A new method for a lattice QCD calculation of $\eta_c \to 2\gamma$

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Introduction

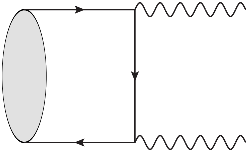
- Heavy quarkonium decay provides an ideal ground for various perturbative and nonperturbative QCD due to its multiscale features.
- The basic decay $\eta_c \to 2\gamma$ tests the validity limit of various approximations and techniques working on both scales, e.g. NRQCD, DSE, lattice method $\cdots$

Exp	$\mathcal{B}r \times 10^5$	Refs
CLEO	$1.4^{+0.7}_{-0.5} \pm 0.3$	PRL 101,101801(2008)
BESIII	$2.7 \pm 0.8 \pm 0.6$	PRD 87,032003(2013)
World average	$1.9^{+0.7}_{-0.6}$	PDG(2021)
Global fit	1.61 ± 0.12	PDG(2021)

  - Experimental difficulty: mixing with other light pseudoscalars.
  - As the largest $\tau-$charm factory, the BESIII has accumulated more than 10^{10} J/ψ events, the new result is forthcoming.

- We present a systematic study on $\eta_c \to 2\gamma$ using a new method.

$\eta_c \to 2\gamma$



- Amplitude:

$$\mathcal{M} = (q_c e)^2 \epsilon_\mu(p)\epsilon_\nu(p') H_{\mu\nu}(p,q)$$

$$H_{\mu\nu}(p,q) = \int d^4x e^{-ipx} \mathcal{H}_{\mu\nu}(x,q), \quad \mathcal{H}_{\mu\nu} = \langle 0|\mathrm{Tr}[J_\mu(x)J_\nu(0)]|\eta_c(q)\rangle$$

- Form factor: $$H_{\mu\nu}(p,q) = \epsilon_{\mu\nu\alpha\beta}p_\alpha q_\beta F(p^2)$$
- Decay width:

$$\Gamma_{\eta_c\gamma\gamma} = \alpha_{\mathrm{em}}^2 \frac{\pi}{4} m_{\eta_c}^3 q_c^2 |F_{\eta_c\gamma\gamma}|^2, \quad F_{\eta_c\gamma\gamma} = F(0)$$

Traditional method for on-shell form factor

- Off-shell form factors by projecting discrete momentum $\vec{p} = \frac{2\pi\vec{n}}{L}$

$$F(p^2) \xrightarrow{\text{Cont.Limit}} F(0)$$

  - Leading to additional computation cost and systematic source.

J. J. Dudek *et al.* Phys.Rev.Lett.**97**,172001(2006)

Chuan Liu *et al.* Phys.Rev.D.**102**,034502(2020)

- Without considering various systematic effects
  - Lattice discretization effect: only one or two lattice spacings used
  - Excited-state contamination: without extrapolation for the distance between hadron and nearest current

Direct approach to on-shell form factor

- Construct a scalar function[infinite volume]

$$
\begin{aligned}
\mathcal{I} \equiv &\quad \epsilon_{\mu\nu\alpha\beta} p_\alpha q_\beta H_{\mu\nu}(p,q) \\
= &\quad m_{\eta_c} \int dt e^{m_{\eta_c} t/2} \int d^3\vec{\boldsymbol{x}} e^{-i\vec{\boldsymbol{p}}\cdot\vec{\boldsymbol{x}}} \epsilon_{\mu\nu\alpha 0} \frac{\partial \mathcal{H}_{\mu\nu}(x,q)}{\partial x_\alpha}
\end{aligned}
$$

- Averaging the $\vec{p}$ directions and projecting $|\vec{p}| = m_{\eta_c}/2$[infinite volume]

$$
\begin{aligned}
F_{\eta_c\gamma\gamma} \quad \equiv &\quad \frac{\mathcal{I}}{[\epsilon_{\mu\nu\alpha\beta} p_\alpha q_\beta][\epsilon_{\mu\nu\rho\sigma} p_\rho q_\sigma]} \\
= &\quad -\frac{1}{2m_{\eta_c}} \int dt e^{m_{\eta_c} t/2} \int d^3\vec{\boldsymbol{x}} \frac{j_1(|\vec{\boldsymbol{p}}||\vec{\boldsymbol{x}}|)}{|\vec{\boldsymbol{p}}||\vec{\boldsymbol{x}}|} \epsilon_{\mu\nu\alpha 0} x_\alpha \mathcal{H}_{\mu\nu}(x,q)
\end{aligned}
$$

