

Application of the Misner's method to the coupled-channel $N\Lambda$ - $N\Sigma$ potential in lattice QCD

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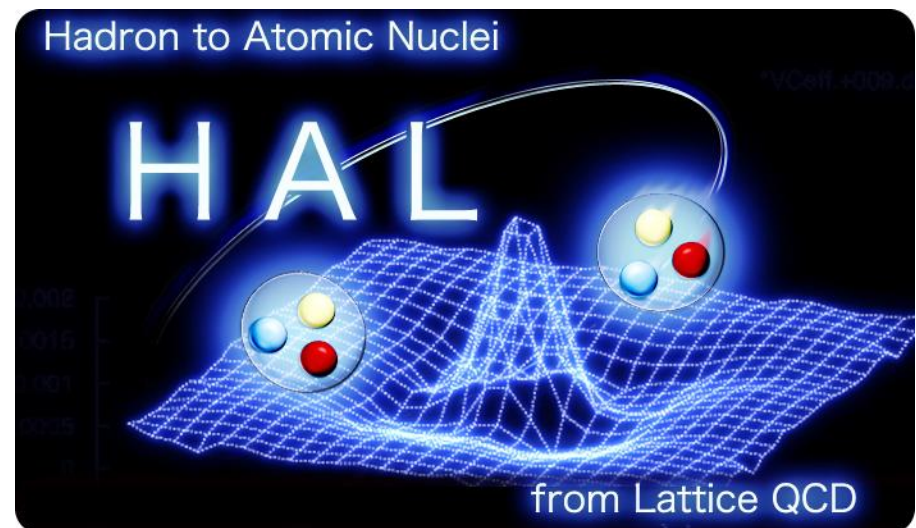
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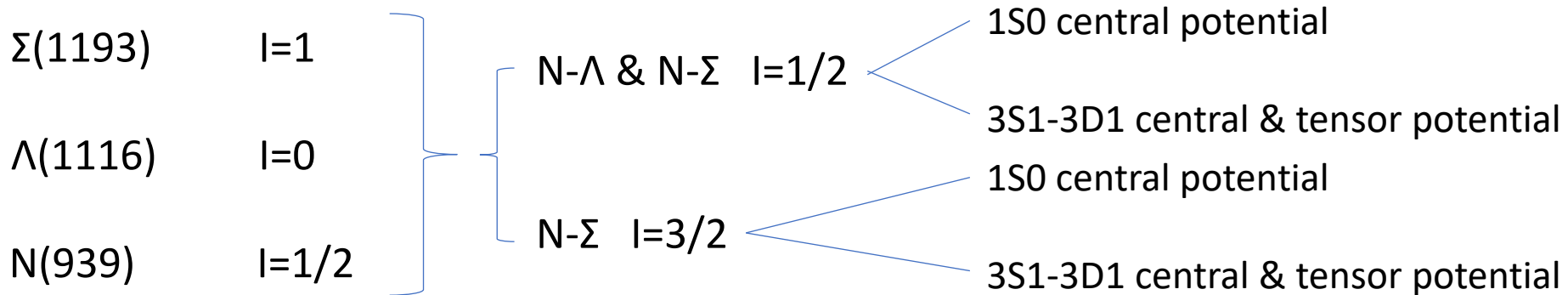
And HAL QCD collaboration.



Target in this study:

Baryon-Baryon interactions in $S=-1$ channel

= $N\Lambda$ and $N\Sigma$ (coupled) channel potentials



HAL QCD method

An efficient method to calculate hadron-hadron potential in the lattice QCD

In the case of NN potential

- Nonlocal NN potential U is defined from NN correlator

$$\left(\frac{1}{8\mu} \frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} + \frac{\nabla^2}{2\mu} \right) R(\mathbf{r}, t) = \int d\mathbf{r}' U(\mathbf{r}, \mathbf{r}') R(\mathbf{r}', t)$$

NN correlator obtained by lattice calc.

Nonlocal NN potential

- Local NN potential is defined by derivative expansion of nonlocal potential

$$U(r) = V_C(r) + V_T(r)S_{12} + V_{LS}(r)\mathbf{L} \cdot \mathbf{S} + \dots$$

NΛ-NΣ(I=1/2) coupled channel central & tensor potentials in 3S1-3D1

S. Aoki et al. (HAL Coll.) Proc.Jpn.Acad.B87(2011)509.

H. Nemura for HAL Coll., AIP Conference Proceedings 2130, 040005 (2019).

$$V_{3S_1} = \Psi^{-1} K$$

$$V_{3S_1} = \begin{pmatrix} V_{3S_1,C}^{NL-NL} & V_{3S_1,T}^{NL-NL} & V_{3S_1,C}^{NL-NS} \Delta_{NS}^{NL} & V_{3S_1,T}^{NL-NS} \Delta_{NS}^{NL} \\ V_{3S_1,C}^{NS-NL} \Delta_{NL}^{NS} & V_{3S_1,T}^{NS-NL} \Delta_{NL}^{NS} & V_{3S_1,C}^{NS-NS} & V_{3S_1,T}^{NS-NS} \end{pmatrix}$$

$$\Psi = \begin{pmatrix} R_S^{NL-NL} & R_D^{NL-NL} & R_S^{NL-NS} & R_D^{NL-NS} \\ 2\sqrt{2}R_D^{NL-NL} & 2\sqrt{2}R_S^{NL-NL} - 2R_D^{NL-NL} & 2\sqrt{2}R_D^{NL-NS} & 2\sqrt{2}R_S^{NL-NS} - 2R_D^{NL-NS} \\ R_S^{NS-NL} & R_D^{NS-NL} & R_S^{NS-NS} & R_D^{NS-NS} \\ 2\sqrt{2}R_D^{NS-NL} & 2\sqrt{2}R_S^{NS-NL} - 2R_D^{NS-NL} & 2\sqrt{2}R_D^{NS-NS} & 2\sqrt{2}R_S^{NS-NS} - 2R_D^{NS-NS} \end{pmatrix}$$

$$K = \begin{pmatrix} \left(\frac{1+3\delta_{NL}^2}{8\mu_{NL}} \frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} + \frac{\nabla^2}{2\mu_{NL}}\right) & 0 & 0 & 0 \\ 0 & \left(\frac{1+3\delta_{NL}^2}{8\mu_{NL}} \frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} + \frac{\nabla^2}{2\mu_{NL}}\right) & 0 & 0 \\ 0 & 0 & \left(\frac{1+3\delta_{NS}^2}{8\mu_{NS}} \frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} + \frac{\nabla^2}{2\mu_{NS}}\right) & 0 \\ 0 & 0 & 0 & \left(\frac{1+3\delta_{NS}^2}{8\mu_{NS}} \frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} + \frac{\nabla^2}{2\mu_{NS}}\right) \end{pmatrix} \Psi$$

NΛ-NΣ(I=1/2) coupled channel central & tensor potentials in 3S1-3D1

S. Aoki et al. (HAL Coll.) Proc.Jpn.Acad.B87(2011)509.

H. Nemura for HAL Coll., AIP Conference Proceedings 2130, 040005 (2019).

R_S, R_D : S-wave(L=0), D-wave(L=2) BB correlator
obtained by partial wave decomposition

$$V_{3S_1} = \Psi^{-1} K$$

$$V_{3S_1} = \begin{pmatrix} V_{3S_1,C}^{NL-NL} & V_{3S_1,T}^{NL-NL} & V_{3S_1,C}^{NL-NS} \Delta_{NS}^{NL} & V_{3S_1,T}^{NL-NS} \Delta_{NS}^{NL} \\ V_{3S_1,C}^{NS-NL} \Delta_{NL}^{NS} & V_{3S_1,T}^{NS-NL} \Delta_{NL}^{NS} & V_{3S_1,C}^{NS-NS} & V_{3S_1,T}^{NS-NS} \end{pmatrix}$$

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$$K = \begin{pmatrix} \left(\frac{1+3\delta_{NL}^2}{8\mu_{NL}} \frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} + \frac{\nabla^2}{2\mu_{NL}} \right) & 0 & 0 & 0 \\ 0 & \left(\frac{1+3\delta_{NL}^2}{8\mu_{NL}} \frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} + \frac{\nabla^2}{2\mu_{NL}} \right) & 0 & 0 \\ 0 & 0 & \left(\frac{1+3\delta_{NS}^2}{8\mu_{NS}} \frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} + \frac{\nabla^2}{2\mu_{NS}} \right) & 0 \\ 0 & 0 & 0 & \left(\frac{1+3\delta_{NS}^2}{8\mu_{NS}} \frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} + \frac{\nabla^2}{2\mu_{NS}} \right) \end{pmatrix} \Psi$$

Partial wave(L=0,2) decomposition on the lattice

Method 1. A_1^+ projection of cubic group

M. Luscher, Nucl. Phys. B 354 (1991), 531.
Aoki, Hatsuda, Ishii, PTEP 123 (2010).

$$R^{A_1^+}(\mathbf{r}) \equiv \frac{1}{48} \sum_{g \in O_h} R(g^{-1}\mathbf{r})$$

: This has dominant contribution from L=0 and small contribution from L=4,6,....

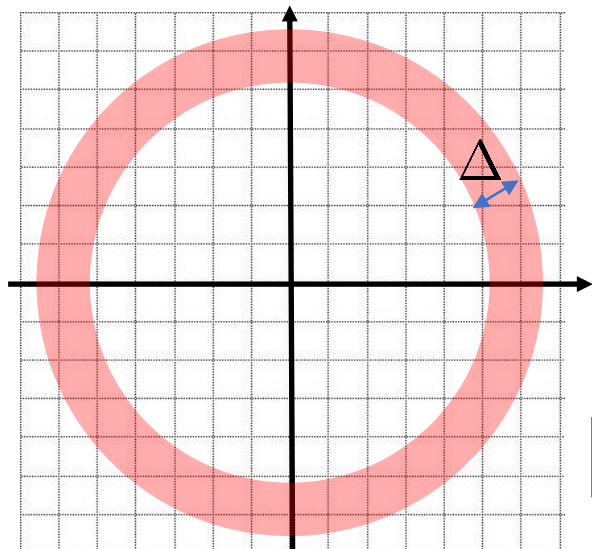


S-wave $R_S(\mathbf{r}) = R^{A_1^+}(\mathbf{r})$

D-wave $R_D(\mathbf{r}) = R(\mathbf{r}) - R^{A_1^+}(\mathbf{r})$

Method 2. Misner's method

C. W. Misner, Class. Quant. Grav. 21 (2004) S243.
T. Miyamoto et al., Phys. Rev. D 101 (2020) 074514.



Use
$$R(\mathbf{r}) = \sum_{n,l,m} c_{nlm}^{\Delta} G_n^{\Delta}(\mathbf{r}) Y_{lm}(\theta, \phi)$$

new basis function in \mathbf{r}

instead of
$$R(\mathbf{r}) = \sum_{l,m} g_{lm}(\mathbf{r}) Y_{lm}(\theta, \phi)$$

sophisticated partial wave decomposition on the lattice

Setup

Configuration: Nearly physical point ($m_\pi \simeq 146[\text{MeV}]$)

Lattice: $L = 96^4$

$a = 0.0846(7) \text{ fm}$ (2333 MeV)

$La = 8.12 \text{ fm}$

414 configurations($\times 2 \times 4 \times 96$)

- forward / backward propagation for correlator
- x,y,z,t rotation for gauge conf
- 96 wall-source point

Baryon mass: N : 0.4093 (955 MeV)

Λ : 0.4884 (1139 MeV)

Σ : 0.5233 (1221 MeV)

$N\Lambda$ and $N\Sigma$ potentials

- $I=1/2$ ($N\Lambda$ - $N\Sigma$)

- $N\Lambda$ - $N\Sigma$ coupled potential

- **1S0 central potential**
- 3S1-3D1 central & tensor potential

threshold

$$E_{\Lambda N\pi}$$

$$E_{\Lambda N\pi}$$

- $I=3/2$ ($N\Sigma$)

- 1S0 central potential
- 3S1-3D1 central & tensor potential

$$E_{\Sigma N\pi}$$

$$E_{\Sigma N\pi}$$

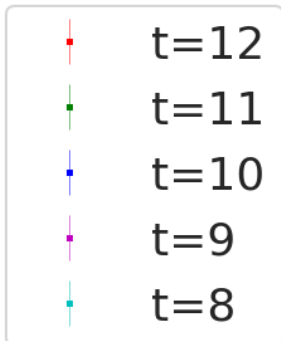
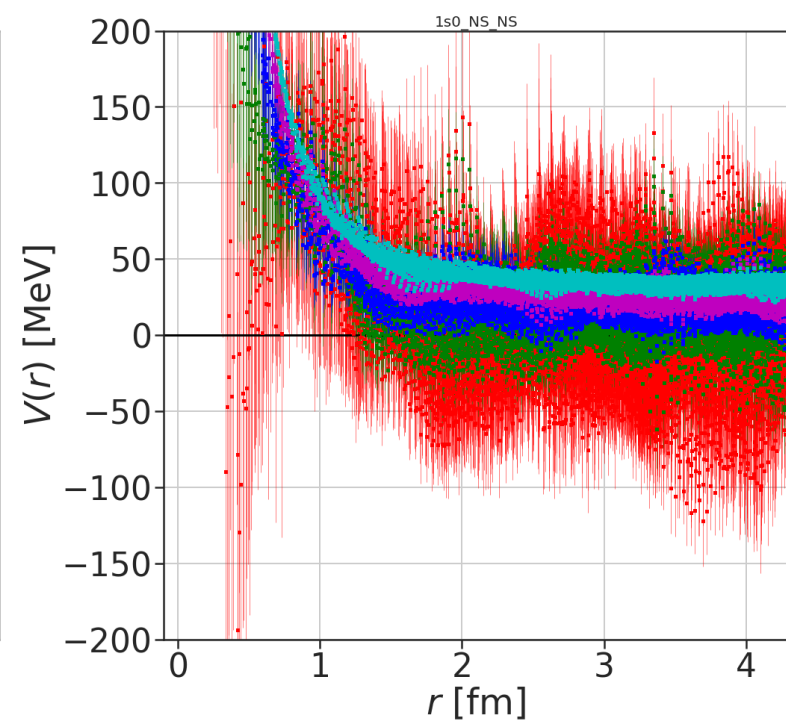
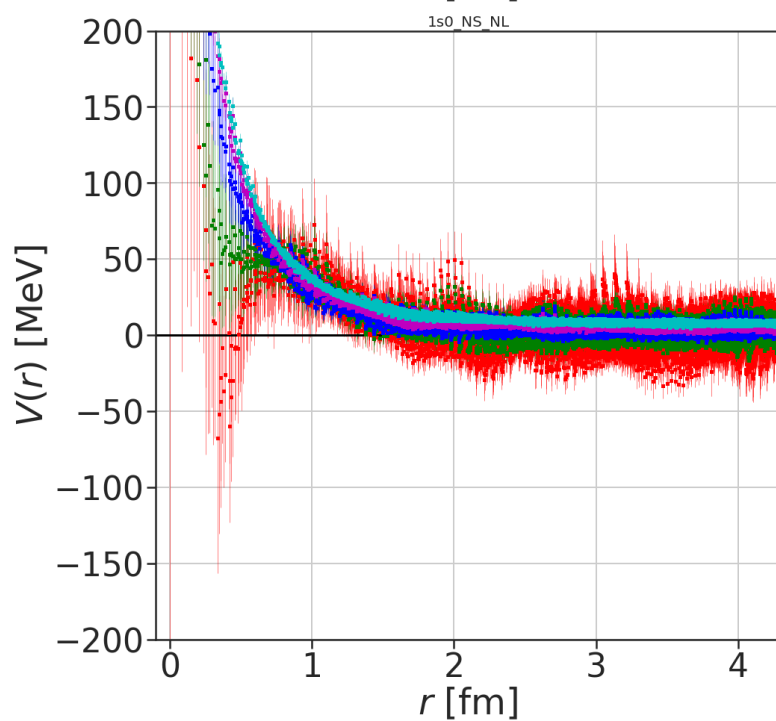
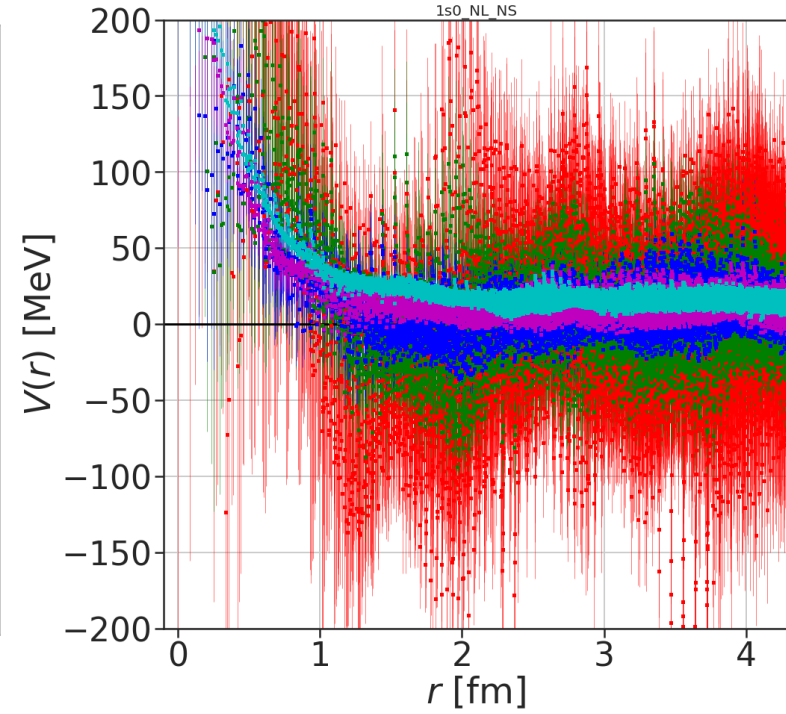
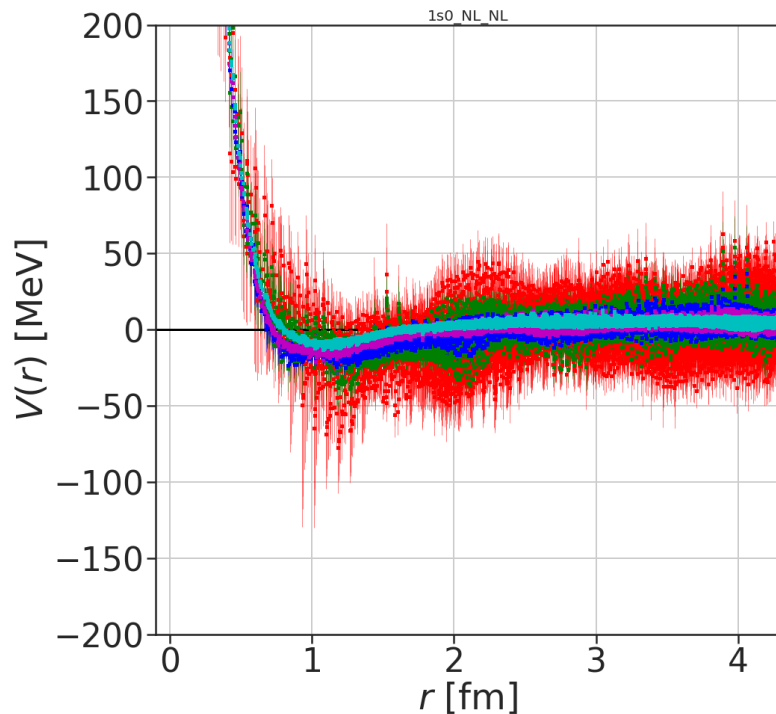
central potential

1s0, l=1/2

t=8~12

binsize=18

w/o Misner
(A_1^+ projection)



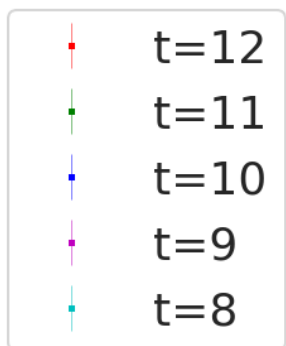
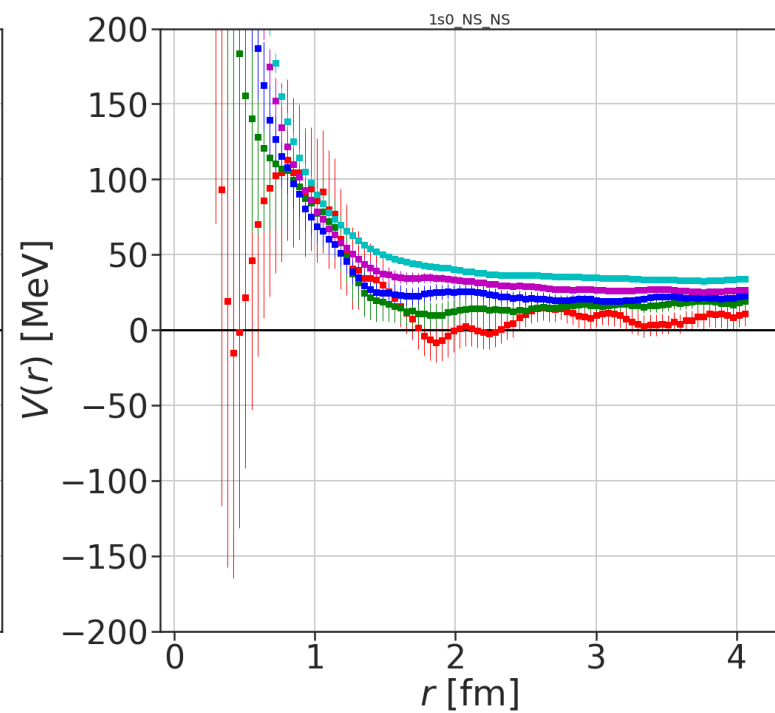
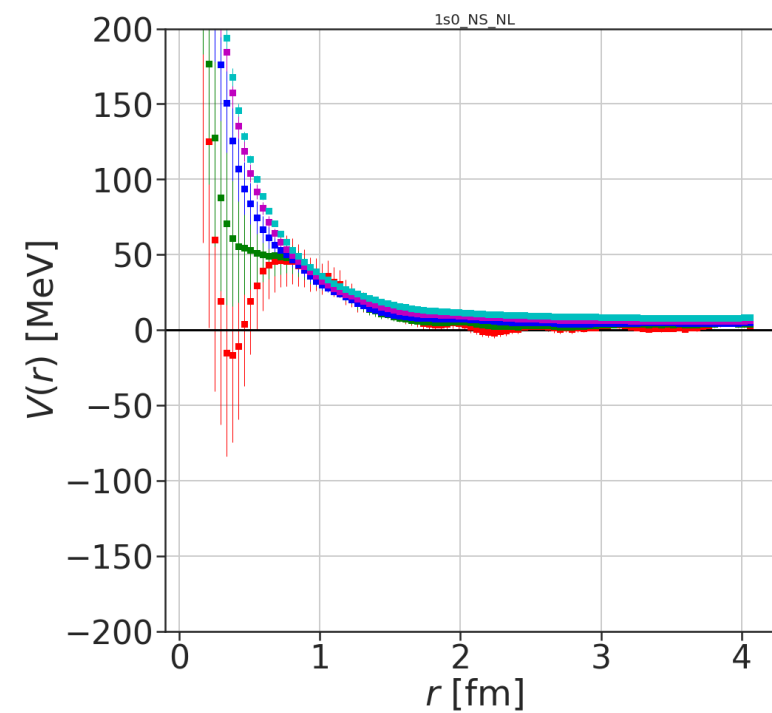
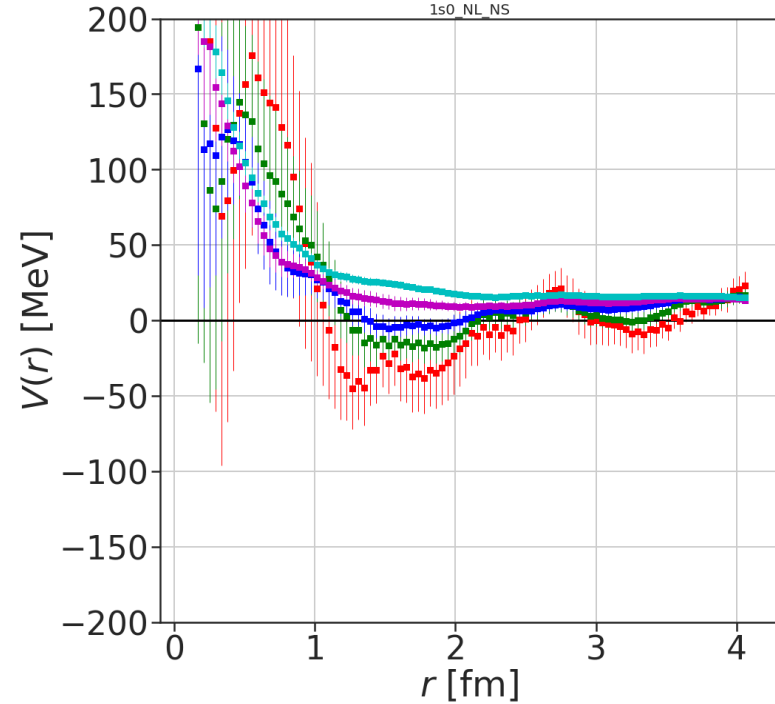
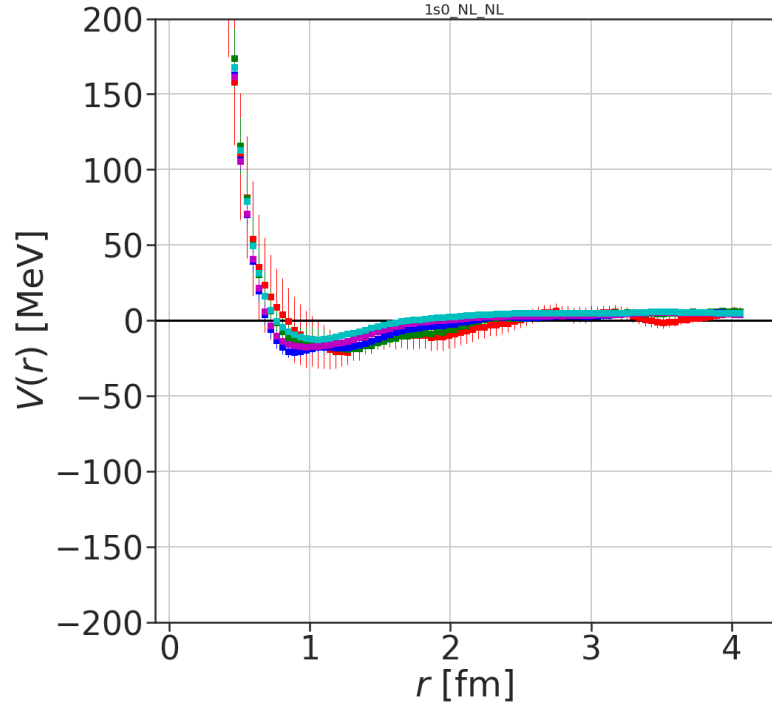
central potential

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w/ Misner



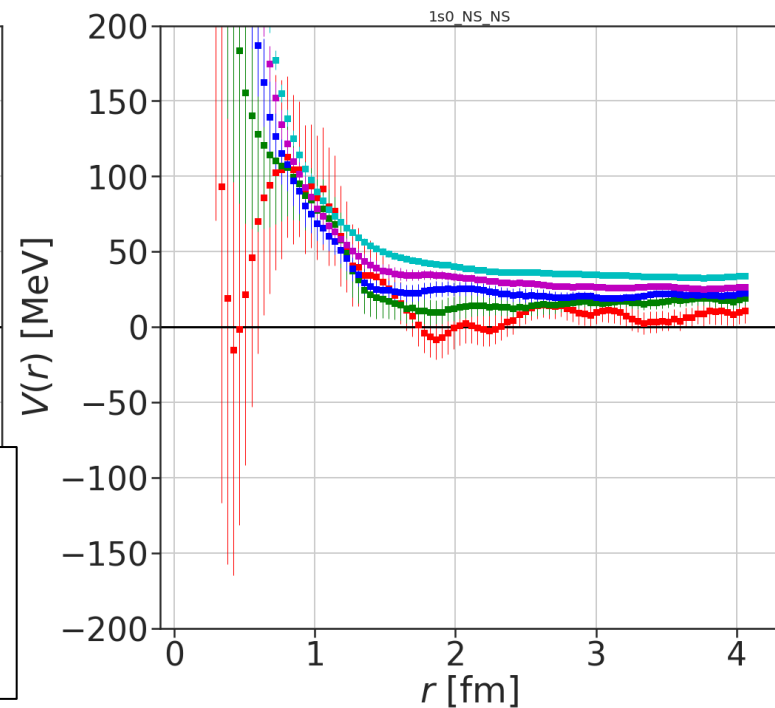
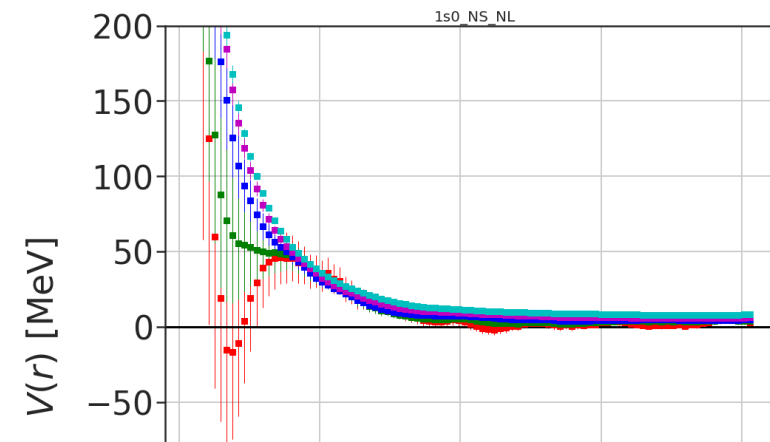
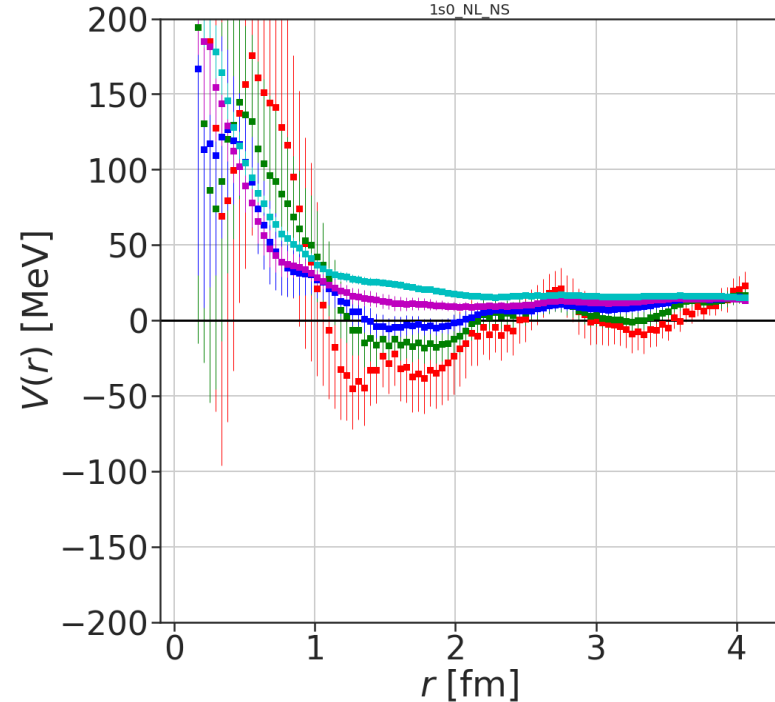
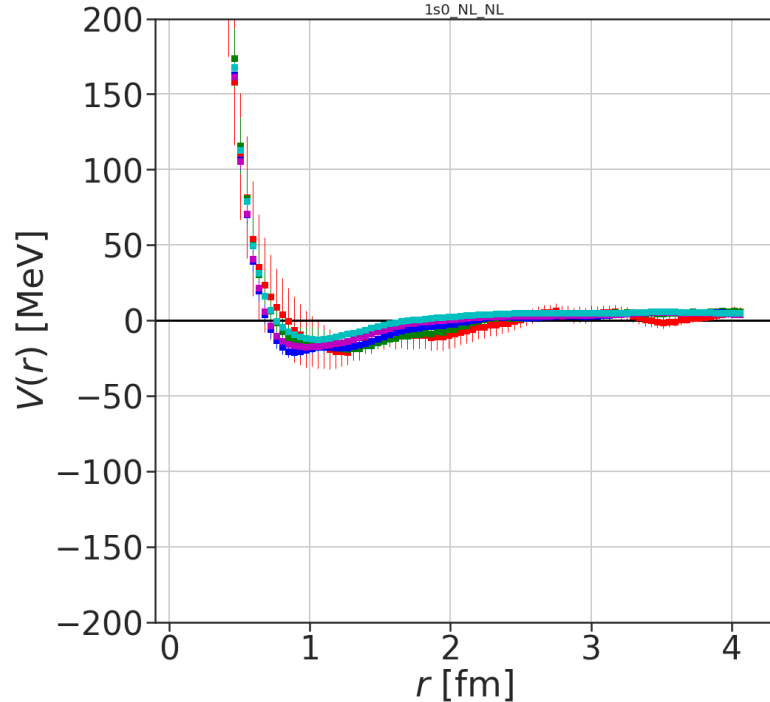
central potential

1s0, l=1/2

t=8~12

binsize=18

w/ Misner



- Misner method makes error bar small.
- Change of the central values is small.
- Potentials at long range do not reach to 0 for any t.
- Wavy behavior is seen at large t.

N Λ and N Σ potentials

- I=1/2 (N Λ -N Σ)

- N Λ -N Σ coupled potential

- 1S0 central potential
- 3S1-3D1 central & tensor potential

- effective N Λ potential

- **1S0 central potential**
- 3S1-3D1 central & tensor potential

- I=3/2 (N Σ)

- 1S0 central potential
- 3S1-3D1 central & tensor potential

threshold

$$E_{\Lambda N \pi}$$

$$E_{\Lambda N \pi}$$

$$E_{\Lambda N}$$

$$E_{\Lambda N}$$

$$E_{\Sigma N \pi}$$

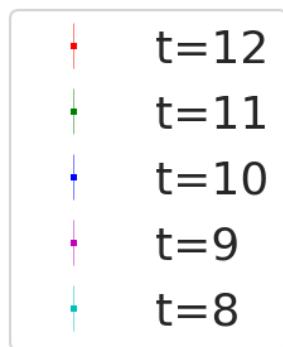
$$E_{\Sigma N \pi}$$

effective $N\Lambda$ central potential

$1s_0, l=1/2$

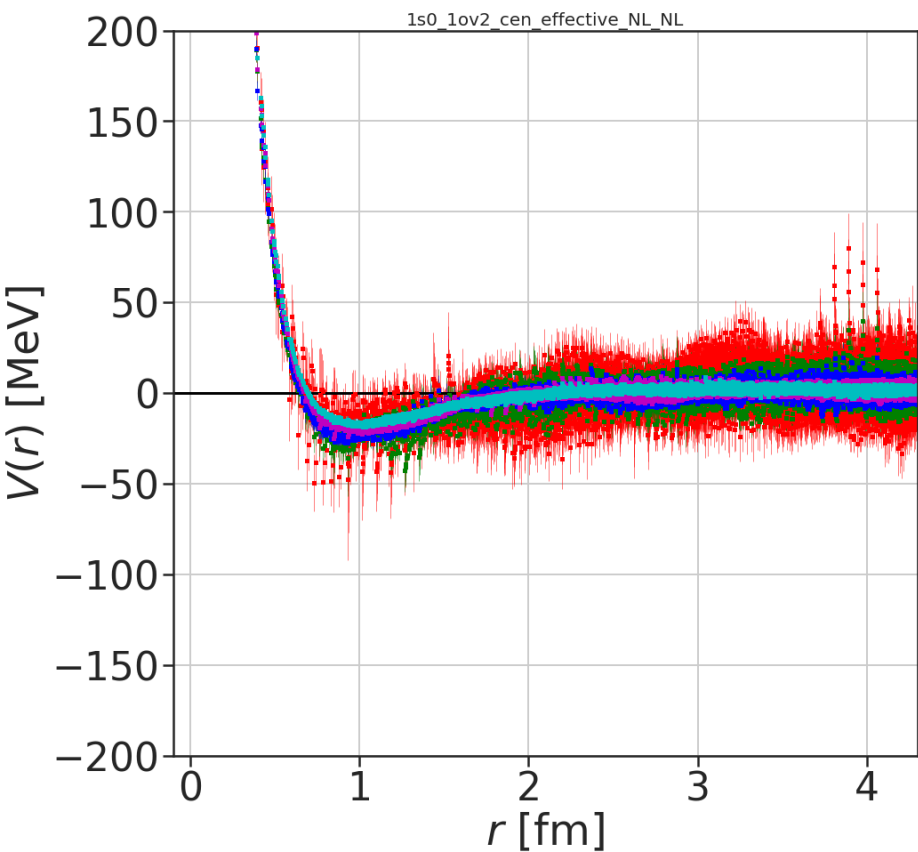
$t=8\sim 12$

binsize=18

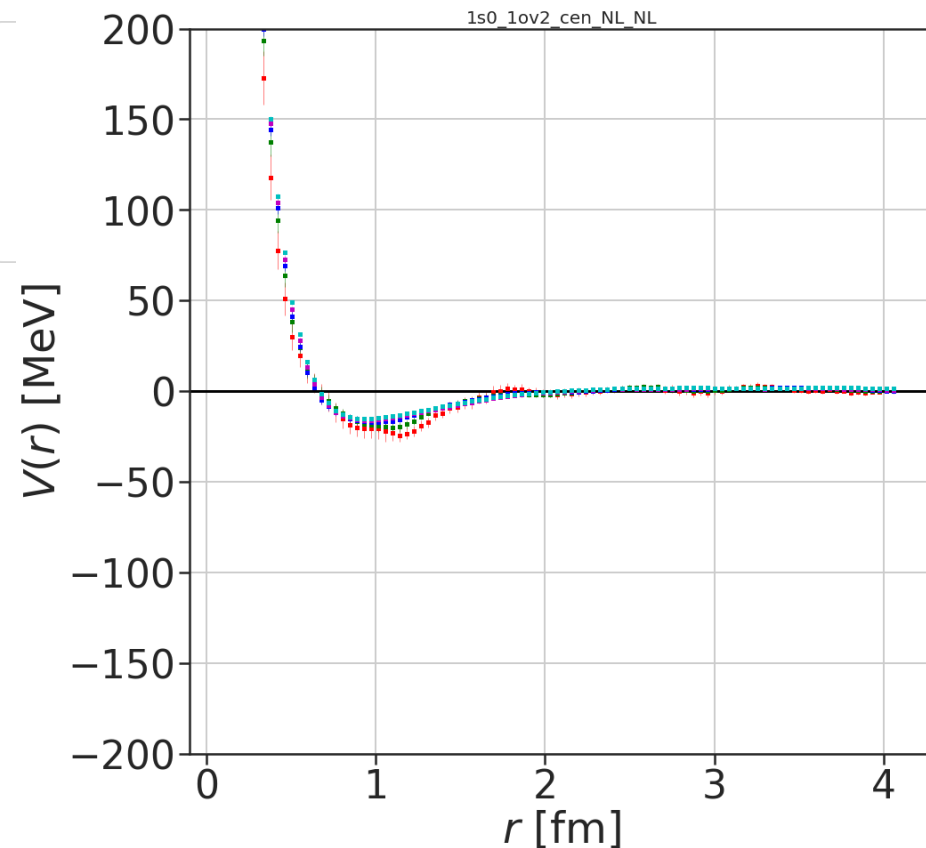


- Potentials at long range reach to 0 for any t .
- Wavy behavior is suppressed.
- At low energy, this effective $N\Lambda$ potential is useful in application of HALQCD potential.

w/o Misner
(A_1^+ projection)



w/ Misner



$N\Lambda$ and $N\Sigma$ potentials

- $I=1/2$ ($N\Lambda$ - $N\Sigma$)

- $N\Lambda$ - $N\Sigma$ coupled potential

- 1S0 central potential
- **3S1-3D1 central & tensor potential**

threshold

$$E_{\Lambda N\pi}$$

$$E_{\Lambda N\pi}$$

- $I=3/2$ ($N\Sigma$)

- 1S0 central potential
- 3S1-3D1 central & tensor potential

$$E_{\Sigma N\pi}$$

$$E_{\Sigma N\pi}$$

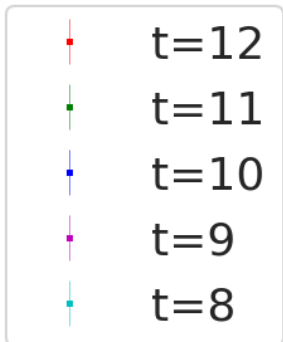
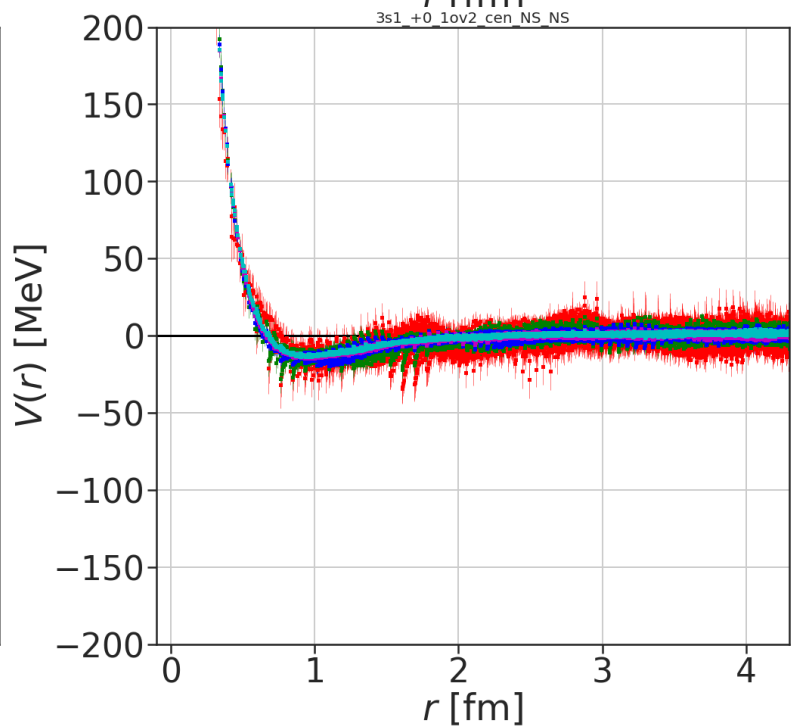
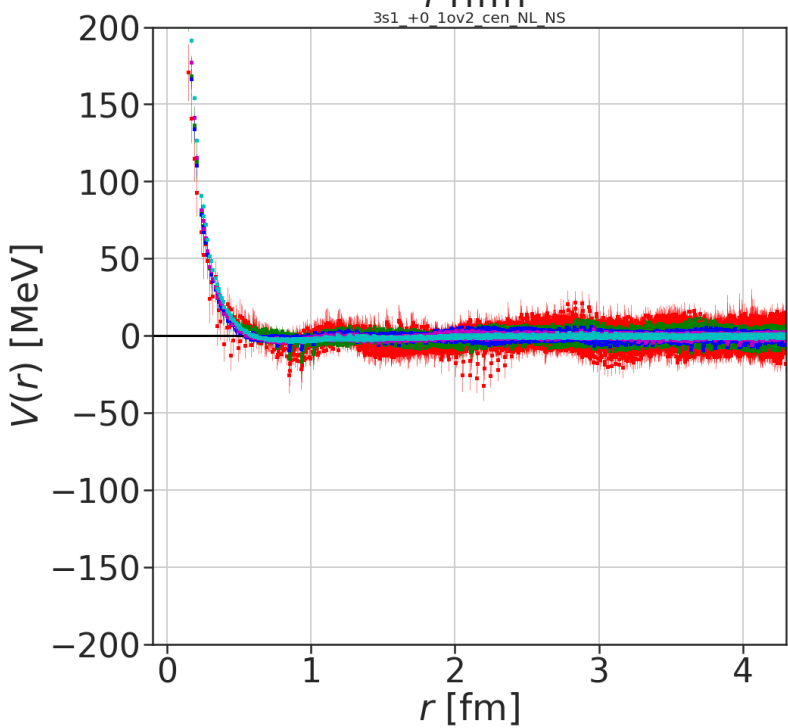
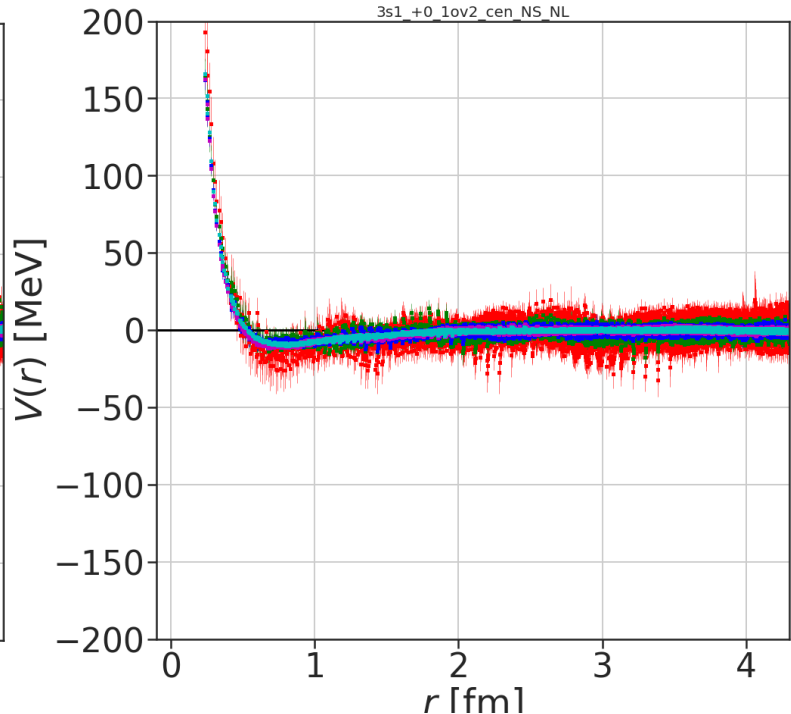
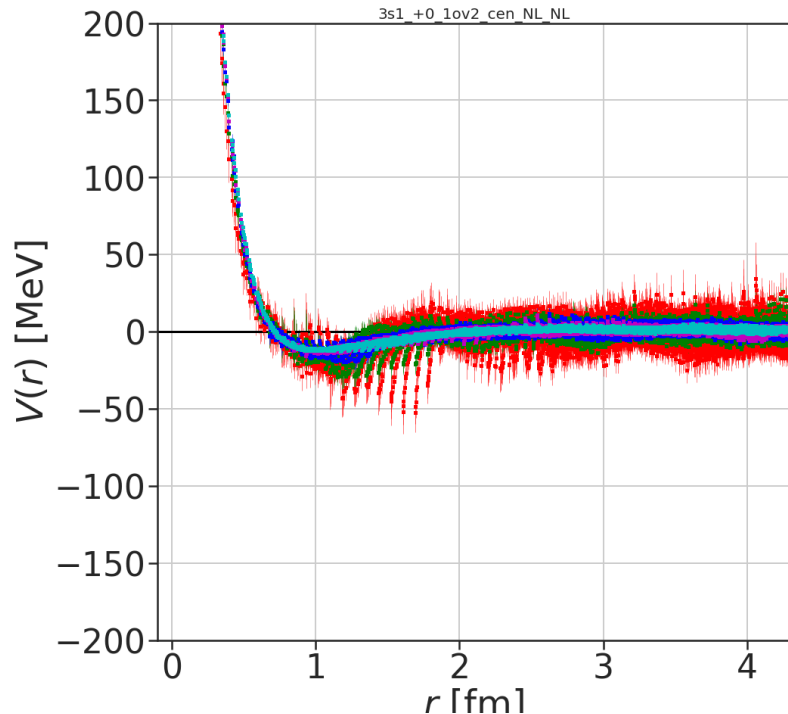
central potential

3s1, l=1/2

t=8~12

binsize=18

w/o Misner
(A_1^+ projection)



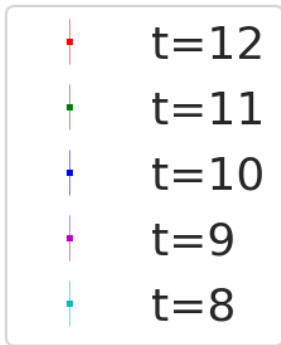
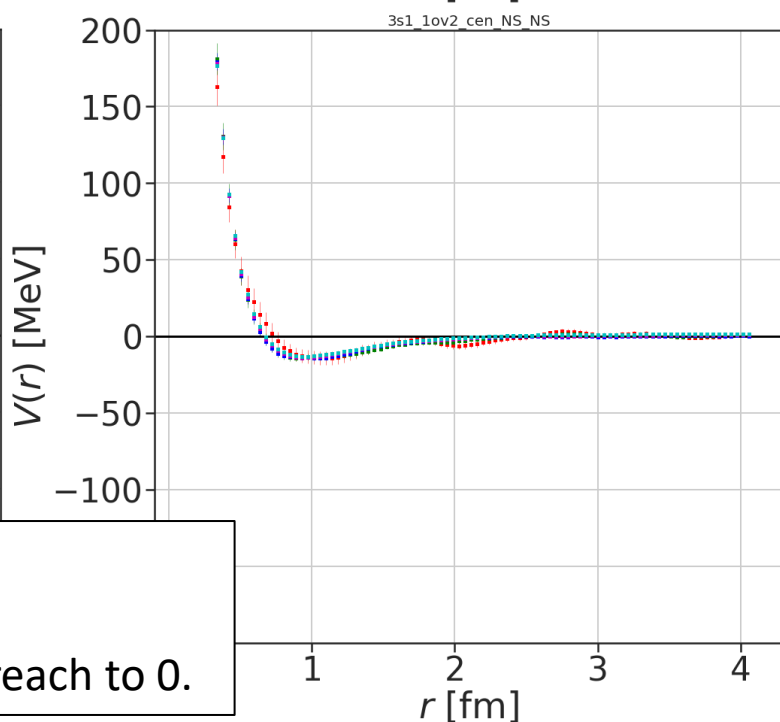
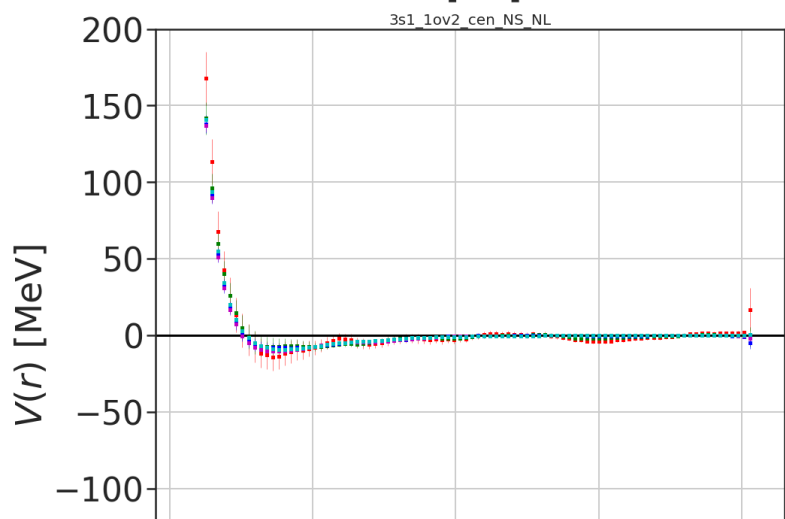
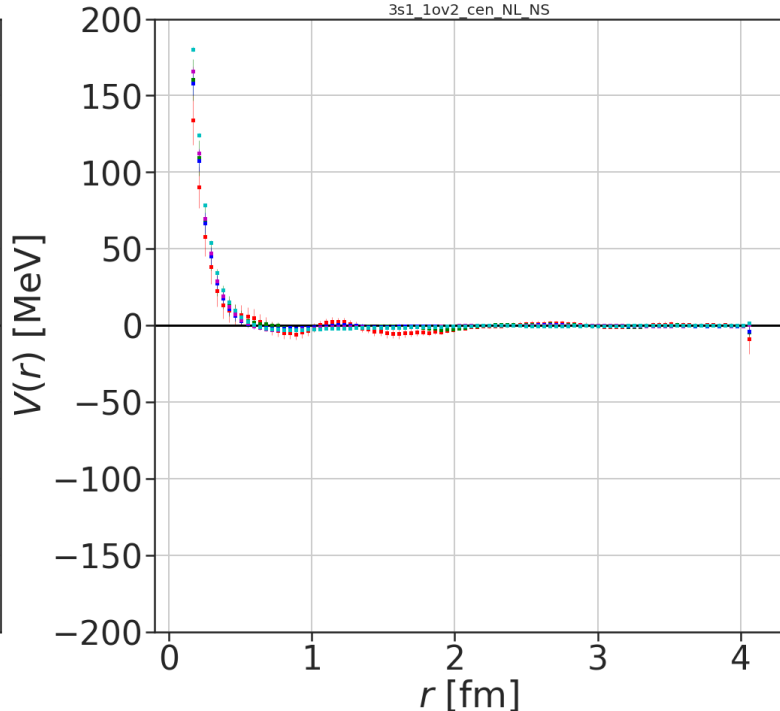
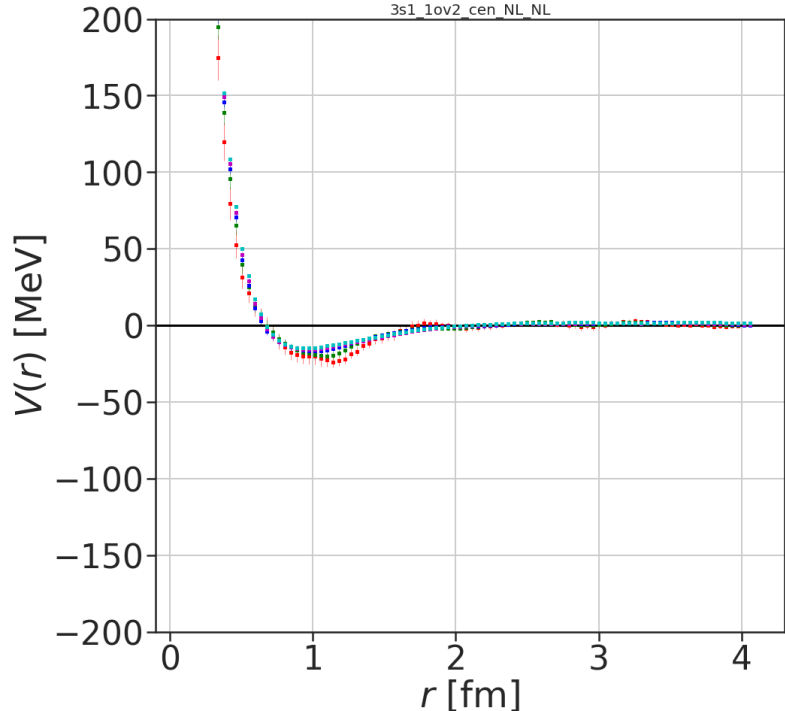
central potential

3s1, l=1/2

t=8~12

binsize=18

w/ Misner



- Misner method makes error bar small.
- Change of the central values is small.
- In 3S1-3D1 channel, central potentials at long range reach to 0.

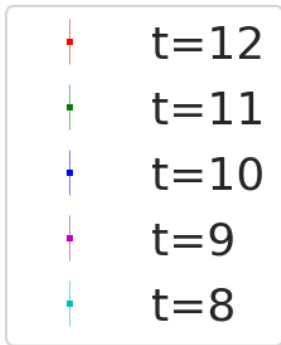
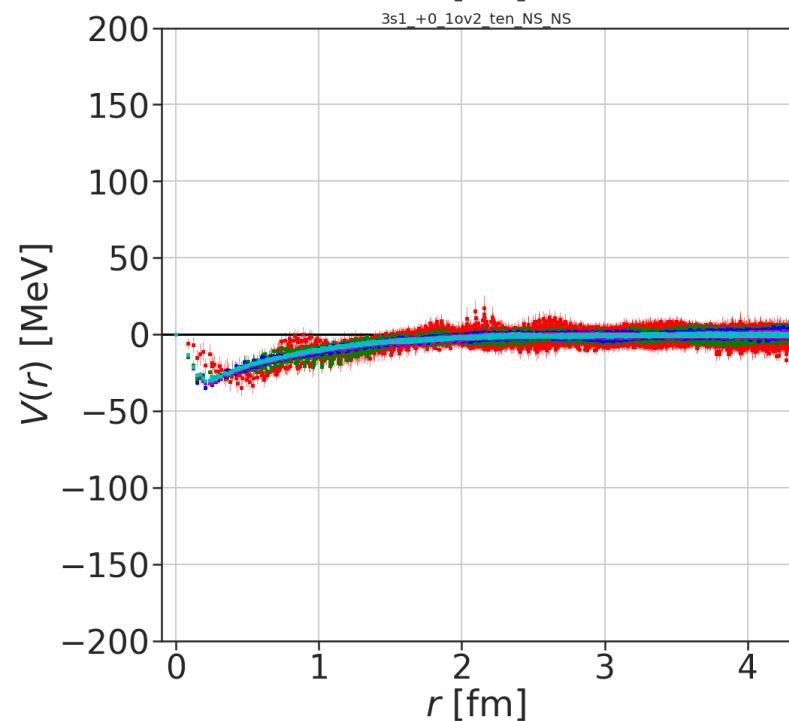
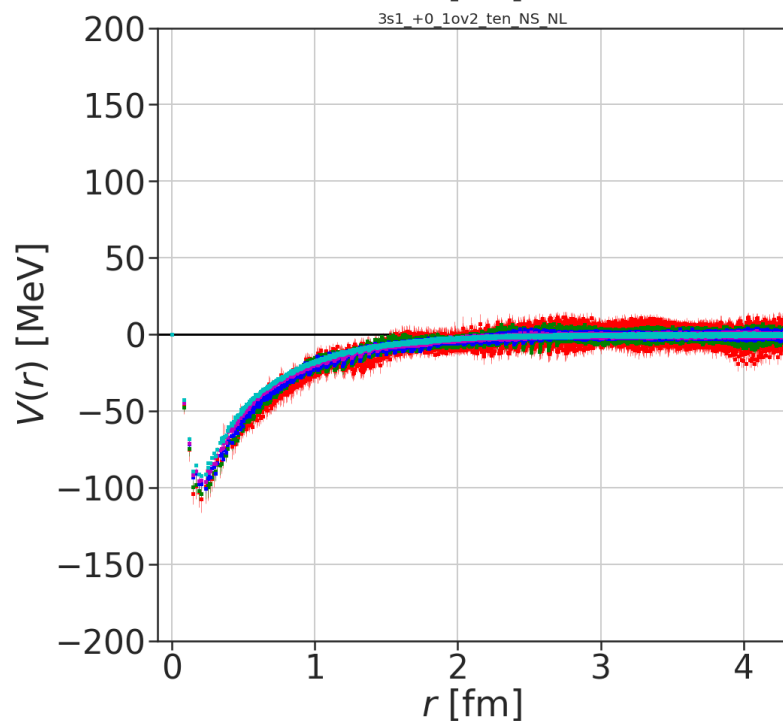
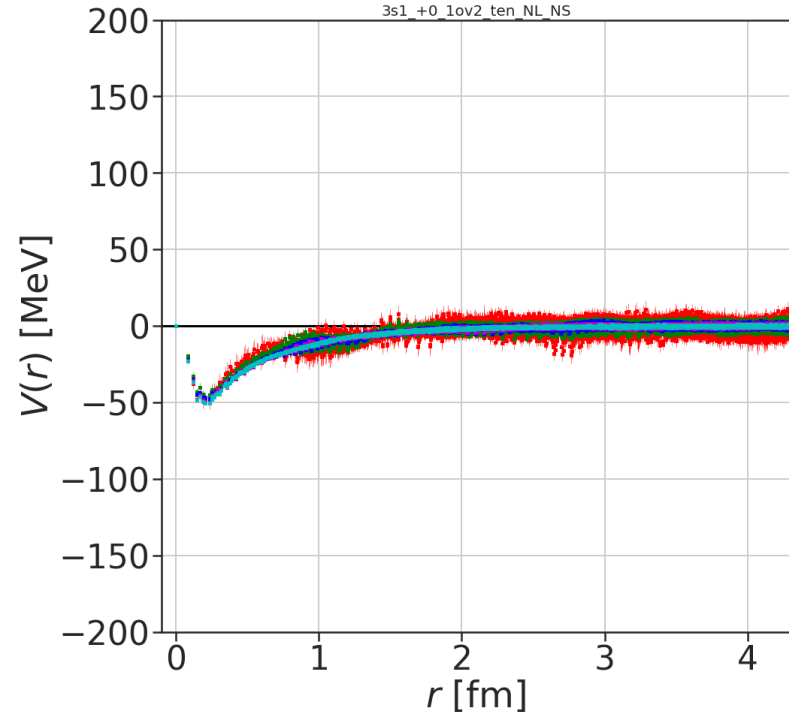
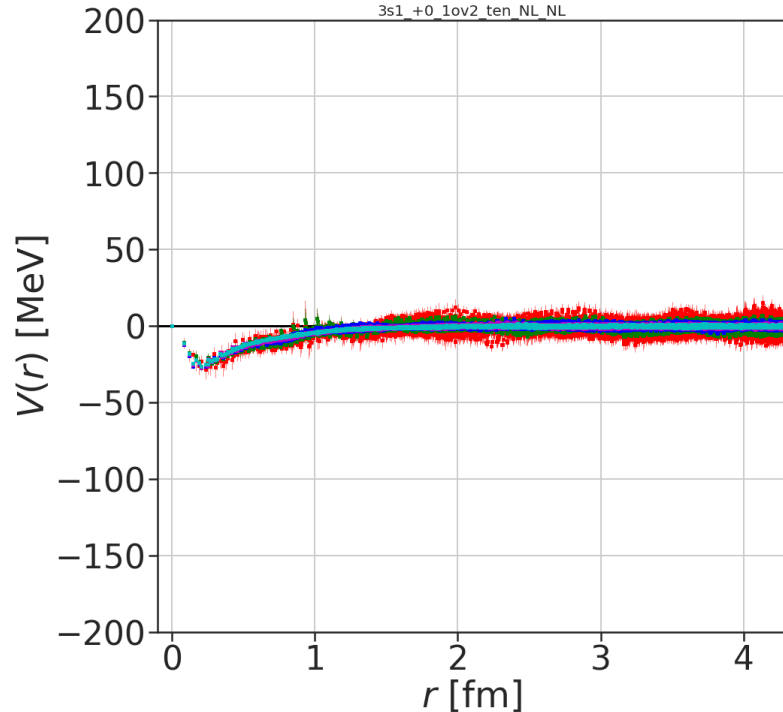
tensor potential

$3s_1, l=1/2$

$t=8\sim 12$

binsize=18

w/o Misner
(A_1^+ projection)



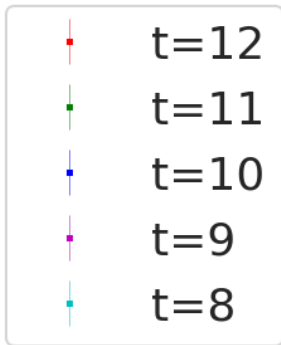
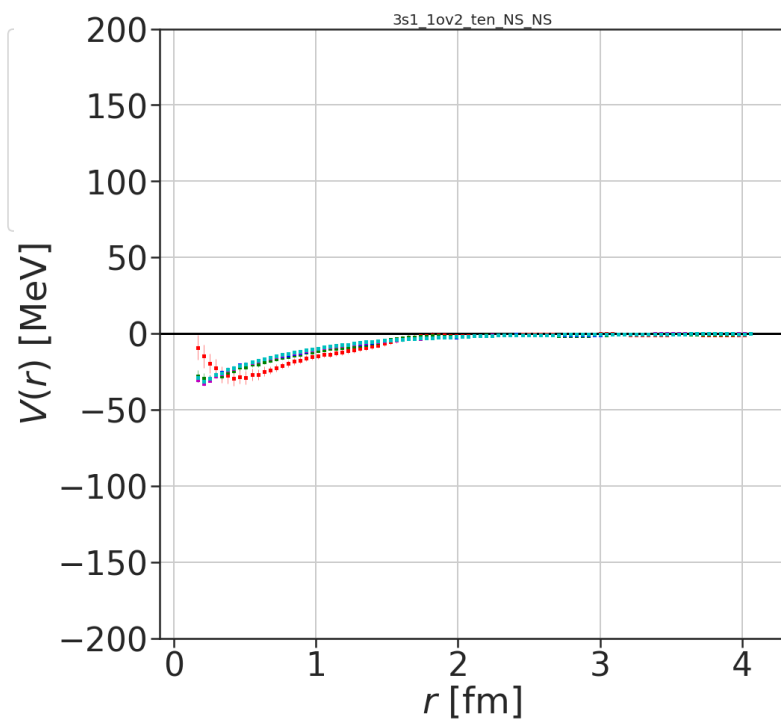
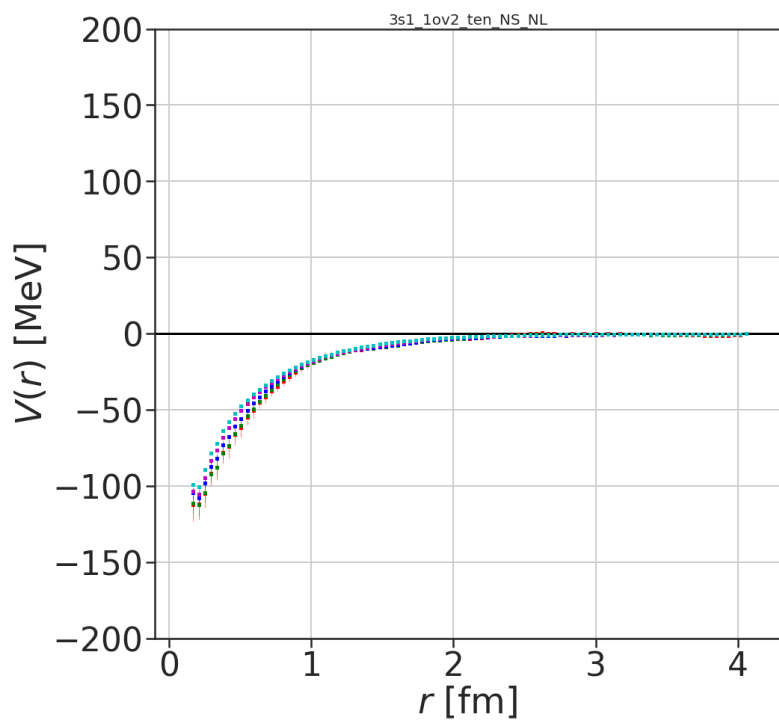
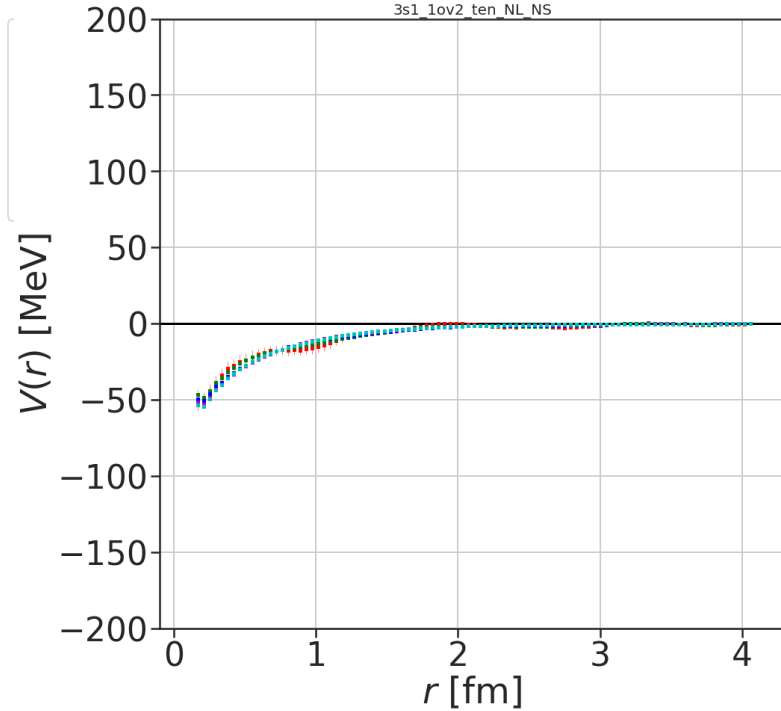
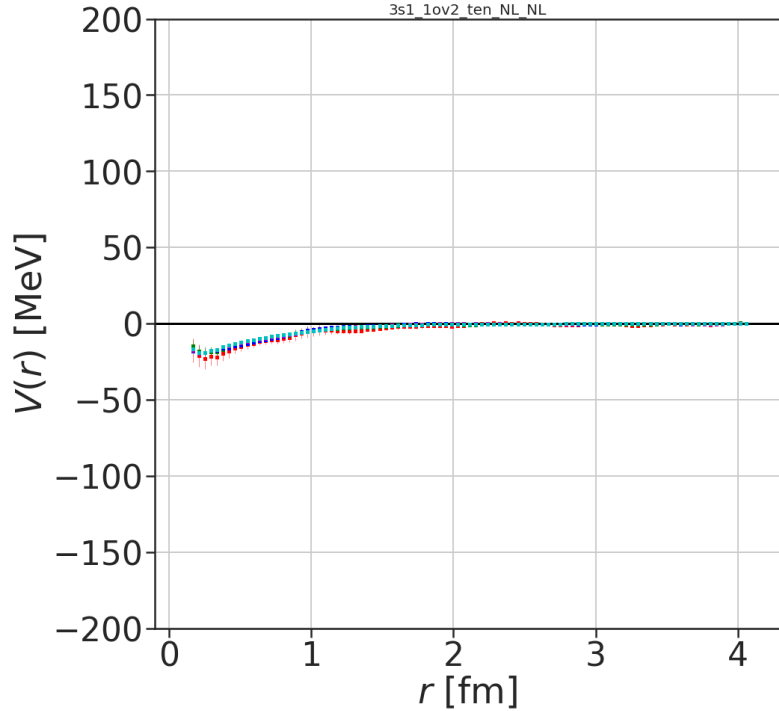
tensor potential

3s1, l=1/2

t=8~12

binsize=18

w/ Misner



$N\Lambda$ and $N\Sigma$ potentials

- $I=1/2$ ($N\Lambda$ - $N\Sigma$)

- $N\Lambda$ - $N\Sigma$ coupled potential

- 1S0 central potential
- 3S1-3D1 central & tensor potential

threshold

$$E_{\Lambda N\pi}$$

$$E_{\Lambda N\pi}$$

- $I=3/2$ ($N\Sigma$)

- **1S0 central potential**
- 3S1-3D1 central & tensor potential

$$E_{\Sigma N\pi}$$

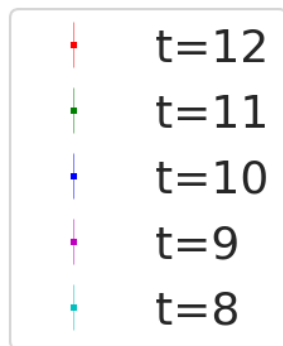
$$E_{\Sigma N\pi}$$

N Σ central potential

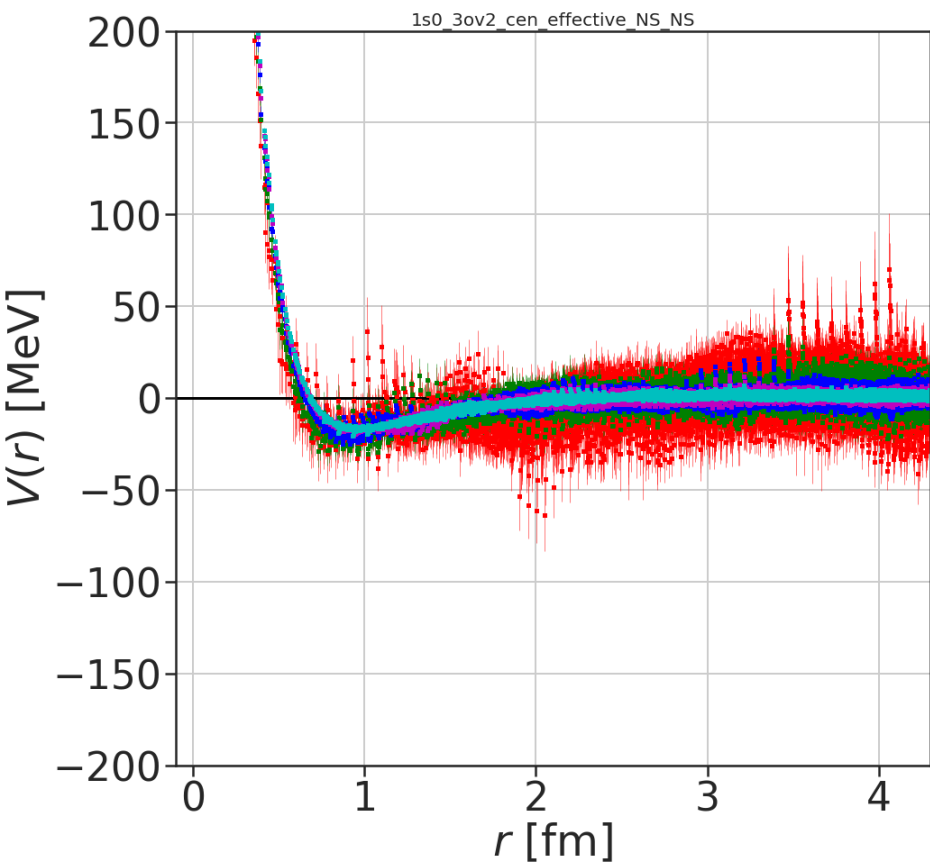
1s0, l=3/2

t=8~12

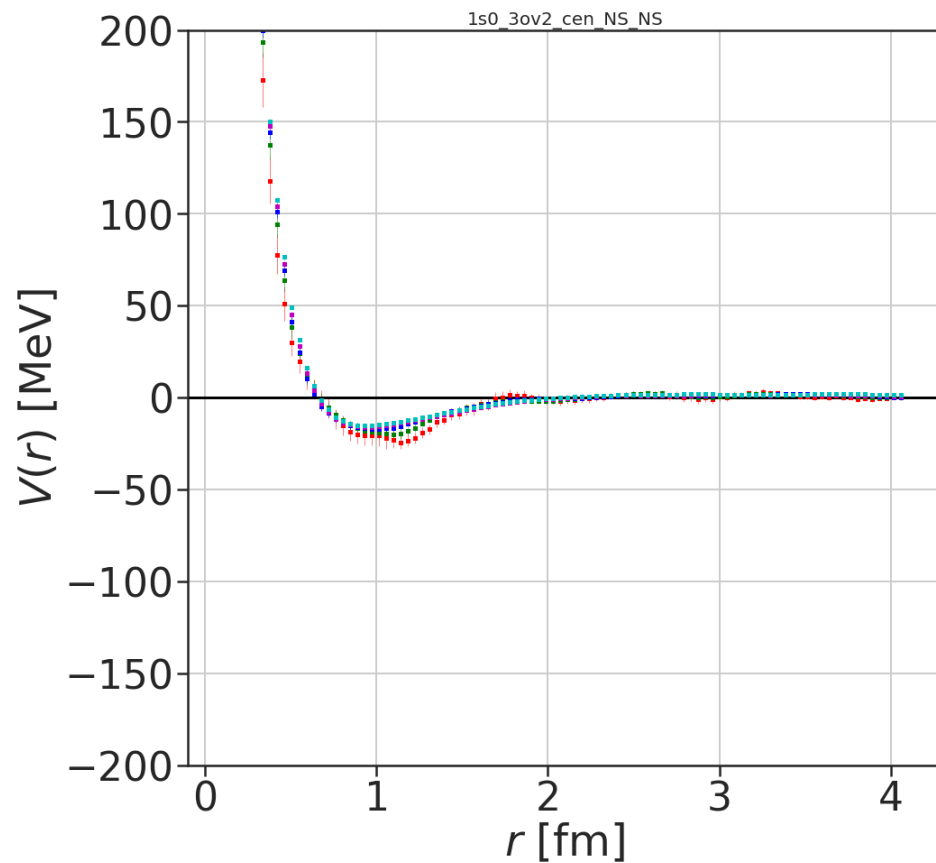
binsize=18



w/o Misner
(A_1^+ projection)



w/ Misner



N Λ and N Σ potentials

- I=1/2 (N Λ -N Σ)

- N Λ -N Σ coupled potential

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threshold

$$E_{\Lambda N \pi}$$

$$E_{\Lambda N \pi}$$

- I=3/2 (N Σ)

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- **3S1-3D1 central & tensor potential**

$$E_{\Sigma N \pi}$$

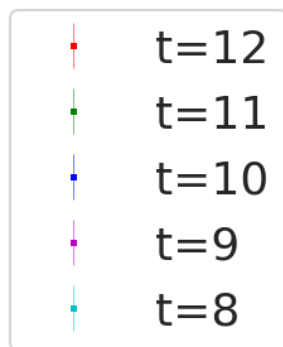
$$E_{\Sigma N \pi}$$

N Σ central potential

3s1, l=3/2

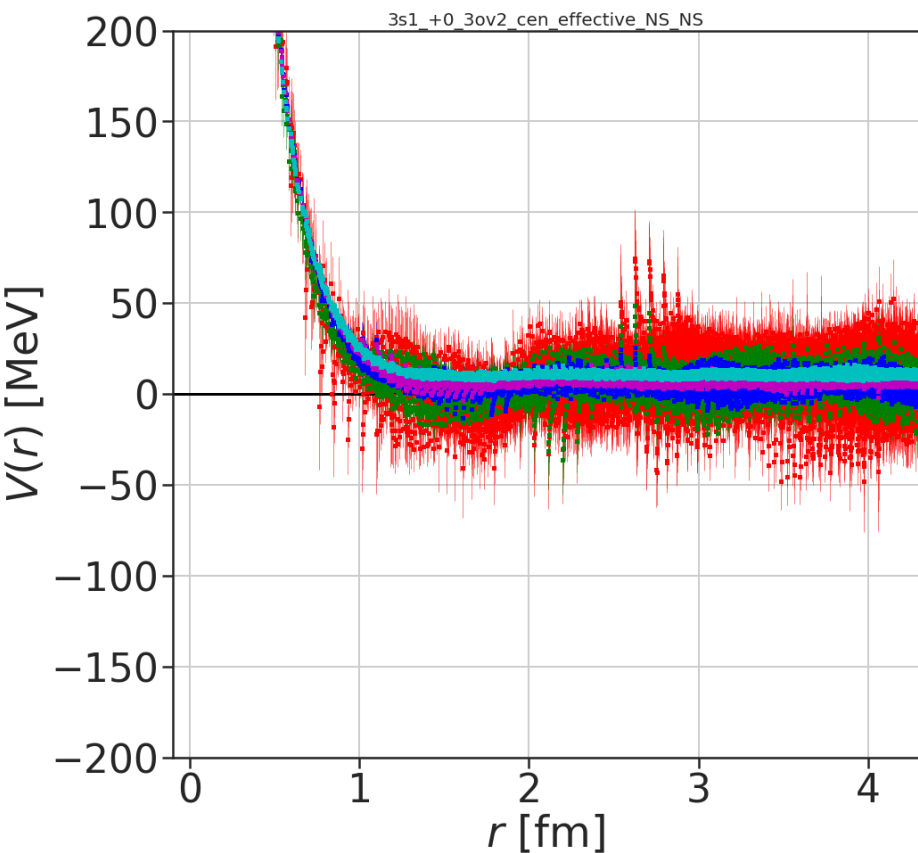
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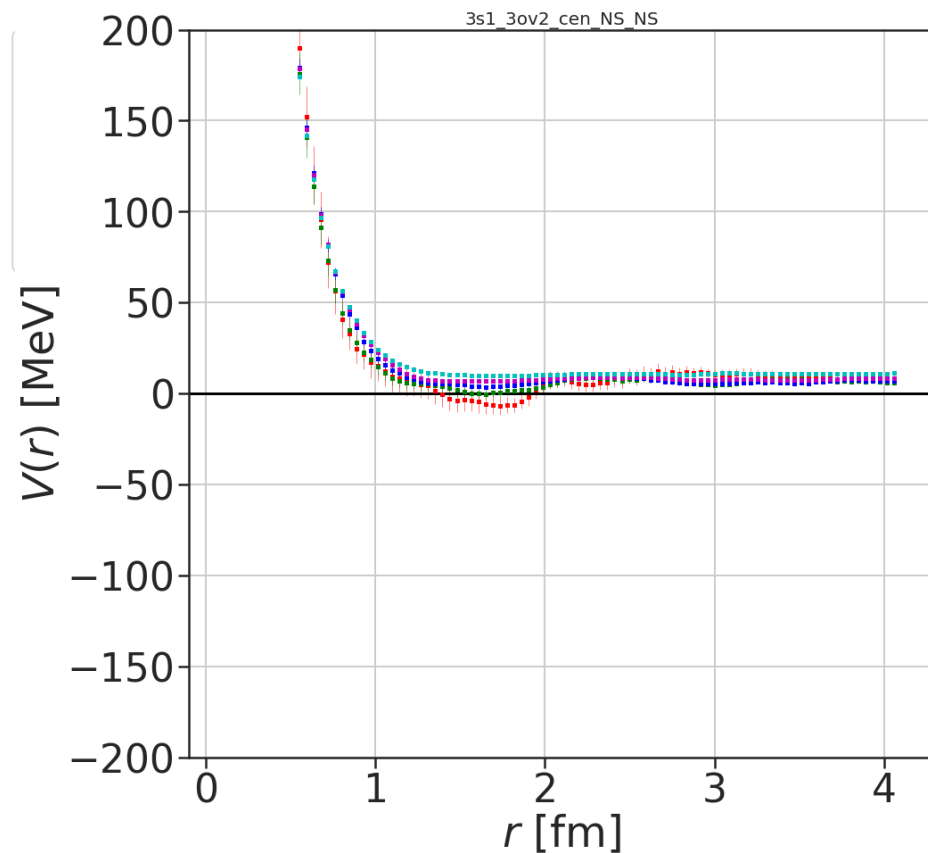


• Central potentials at long range does not reach to 0.

w/o Misner
(A_1^+ projection)



w/ Misner

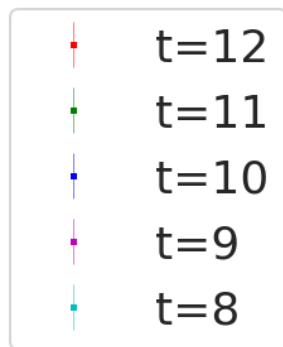


$N\Sigma$ tensor potential

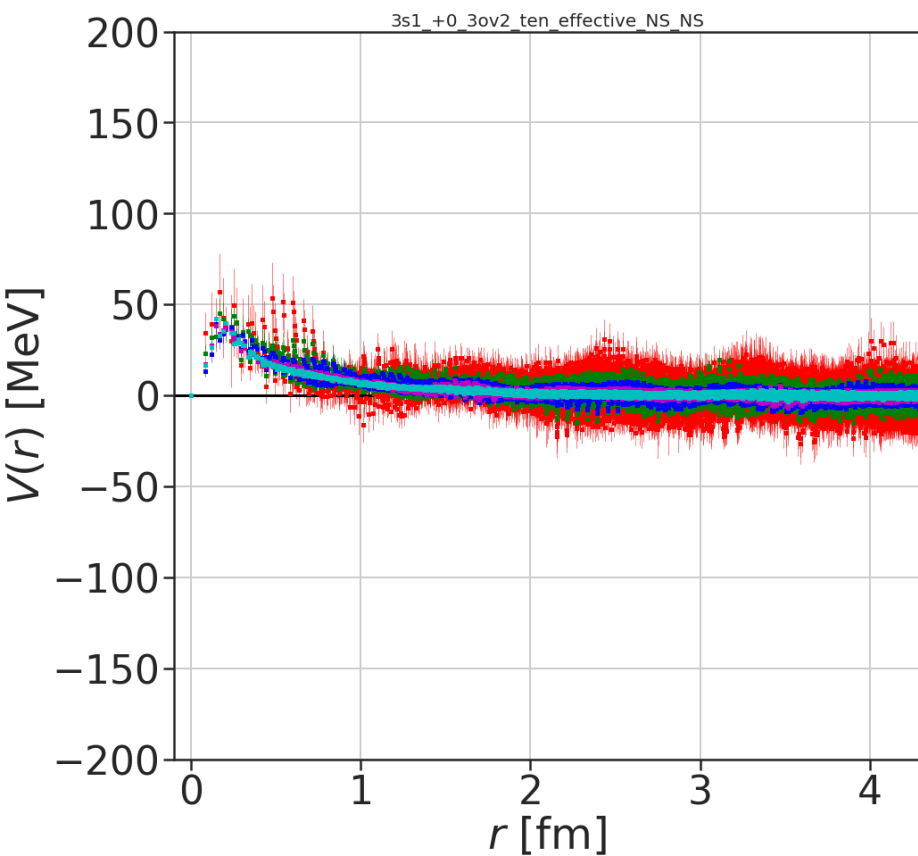
$3s1, l=3/2$

$t=8\sim 12$

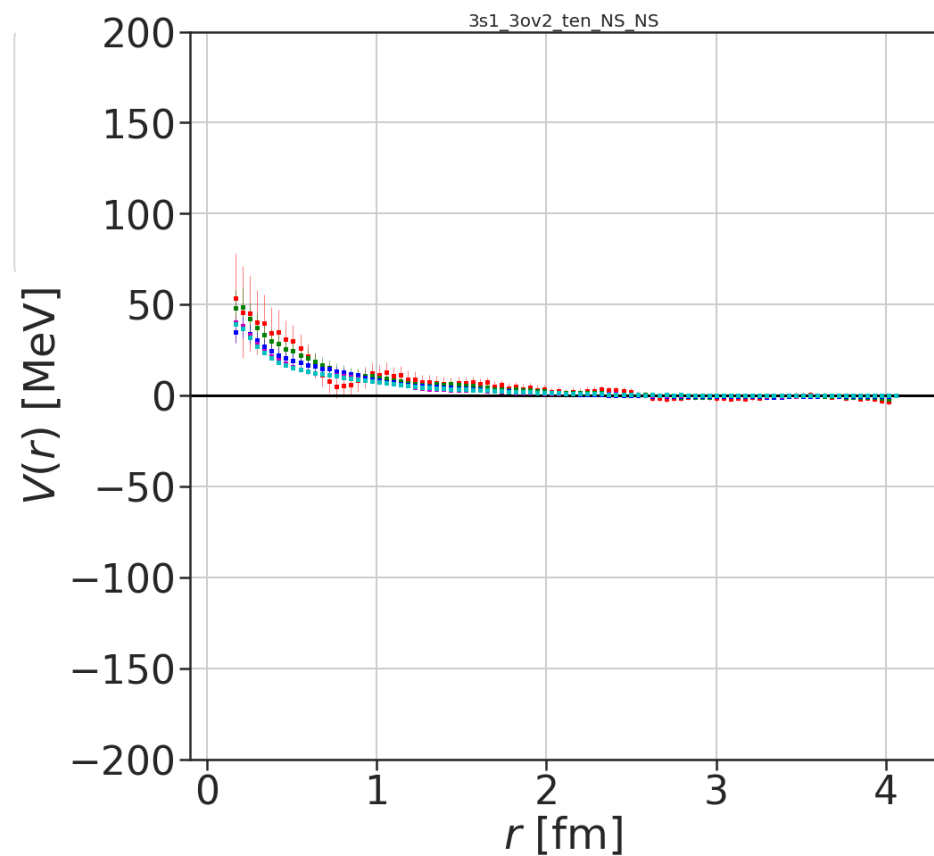
binsize=18



**w/o Misner
(A_1^+ projection)**



w/ Misner



Summary

- $N\Lambda$ & $N\Sigma$ potentials are calculated by HAL QCD method at almost physical point.
 - $l=1/2$, $1S0$ $N\Lambda$ - $N\Sigma$ coupled channel central potential
 - $l=1/2$, $3S1$ - $3D1$ $N\Lambda$ - $N\Sigma$ coupled channel central & tensor potential
 - $l=3/2$, $1S0$ $N\Sigma$ single channel central potential
 - $l=3/2$, $3S1$ - $3D1$ $N\Sigma$ single channel central & tensor potential
- As for partial wave decomposition on the lattice, Misner method works, and Misner method is better than the method by the projection of cubic group.

Future

- Calculate observables(phase shifts).
 - Fit potentials
 - Investigate the wavy behavior at large t
- Potentials are applied to many body calculations of NY system.

Appendix

HAL QCD method

An efficient method to calculate hadron-hadron potential in the lattice QCD

Properties of HAL-QCD potential

- Potential itself is not observable.
- Observables (phase shift, binding energy, ...) are correctly reproduced by HAL-QCD potential.
- Ground state saturation is NOT required.
- Elastic excited states $E \leq E_{NN\pi}$ are signals in HAL QCD method.
- HAL QCD potentials can be applied to many-body calculations of hadrons.

NΛ-NΣ(I=1/2) coupled channel central & tensor potentials in 3S1-3D1

S. Aoki et al. (HAL Coll.) Proc.Jpn.Acad.B87(2011)509.

H. Nemura for HAL Coll., AIP Conference Proceedings 2130, 040005 (2019).

$$V_{3S_1} = \Psi^{-1} K$$

generalization of
NN case

$$V_C = \frac{1}{R^{NN}} \left(\frac{1}{8\mu} \frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} + \frac{\nabla^2}{2\mu} \right) R^{NN}$$

$$V_{3S_1} = \begin{pmatrix} V_{3S_1,C}^{NL-NL} & V_{3S_1,T}^{NL-NL} & V_{3S_1,C}^{NL-NS} \Delta_{NS}^{NL} & V_{3S_1,T}^{NL-NS} \Delta_{NS}^{NL} \\ V_{3S_1,C}^{NS-NL} \Delta_{NL}^{NS} & V_{3S_1,T}^{NS-NL} \Delta_{NL}^{NS} & V_{3S_1,C}^{NS-NS} & V_{3S_1,T}^{NS-NS} \end{pmatrix}$$

$$\Psi = \begin{pmatrix} R_S^{NL-NL} & R_D^{NL-NL} & R_S^{NL-NS} & R_D^{NL-NS} \\ 2\sqrt{2}R_D^{NL-NL} & 2\sqrt{2}R_S^{NL-NL} - 2R_D^{NL-NL} & 2\sqrt{2}R_D^{NL-NS} & 2\sqrt{2}R_S^{NL-NS} - 2R_D^{NL-NS} \\ R_S^{NS-NL} & R_D^{NS-NL} & R_S^{NS-NS} & R_D^{NS-NS} \\ 2\sqrt{2}R_D^{NS-NL} & 2\sqrt{2}R_S^{NS-NL} - 2R_D^{NS-NL} & 2\sqrt{2}R_D^{NS-NS} & 2\sqrt{2}R_S^{NS-NS} - 2R_D^{NS-NS} \end{pmatrix}$$

$$K = \begin{pmatrix} \left(\frac{1+3\delta_{NL}^2}{8\mu_{NL}} \frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} + \frac{\nabla^2}{2\mu_{NL}} \right) & 0 & 0 & 0 \\ 0 & \left(\frac{1+3\delta_{NL}^2}{8\mu_{NL}} \frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} + \frac{\nabla^2}{2\mu_{NL}} \right) & 0 & 0 \\ 0 & 0 & \left(\frac{1+3\delta_{NS}^2}{8\mu_{NS}} \frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} + \frac{\nabla^2}{2\mu_{NS}} \right) & 0 \\ 0 & 0 & 0 & \left(\frac{1+3\delta_{NS}^2}{8\mu_{NS}} \frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} + \frac{\nabla^2}{2\mu_{NS}} \right) \end{pmatrix} \Psi$$

Partial wave(L=0,2) decomposition on the lattice

Method 1. A_1^+ projection of cubic group

M. Luscher, Nucl. Phys. B 354 (1991), 531.
Aoki, Hatsuda, Ishii, PTEP 123 (2010).

$$R^{A_1^+}(\mathbf{r}) \equiv \frac{1}{48} \sum_{g \in O_h} R(g^{-1}\mathbf{r})$$

: This has dominant contribution from L=0 and small contribution from L=4,6,....

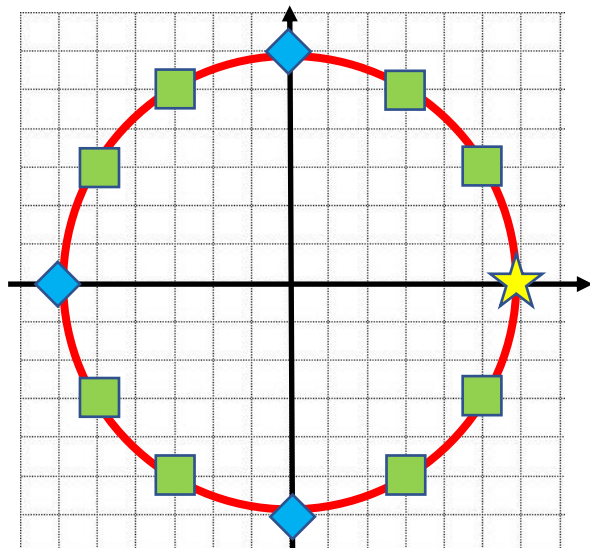


S-wave $R_S(\mathbf{r}) = R^{A_1^+}(\mathbf{r})$

D-wave $R_D(\mathbf{r}) = R(\mathbf{r}) - R^{A_1^+}(\mathbf{r})$

Method 2. Misner's method

C. W. Misner, Class. Quant. Grav. 21 (2004) S243.
T. Miyamoto et al., