Composition of the inclusive semi-leptonic decay of B meson

Shoji Hashimoto, Takashi Kaneko (KEK, SOKENDAI), Gabriela Bailas (U. of Tsukuba), Paolo Gambino, Sandro Machler (U. of Torino)

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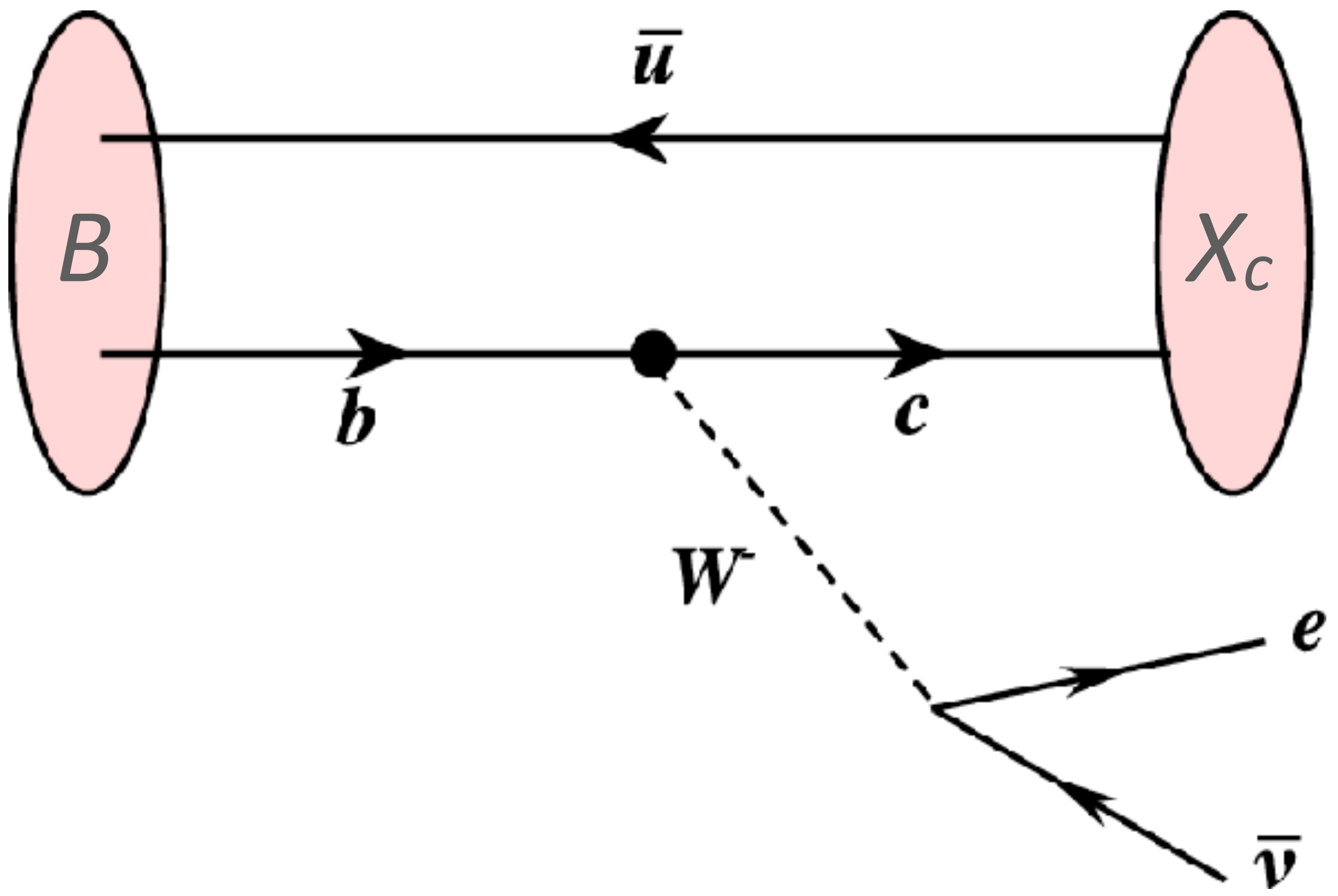
Jul 28, 2021

Semi-leptonic B decay



Exclusive: specific final states (D, D*, …)

Inclusive: sum over all final states,
can be computed using PT (or OPE)

A new method to compute the “sum” in LQCD

Gambino and SH, arXiv:2005.13730

from the forward-Compton amplitude.

$$\langle B(\mathbf{0})|\tilde{J}^\dagger_\mu(-\boldsymbol{q};t)\,\vdots\,\tilde{J}_\nu(\boldsymbol{q};0)|B(\mathbf{0})\rangle$$

all possible states contribute

Composition of inclusive decays (exp't):

Bernlochner @ ICHEP 2012

Charm state X_c	$\mathcal{B}(B^+ \to X_c\, \ell^+\, \nu)$
D	$(2.31 \pm 0.09)\,\%$
D^*	$(5.63 \pm 0.18)\,\%$
$\sum D^{(*)}$	$(7.94 \pm 0.20)\,\%$
$D_0^* \to D\,\pi$	$(0.41 \pm 0.08)\,\%$
$D_1^* \to D^*\,\pi$	$(0.45 \pm 0.09)\,\%$
$D_1 \to D^*\,\pi$	$(0.43 \pm 0.03)\,\%$
$D_2^* \to D^{(*)}\,\pi$	$(0.41 \pm 0.03)\,\%$
$\sum D^{**} \to D^*\,\pi$	$(1.70 \pm 0.12)\,\%$
$D\,\pi$	$(0.66 \pm 0.08)\,\%$
$D^*\,\pi$	$(0.87 \pm 0.10)\,\%$
$\sum D^*\,\pi$	$(1.53 \pm 0.13)\,\%$
$\sum D^{(*)} + \sum D^*\,\pi$	$(9.47 \pm 0.24)\,\%$
$\sum D^{(*)} + \sum D^{**} \to D^{(*)}\pi$	$(9.64 \pm 0.23)\,\%$
Inclusive X_c	$(10.92 \pm 0.16)\,\%$

