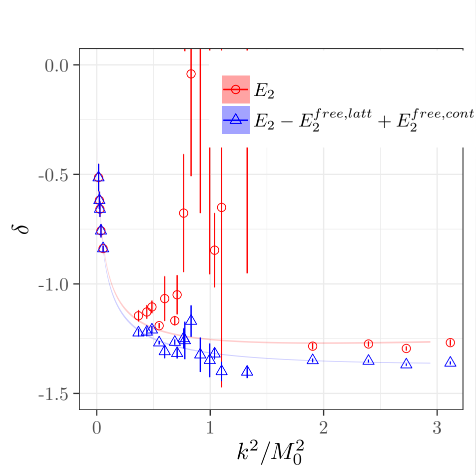

- $k \cot \delta = \frac{1}{a_0} + \frac{r_0 k^2}{2}$
- $a_0 m_0 = -4.58(7)$
  $r_0 m_0 = -0.220(5)$
  $\chi^2/d.o.f \sim 3.9$
- $a_0 m_0 = -4.79(7)$
  $r_0 m_0 = -0.105(3)$
  $\chi^2/d.o.f \sim 1.3$

Conclusion

- In the ϕ^4 model considered we found that [M. Bruno, M. T. Hansen (2021)] method produces results compatible with [M. Lüscher (1986)]
- At large momentum we need to consider discretization effect in the lattice dispersion relation to determine the scattering phase shifts δ, as already observed in [K. Rummukainen and S. A. Gottlieb (1995)]

Outlook

- add a $g\phi_0^3\phi_1$ term in the Lagrangian to induce decay $\phi_1 \to 3\phi_0$ and study resonance

Thank you for your attention