Coulomb Corrections to $\pi - \pi$ Scattering

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Introduction

- $K \to \pi\pi$ decay and direct CP violation *[R. Abbott et al, arXiv:2004.09440]*

$$\epsilon' = \frac{ie^{\delta_2 - \delta_0}}{\sqrt{2}} \frac{\mathrm{Re}A_2}{\mathrm{Re}A_0} \left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} \right) \tag{1}$$

- $\Delta I = 1/2$ rule, where $\frac{\mathrm{Re}A_2}{\mathrm{Re}A_0} \sim 20$, amplifies possible corrections
- Naïvely 1% isospin breaking, such as QED and light quark mass difference, can be $\sim 20\%$ corrections
- In order to study how to include QED in a finite box with two hadrons, begin with the simpler system of $\pi - \pi$ scattering

QED_L in a finite box

- Lüscher quantization works for interactions with a finite range, such as QCD unlike QED
- QED_L is a popular way of including QED into a finite box by removing the zero modes. $V_{\mathrm{QED}_L}(x) = \sum_{|k|\neq 0} \frac{e^{ik\cdot x}}{k^2}$
  - Adds new $1/L$ power law corrections to be determined
- Relation between finite volume energies with QED_L and scattering phase shift, in non-relativistic regime, derived by Beane and Savage *[S. Beane and M. Savage, arXiv:1407.4846]* and implemented by NPLQCD and QCDSF *[S. Beane et al, arXiv:2003.12130]* at heavy pion masses for understanding nucleon scattering

Splitting up QED

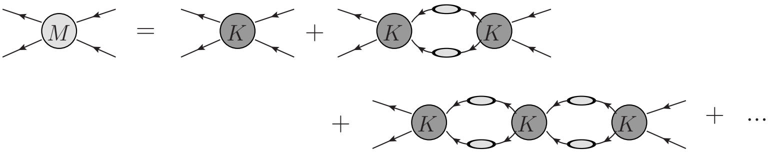
- First, choose the Coulomb gauge
  - Separates transverse radiation and instantaneous Coulomb interaction
  - The non-Lorentz covariance of the gauge is not issue. Use rest frame of the Kaon
- Second, truncate the Coulomb interaction to finite range R
  - The truncated interaction works perfectly with Lüscher quantization in a finite volume
  - Gives residual power law corrections in R
  - The remainder of the Coulomb interaction, and the power law corrections in R, can be fixed after the fact
- Three pieces
  - Truncated Coulomb potential $V_{TC} = \frac{e^2}{r}\theta(R-r)$ [Numerically]
  - Complement of the truncated Coulomb potential $V_{\overline{TC}} = \frac{e^2}{r}\theta(r-R)$ [Analytically]
  - Transverse Radiation [Neglected for now]

Truncated Coulomb potential

- Truncated Coulomb potential works perfectly well with Lüscher quantization given
  - $R < L/2$ so that the potential remains within the finite volume
  - $R >\sim \Lambda_{\mathrm{QCD}}^{-1}$ so that $V_{\overline{TC}}$ can be corrected with out needing quark dynamics
  - Limits need to be studied realistic calculations
- Unlike QED_L, the only $1/L$ power law corrections are those from neglected higher partial waves, instead have $1/R$ power law corrections
  - $1/R$ can be studied without changing the ensemble!
  - $1/R$ can also be corrected analytically

Calculating Corrections

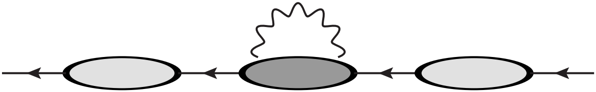
- Want to remove all power corrections in R from the truncation, neglecting exponentially suppressed terms
- With sufficiently large R, the interacting pions can be treated as elementary particles in infinite volume
- Include $V_{\overline{TC}}$ into the Lippmann-Schwinger series



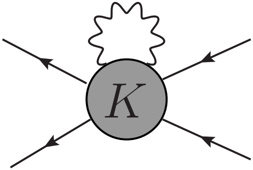
Calculating Corrections

- (a) Self Energy Correction [Suppressed]
- (b) 2-PI Kernel Correction [Suppressed]
- (c) Exchange Diagrams Correction [Dominant]

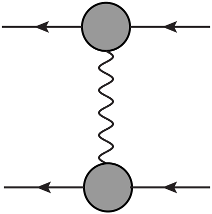
(a)



(b)



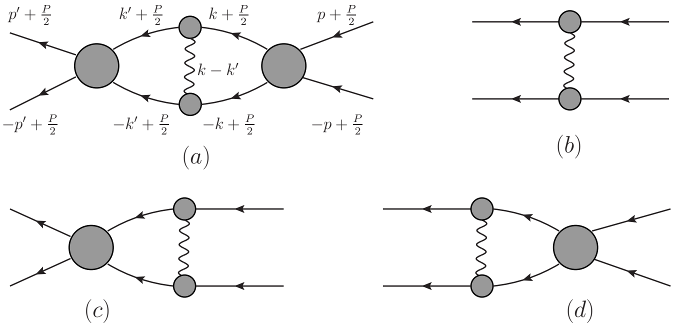
(c)



Exponentially Suppressed Corrections

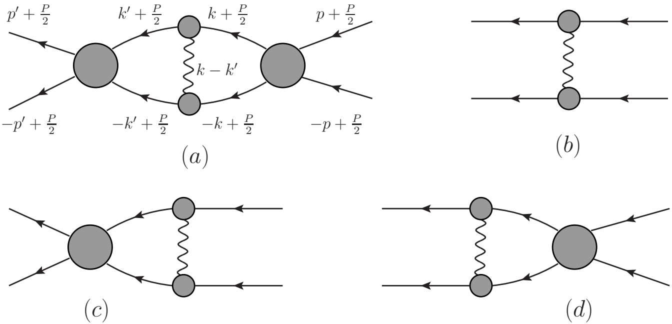
- When the two pion energy is below the 4 pion threshold, the self energy and 2-PI Kernel Correction can be calculated in Euclidean space and analytically continued to Minkowski space
- These diagrams will be dominated by short range interactions due to exponential decay of the propagators
- When $V_{\overline{TC}}$ is included, at least 2 propagators must travel the large distance R giving exponential suppression of the diagram

The Exchange Diagrams Correction



- Diagrams involve off-shell scattering kernel and off-shell pion EM vertex
- $V_{\overline{TC}}$ restricts to long distance regions where they go on shell
- On shell quantities are obtainable from LQCD or combination of experiment and phenomenology

The Exchange Diagrams Correction

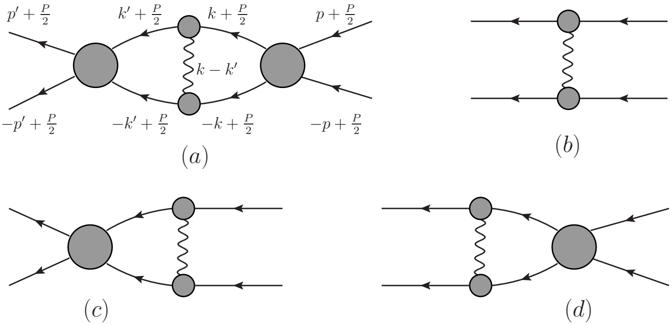


$$M_{\overline{TC},l} =$$

$$\frac{1}{2l+1} \sum_{m=-l}^{l} \int \int d^4k' d^4k \Psi_{lm}^{\text{out}}(k', P)^* K_{\overline{TC}}(k', k, P) \Psi_{lm}^{\text{in}}(k, P) \quad (2)$$

$$\Psi_{lm}^{\text{in/out}}(k, P) = \psi_{lm}^{0}(k, P) + \psi_{lm}^{\text{in/out}}(k, P)$$

The Exchange Diagrams Correction



$$\delta_l^{\overline{TC}} = \frac{\omega_p}{2p} \frac{1}{2l+1} \sum_{m=-l}^{l} \sum_{l'=0}^{\infty} \sum_{m'=-l'}^{l'} \int_0^{\infty} dz \iint d\Omega_{\hat{p}'} d\Omega_{\hat{p}} V_{\overline{TC}}(|\vec{z}|)$$

$$\cdot \left[Y_{lm}(\hat{p}')^* Y_{l'm'}(\hat{p}')^* Y_{lm}(\hat{p}) Y_{l'm'}(\hat{p}) \right] F(|\vec{p} - \vec{p}'|)^2 \sin^2\left(p|\vec{z}| - \frac{\pi l}{2} + \delta_l\right)$$

Conclusions

- Can add Coulomb interactions to Lüscher finite volume quantization without new $1/L$ power law corrections, at cost of new $1/R$ corrections
- If R is within appropriate limits, $1/R$ corrections can be calculated analytically in the infinite volume
- Need to study R dependence in realistic lattice calculation to study the limits and R independence of final phase shifts
- Need to add back in the transerve radiation to solve full relativistic problem