broad states $(0.86 \pm 0.12)\,\%$

narrow states $(0.84 \pm 0.04)\,\%$

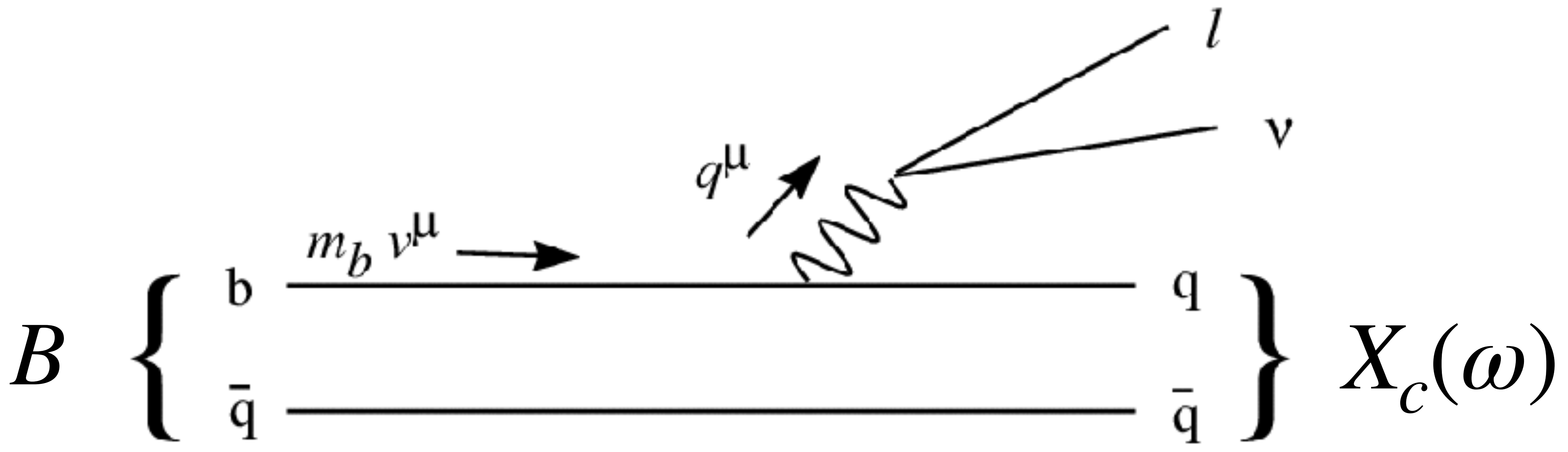
All values from [HFAG 2010]. For the values of $B \to D\,\pi\,\ell\,\bar{\nu}_\ell$ and $B \to D^*\,\pi\,\ell\,\bar{\nu}_\ell$ an uncertainty weighted average of both isospin modes was calculated assuming a 100% correlation between both values.

$\Rightarrow$ **'Gap'** of $(1.45 \pm 0.29)\,\%$ emerges which is not accounted for

Sum of all identified states is less than **ALL**.

Can we understand why using lattice data? Need an access to both incl and excl.

Inclusive rate



Differential decay rate:

$$d\Gamma \sim |V_{cb}|^2 l^{\mu\nu} W_{\mu\nu}$$

Structure function:

$$W_{\mu\nu} = \sum_X (2\pi)^2 \delta^4(p_B - q - p_X) \frac{1}{2M_B} \langle B(p_B)|J_\mu^\dagger(0)|X\rangle\langle X|J_\nu(0)|B(p_B)\rangle$$

$$\langle B(\mathbf{0})|\tilde{J}_\mu^\dagger(-\boldsymbol{q};t)\ \delta(\omega - \hat{H})\ \tilde{J}_\nu(\boldsymbol{q};0)|B(\mathbf{0})\rangle$$

Decay rate:

$$\Gamma \propto \int_0^{\boldsymbol{q}^2_{\max}} d\boldsymbol{q} \int_{\sqrt{m_D^2+\boldsymbol{q}^2}}^{m_B-\sqrt{\boldsymbol{q}^2}} d\omega\, K(\omega;\boldsymbol{q}^2)\langle B(\mathbf{0})|\tilde{J}^\dagger(-\boldsymbol{q})\delta(\omega-\hat{H})\tilde{J}(\boldsymbol{q})|B(\mathbf{0})\rangle$$