- Replaced by finite-volume version

$$
\epsilon_{\mu\nu\alpha 0} x_\alpha \mathcal{H}_{\mu\nu}(x,q) \to \epsilon_{\mu\nu\alpha 0} x_\alpha \mathcal{H}^L_{\mu\nu}(x,q)
$$

  - Exponentially supressed with the distance

Infinite volume reconstruction [1812.0981]

- Divide the time integral into two parts: short distance calculated on lattice, long distance reconstructed with the short-distance hadroinc function:

$$\mathcal{H}_{\mu\nu}(\vec{x},t)|_{t>t_s} = \int \frac{d^3\vec{p}}{(2\pi)^3} \int d^3\vec{x'}\mathcal{H}_{\mu\nu}(\vec{x'},t_s)e^{-E_{\vec{p}}(t-t_s)+i\vec{p}\cdot(\vec{x}-\vec{x'})}$$

- Long-distance form factor:

$$F^{(\infty)}_{\eta_c\gamma\gamma} = -\frac{1}{m_{\eta_c}}\frac{e^{|\vec{\boldsymbol{p}}|t_s}}{\sqrt{m^2_{J/\psi}+|\vec{\boldsymbol{p}}|^2}-|\vec{\boldsymbol{p}}|}\int d^3\vec{\boldsymbol{x}}\frac{j_1(|\vec{\boldsymbol{p}}||\vec{\boldsymbol{x}}|)}{|\vec{\boldsymbol{p}}||\vec{\boldsymbol{x}}|}\epsilon_{\mu\nu\alpha 0}x_\alpha\mathcal{H}_{\mu\nu}(t_s,\vec{\boldsymbol{x}})$$

- The total:

$$F_{\eta_c\gamma\gamma} = F^{(L)}_{\eta_c\gamma\gamma} + F^{(\infty)}_{\eta_c\gamma\gamma}$$

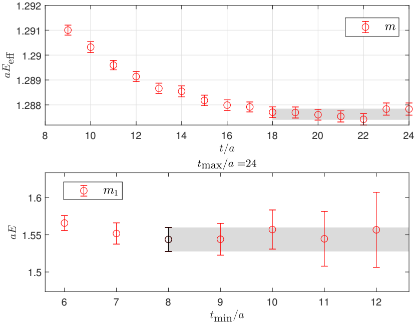
Extraction of η_c ground-state contribution

- Ground-state contribution

$$\langle J_\mu(\vec{x},t)J_\nu(0)\mathcal{O}^\dagger_{\eta_c}(-\Delta t)\rangle = \frac{Z_0}{2m}\mathcal{H}^{(0)}_{\mu\nu}e^{-m\Delta t} + \frac{Z_1}{2m_1}\mathcal{H}^{(1)}_{\mu\nu}e^{-m_1\Delta t}$$

$$\mathcal{H}_{\mu\nu} = \mathcal{H}^{(0)}_{\mu\nu} + \frac{Z_1 m}{Z_0 m_1}\mathcal{H}^{(1)}_{\mu\nu} \times e^{-(m_1-m_0)\Delta t}$$

$$F_{\eta_c\gamma\gamma} = F^{(0)}_{\eta_c\gamma\gamma} + C_F \times e^{-(m_1-m)\Delta t}$$



- Two-state fit:

$$C_2(t) = A_1\left(e^{-mt} + e^{-m(T-t)}\right) + A_2\left(e^{-m_1 t} + e^{-m_1(T-t)}\right), m_1 \geq m$$

$N_f = 2$ twisted-mass

Ens	a(fm)	V/a^4	$a\mu_{sea}$	$N_{\rm conf} \times T_s$	$\Delta t/a$	Z_V
a98	0.098	$24^3 \times 48$	0.0060	236×48	10:1:18	0.6056(46)
a85	0.085	$24^3 \times 48$	0.0040	200×48	10:1:18	0.6297(17)
a67	0.0667	$32^3 \times 64$	0.0030	197×64	10:2:24	0.6523(10)

- All ensembles have similar physical spatial volume: $2.04 \sim 2.35$ fm.
- All ensembles have similar pion mass: $300 \sim 360$ MeV.
- A series of Δt to extract the ground-state contribution for $\eta_c \to 2\gamma$.
- Charm quark mass is tuned by physical η_c and J/ψ mass, respectively, and take the difference as our systematic error.
- Connected diagram only.

Renormalization constant

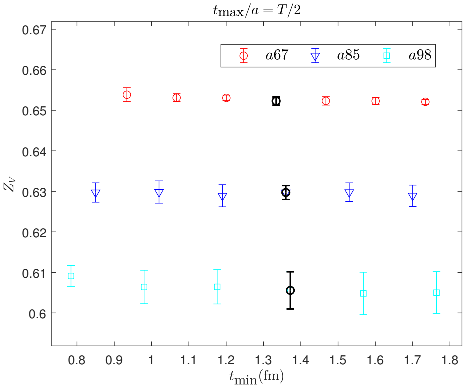
- Modified ratio:

$$\frac{\Gamma^{(2)}_{\eta_c\eta_c}(\boldsymbol{k}, t_f = t, t_i = 0)}{\Gamma^{(3)}_{\eta_c\gamma_0\eta_c}(\boldsymbol{k}, t_f = t, t/2, t_i = 0)} = Z_V \cdot \left[1 + e^{-m(T-2t)}\right]$$

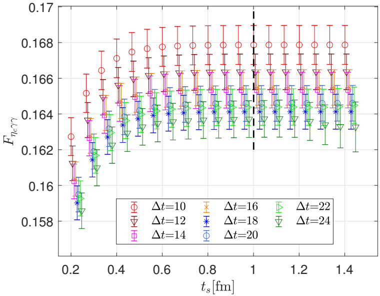
- A series of t_f to control possible excited-state effect.

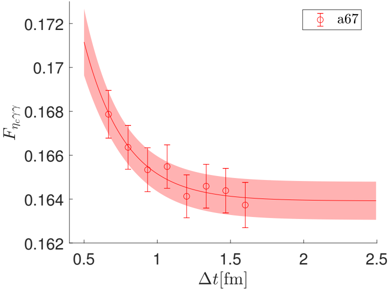
- The right: fiting results for Z_V with time window

$$[t_{\min}, t_{\max}]$$



Form factors on lattice

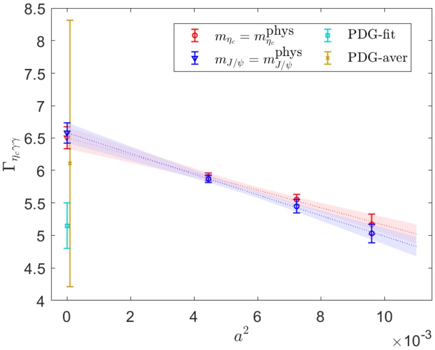




- An obvious dependene on excited-state of η_c.
- Dashed black line: a suitable time truncation $t_{\rm cut} \sim 1$ fm.
- The right: an extrapolation for the form factors at $t_{\rm cut}$.

Continuous limit

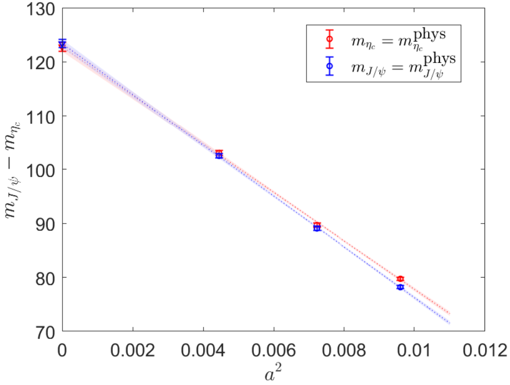
- Automatic $O(a)$ improved;
- PDG-fit: global fit of PDG;
- PDG-aver: world average of PDG;
- Consistency of the different tunings in continuous limit.



$$\Gamma(\eta_c \to 2\gamma) = \begin{cases} 6.51(17)\,\text{keV} & m_{\eta_c} = m_{\eta_c}^{\rm phys} \\ 6.58(16)\,\text{keV} & m_{J/\psi} = m_{J/\psi}^{\rm phys} \\ 5.15(35)\,\text{keV} & \text{PDG-fit} \\ 6.11^{+2.2}_{-1.9}\,\text{keV} & \text{PDG-aver} \end{cases}$$

- The discrepancy bewteen PDG-fit and lattice result is beyond 3 σ

Hyperfine splitting



$$m_{J/\psi} - m_{\eta_c} = \begin{cases} 122.76(85)\,\text{MeV} & m_{\eta_c} = m_{\eta_c}^{\text{phys}} \\ 123.36(78)\,\text{MeV} & m_{J/\psi} = m_{J/\psi}^{\text{phys}} \end{cases}$$

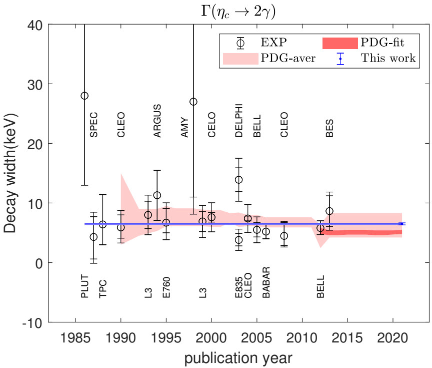
- Both smaller difference of mass tunings indicate a reliable Cont.Limit.

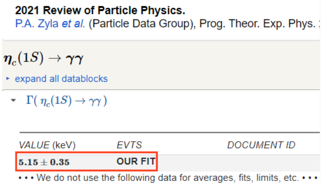
Systematic study

- Excited-state effect → A series of Δt
- Lattice discretization effect → Three lattice spacings
- Scale setting effect → Two mass tunings
- Finite volume effect → Infinite volume reconstruction

Systematic effects we have considered

Lattice & Experiments





- OUR FIT: a minimum χ^2-fit for the branching ratios from lots of experimental measurements on different decay channels.
- The fitting errors are consistent with world average for decay channels used, but except for $\eta_c \to 2\gamma$.
- We suggest PDG to fit with our lattice value, to see the effects on relevant channels.

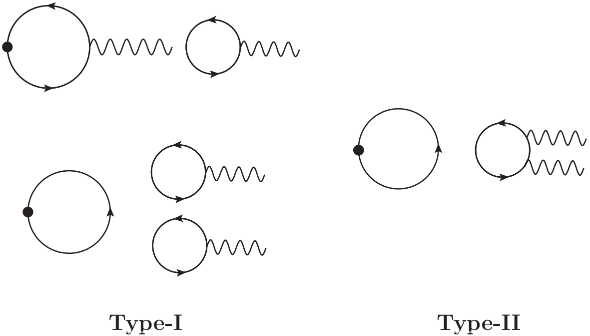
Conclusion

- Propose a direct method to calculate the on-shell form factor of $\eta_c \to 2\gamma$.
- The method can be applied for processes involving the leptonic or radiative particles in the final states:
  - $\pi^0 \to 2\gamma$
  - $J/\psi \to 3\gamma$
  - $J/\psi \to \gamma\eta_c$
  - $J/\psi \to \gamma\nu\bar{\nu}$
- Various systematic effects are examined carefully.
- In agreement with the world average of PDG, but different from the global fit beyond 3σ.

The most precise analysis from BESIII with $10^{10} J/\psi$ is underway.

Back up

Disconnected diagrams



- Much smaller contribution due to
  - Type-I: SU(3) asymptotic symmetry with $m_{u,d,s} \ll m_c$
  - Type-II: OZI supressed with $\alpha_s(m_c)$

PDG global fit

CONSTRAINED FIT INFORMATION

show precise values? ☐

An overall fit to total width, 8 combinations of particle width obtained from integrated cross section, 19 branching ratios uses 93 measurements and one constraint to determine 13 parameters. The overall fit has a χ^2 = 117.8 for 81 degrees of freedom.

The following *off-diagonal* array elements are the correlation coefficients $\langle\delta x_i \delta x_j\rangle / (\delta x_i \cdot \delta x_j)$, in percent, from the fit to parameters p_i, including the branching fractions, $x_i = \Gamma_i / \Gamma_{total}$ The fit constrains the x_i whose labels appear in this array to sum to one.

x_4	100												
x_7	16	100											
x_{15}	3	5	100										
x_{27}	18	35	6	100									
x_{28}	9	17	3	47	100								
x_{31}	10	18	3	21	10	100							
x_{35}	7	13	2	21	10	8	100						
x_{38}	12	22	4	25	12	14	10	100					
x_{41}	11	20	4	27	13	12	10	15	100				
x_{43}	3	5	1	6	3	3	2	4	23	100			
x_{49}	-27	-51	-9	-59	-28	-32	-23	-38	-38	-9	100		
Γ	-1	-3	0	-3	-1	-2	-1	-2	6	1	-27	100	
x_{999}	0	0	0	0	0	0	0	0	0	0	0	0	100
	x_4	x_7	x_{15}	x_{27}	x_{28}	x_{31}	x_{35}	x_{38}	x_{41}	x_{43}	x_{49}	Γ	x_{999}

	Mode	Rate (MeV)	Scale factor
Γ_4	$\eta_c(1S) \to K^*(892)\overline{K}^*(892)$	0.0069 ± 0.0013	
Γ_7	$\eta_c(1S) \to \phi\phi$	0.00174 ± 0.00019	
Γ_{15}	$\eta_c(1S) \to f_2(1270) f_2(1270)$	0.0098 ± 0.0025	
Γ_{27}	$\eta_c(1S) \to K\overline{K}\pi$	0.073 ± 0.004	
Γ_{28}	$\eta_c(1S) \to K\overline{K}\eta$	0.0136 ± 0.0015	
Γ_{31}	$\eta_c(1S) \to K^+K^-\pi^+\pi^-$	0.0066 ± 0.0011	
Γ_{35}	$\eta_c(1S) \to 2(K^+K^-)$	0.00143 ± 0.00030	
Γ_{38}	$\eta_c(1S) \to 2(\pi^+\pi^-)$	0.0091 ± 0.0012	
Γ_{41}	$\eta_c(1S) \to p\overline{p}$	0.00144 ± 0.00014	
Γ_{43}	$\eta_c(1S) \to \Lambda\overline{\Lambda}$	0.00106 ± 0.00023	
Γ_{49}	$\eta_c(1S) \to \gamma\gamma$	$(1.61 \pm 0.12) \times 10^{-4}$	

- The smaller errros of fitting may due to: (i)large uncertainties of $\eta_c \to 2\gamma$; (ii)Highly correlated with other decay channels ; (iii)The other channels have high precisions.

Renormalization constant from ETM

Ens	Methods	Z_V	Z_A	Refs
a98	RM-1	0.604(07)	0.746(11)	ETM
	RM-2	0.623(05)	0.727(07)	
	Alt.	0.5816(02)	0.747(22)	
	Ratio	0.6056(46)		Our
a85	RM-1	0.624(04)	0.746(06)	ETM
	RM-2	0.634(03)	0.730(03)	
	Alt.	0.6103(03)	0.743(18)	
	Ratio	0.6297(17)		Our
a67	RM-1	0.659(04)	0.772(06)	ETM
	RM-2	0.662(03)	0.758(04)	
	Alt.	0.6451(03)	0.746(18)	
	Ratio	0.6523(10)		Our

ETM:1004.1115