known kinematical factor

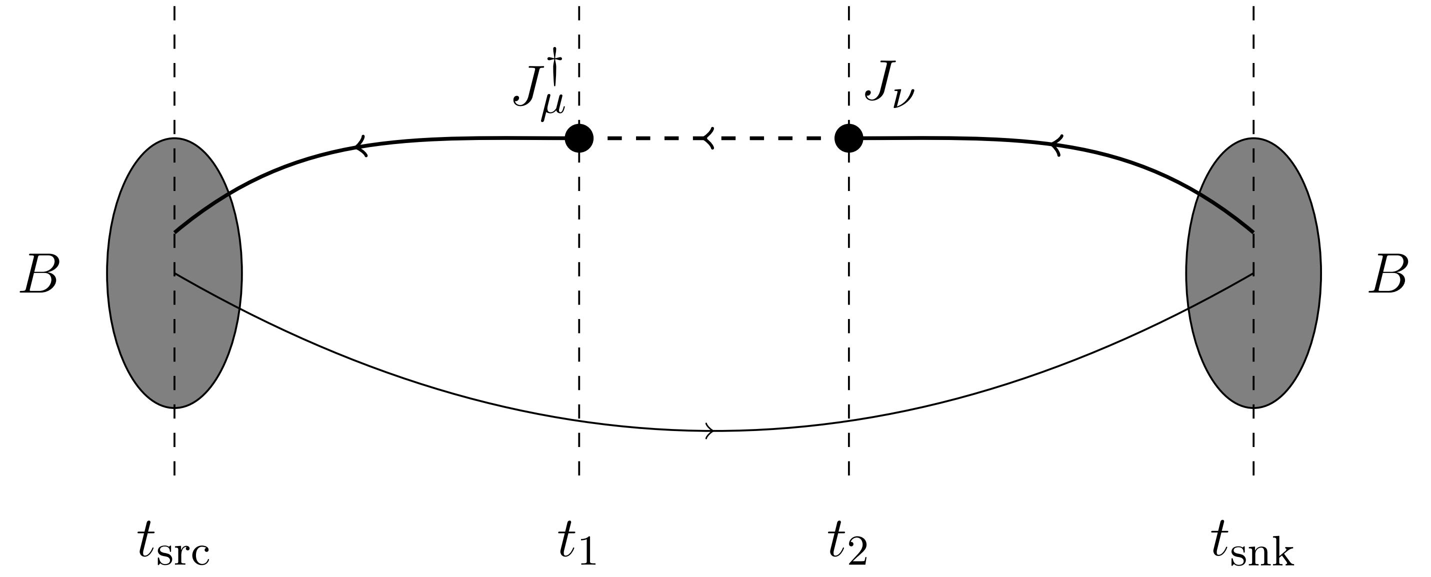
Energy integral

$$\Gamma \propto \int_0^{\boldsymbol{q}^2_{\max}} d\boldsymbol{q} \int_{\sqrt{m_D^2+\boldsymbol{q}^2}}^{m_B-\sqrt{\boldsymbol{q}^2}} d\omega\, K(\omega;\boldsymbol{q}^2)\langle B(\mathbf{0})|\tilde{J}^\dagger(-\boldsymbol{q})\delta(\omega-\hat{H})\tilde{J}(\boldsymbol{q})|B(\mathbf{0})\rangle$$

$$= \langle B(\mathbf{0})|\tilde{J}^\dagger(-\boldsymbol{q})K(\hat{H};\boldsymbol{q}^2)\tilde{J}(\boldsymbol{q})|B(\mathbf{0})\rangle$$

Lattice Compton amplitude:

$$\langle B(\mathbf{0})|\tilde{J}^\dagger_\mu(-\boldsymbol{q};t)\ \tilde{J}_\nu(\boldsymbol{q};0)|B(\mathbf{0})\rangle \longrightarrow \langle B(\mathbf{0})|\tilde{J}^\dagger(-\boldsymbol{q})e^{-\hat{H}t}\tilde{J}(\boldsymbol{q})|B(\mathbf{0})\rangle$$



Approximation:

$$K(\hat{H}) = k_0 + k_1 e^{-\hat{H}} + k_2 e^{-2\hat{H}} + \cdots + k_N e^{-k_N\hat{H}}$$

Chebyshev approx:

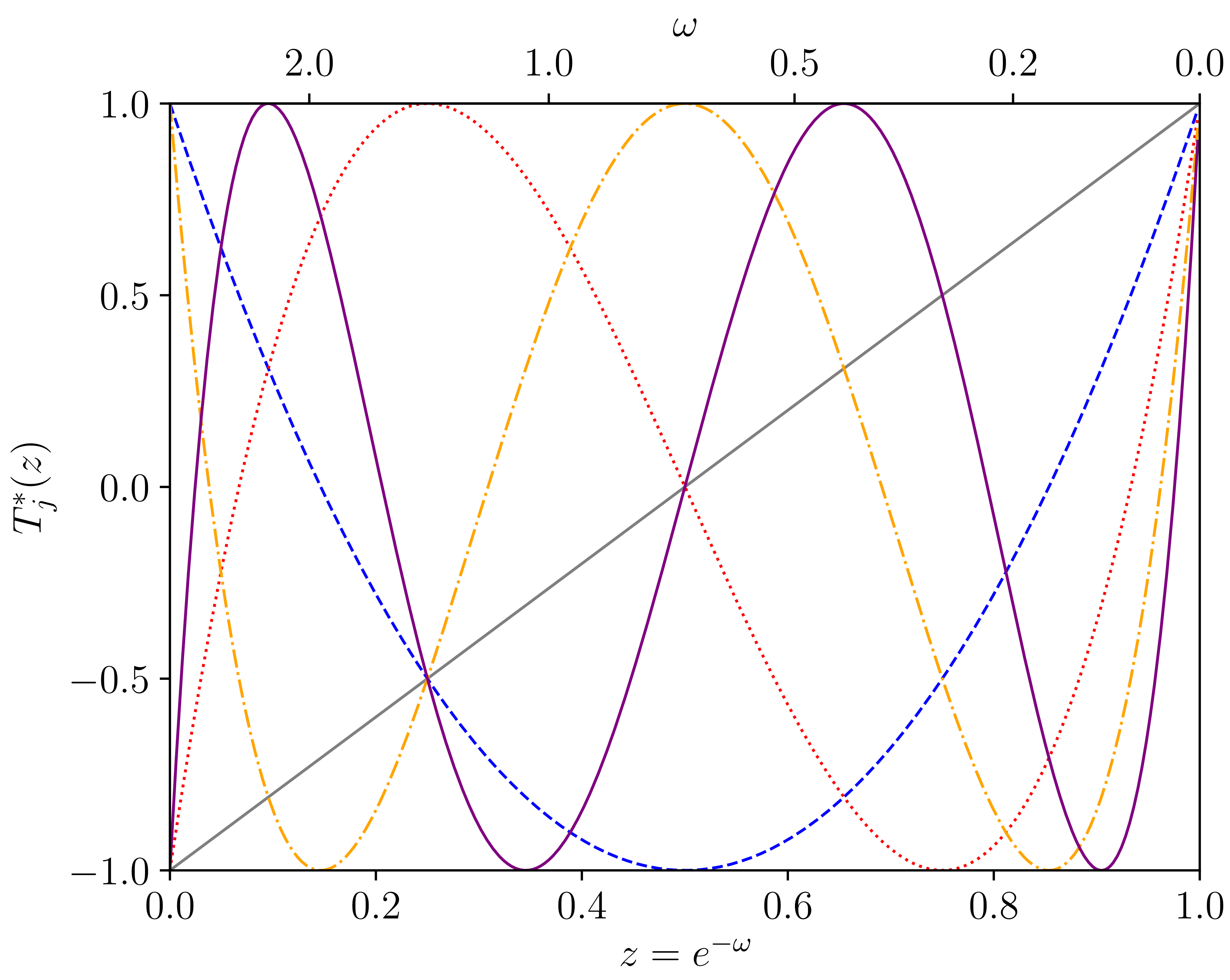
$$K(\omega) \simeq \frac{c_0}{2} + \sum_{j=1}^{N} c_j^* T_j^*(e^{-\omega})$$

Bailas, Ishikawa, SH, arXiv:2001.11779

(shifted) Chebyshev polynomials

$$T_0^*(x) = 1$$
$$T_1^*(x) = 2x - 1$$
$$T_2^*(x) = 8x^2 - 8x + 1$$
$$\vdots$$
$$T_{j+1}^*(x) = 2(2x-1)T_j^*(x) - T_{j-1}^*(x)$$

each term corresponds to the correlator, because x = e^{-H}



"Best" approximation can be obtained with

$$c_j^* = \frac{2}{\pi}\int_0^{\pi} d\theta\, S\left(-\ln\frac{1+\cos\theta}{2}\right)\cos(j\theta)$$

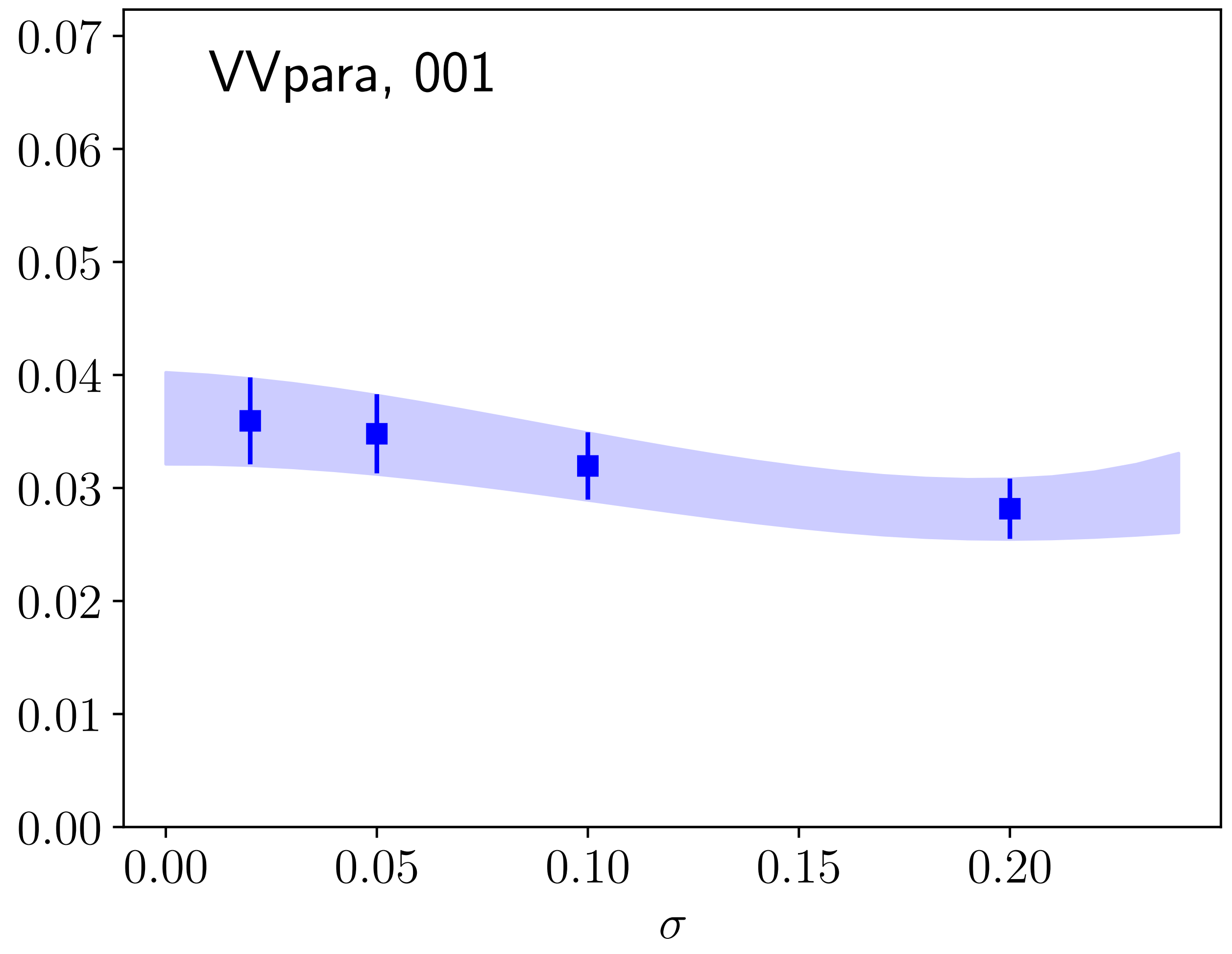
"best" = maximal deviation is minimal

Kernel to approximate

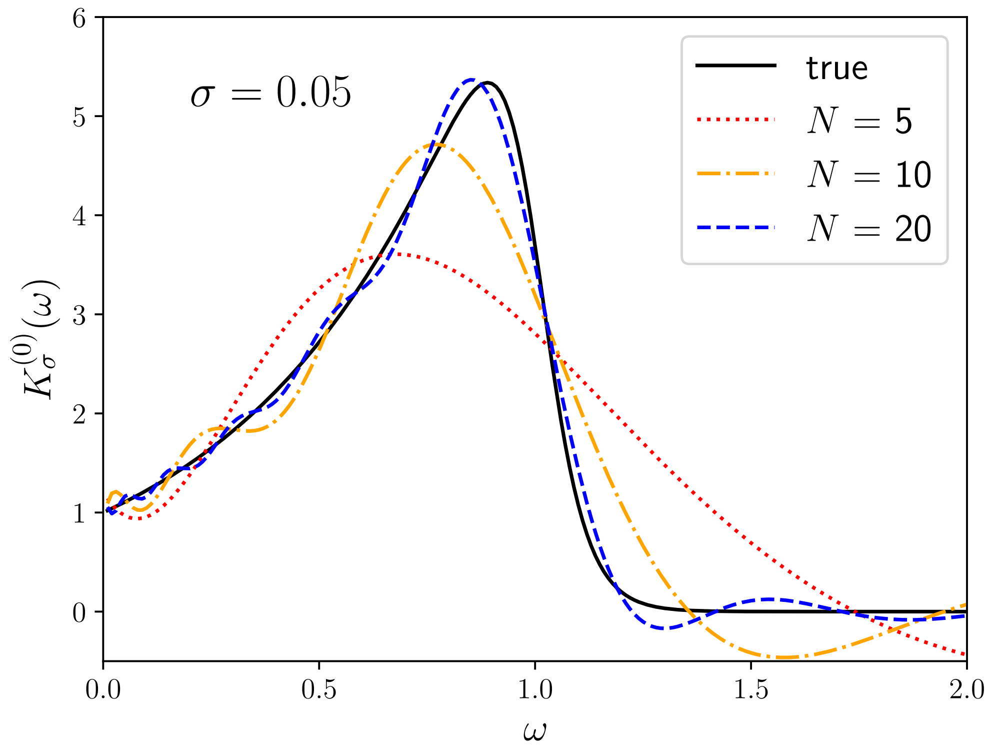
To implement the upper limit of integ

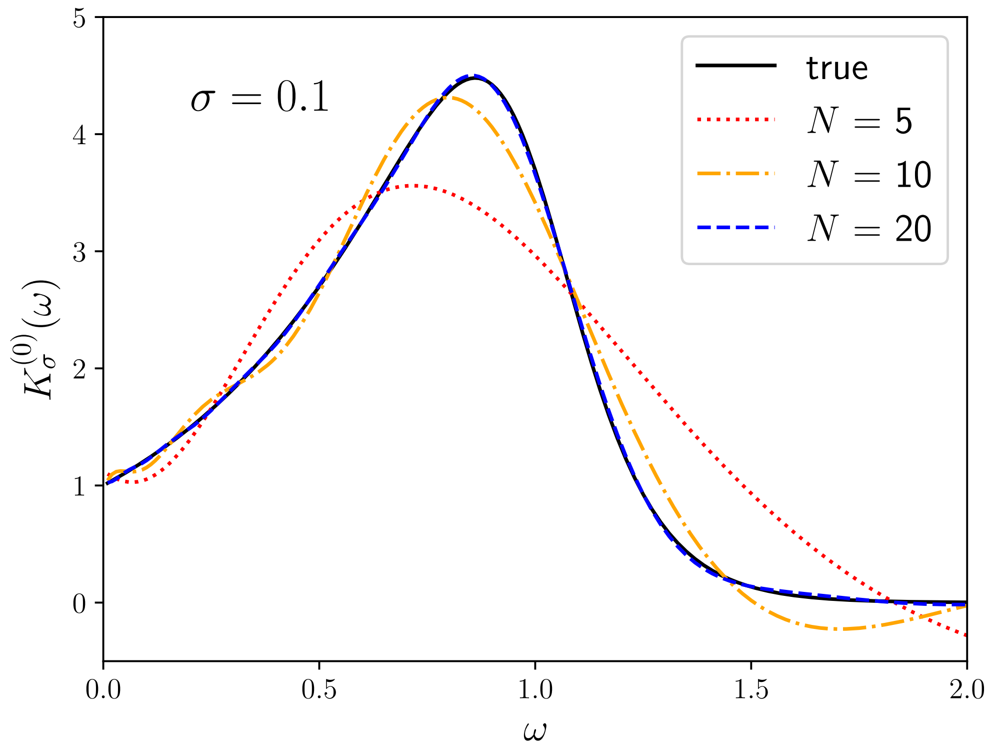
$$K(\omega) \sim e^{2\omega t_0} \underbrace{(m_B - \omega)^l}_{\text{kinematical factor}} \boxed{\theta(m_B - |\mathbf{q}| - \omega)}$$

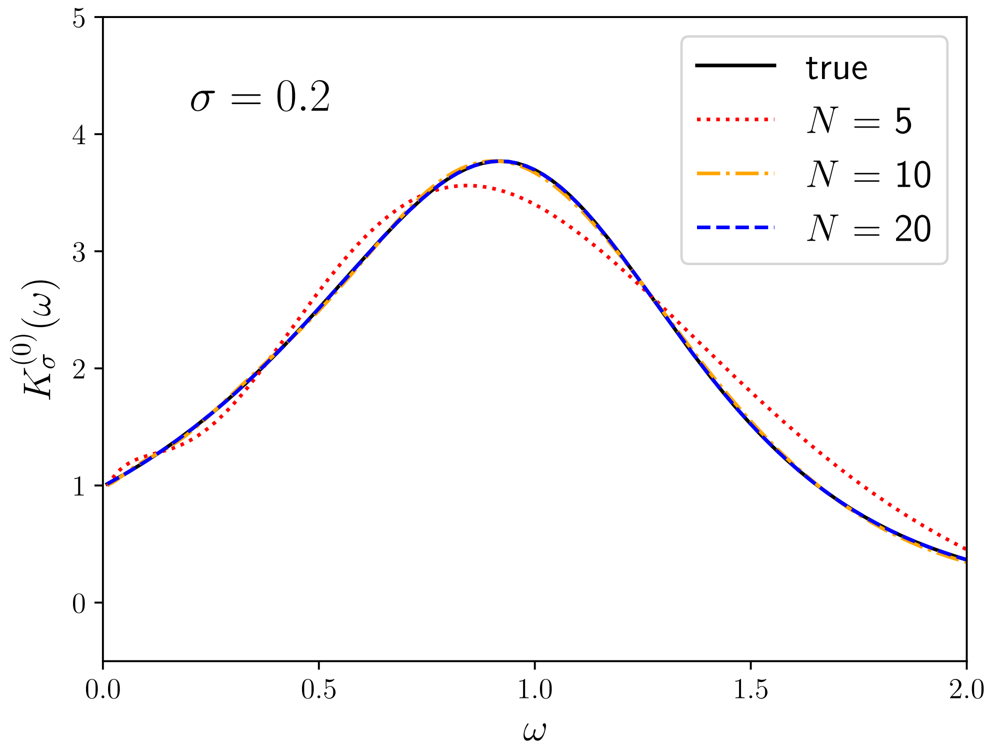
Smear by "sigmoid" with a width σ
Need to take a limit of σ→ 0



narrow ⟷ wide

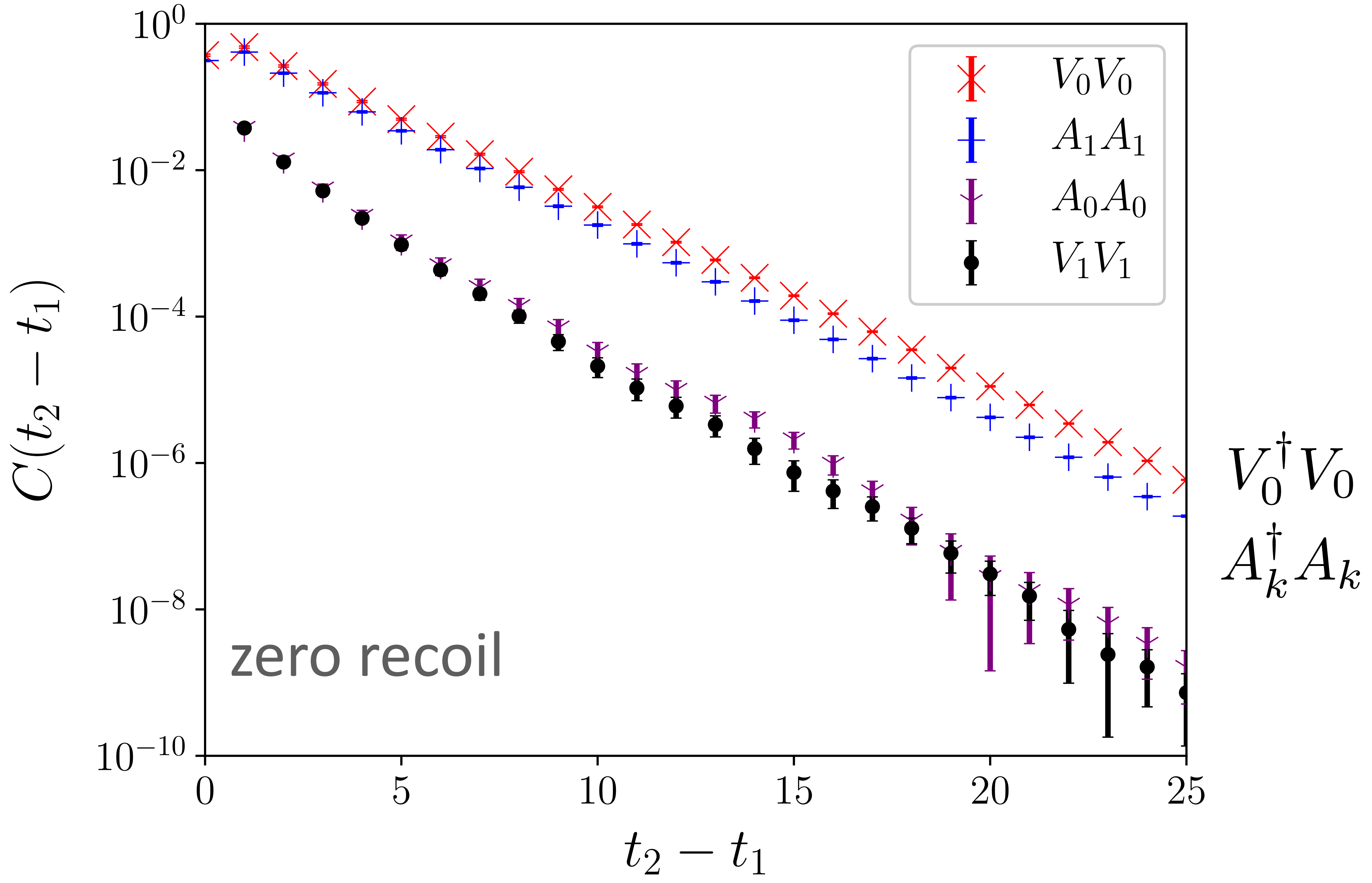






Compton amplitude (S-wave)

$$\langle B(\mathbf{0})|\tilde{J}^{\dagger}_{\mu}(-\boldsymbol{q};t)\ \ \tilde{J}_{\nu}(\boldsymbol{q};0)|B(\mathbf{0})\rangle$$



Pilot lattice computation [JLQCD setup]

- On a lattice of 48^3x96 at $1/a$ = 3.6 GeV
- Strange spectator quark
- physical charm quark mass
- (unphysically) light b quark ~ 2.7 GeV
- 100 configs x 4 src

S-wave (D and D*)

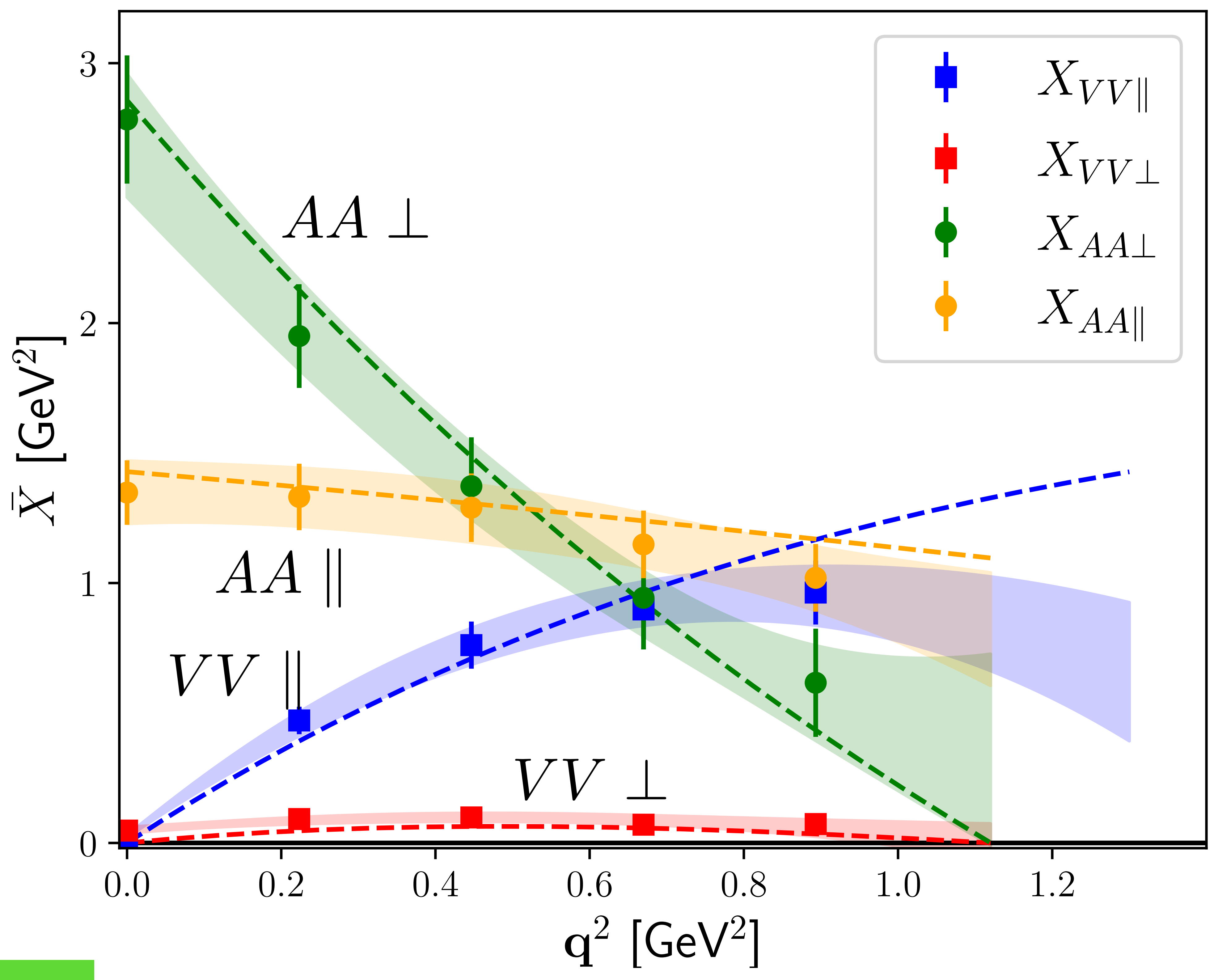
- Very well approximated by a single-exp = no sign of excited state contrib.
- No wave function overlap of excited states when m_b=m_c and zero recoil (V_0V_0)

Inclusive decay rate

- Breakdown to individual channels: VV and AA; parallel and perp with respect to the recoil momentum
- Compared to exclusive contributions estimated from B→D$^{(*)}$ form factors (dashed line), that are separately calculated.

  - VV$_{||}$ dominated by B→D
  - All others by B→D*

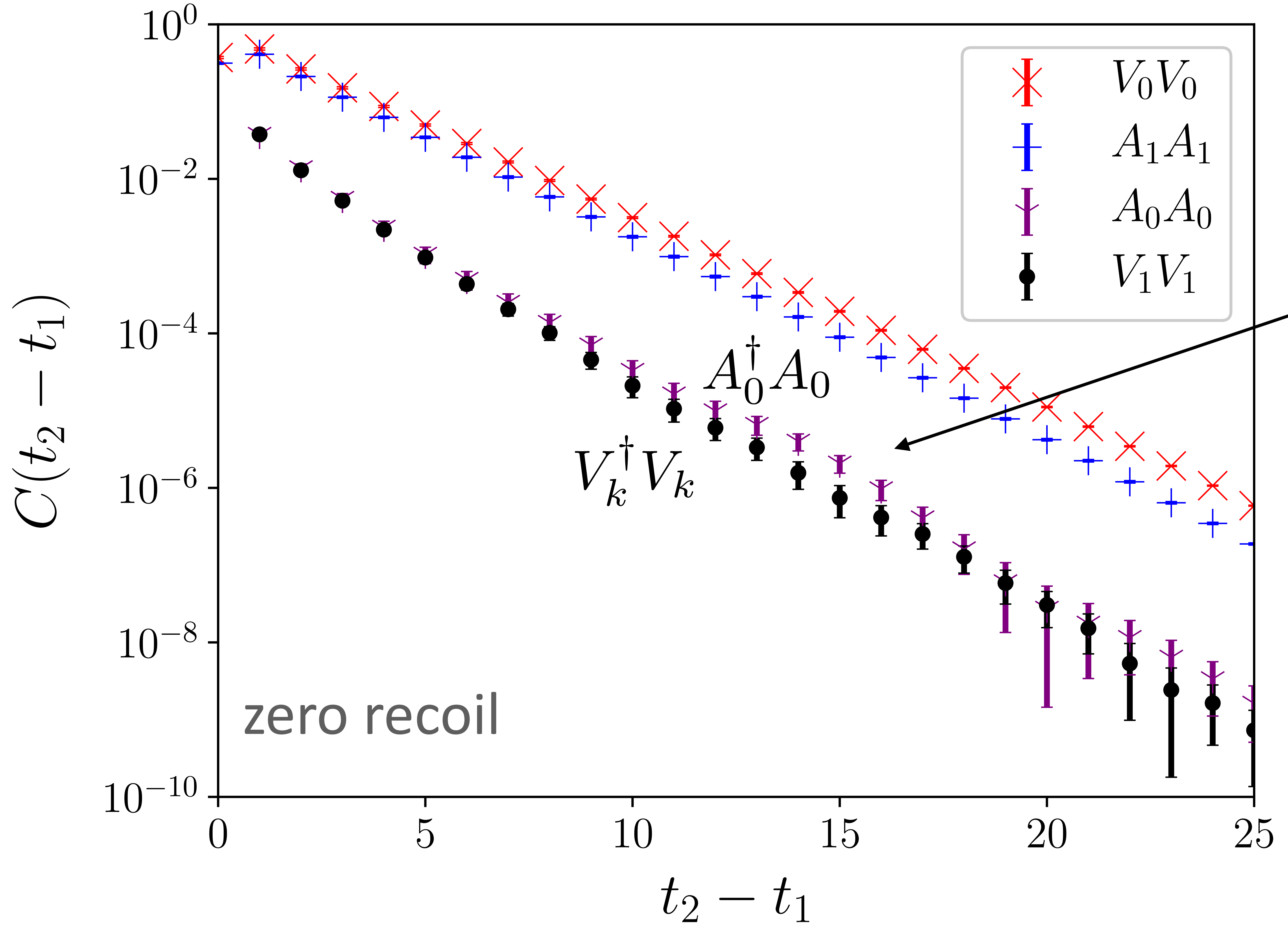
differential decay rate / |**q**|



Inclusive rate is dominated by the ground states, naturally because of the unphysical b quark mass.

Compton amplitude (P-wave)

$$\langle B(\mathbf{0})|\tilde{J}_\mu^\dagger(-\boldsymbol{q};t)\ \ \tilde{J}_\nu(\boldsymbol{q};0)|B(\mathbf{0})\rangle$$



P-wave

- Opposite parity; can’t be S-wave.
- Slightly heavier; Amplitude is ~ 50x smaller.

$$|g_+(1)|^2 e^{-m_{D_0^*}t}$$

HQET Leibovich, Ligeti, Stewart, Wise (1998)

$$g_+(1) = -\frac{3}{2}\left(\frac{1}{2m_c}+\frac{1}{2m_b}\right)(\bar{\Lambda}^*-\bar{\Lambda})\zeta(1)$$

0.35 GeV ~ 0.5

Isgur-Wise func for P-wave (0^+)

Summary

- Framework to compute inclusive decay rate is now available. The energy integral can be reconstructed from Euclidean lattice correlators.
- Compton amplitudes contain the excited state contributions. They are suppressed.
  - S-wave: small wave function overlap.
  - P-wave: starts from 1/m or non-zero recoil
- Decomposition into different channels. Will be compared with pQCD/OPE. See,
  - Sandro Maechler "Comparison of lattice QCD results for inclusive semi-leptonic decays of B meson with OPE", Thu 6:30am
- See also the next talk (Jun-Sik Yoo), about an application to inelastic ℓN scattering.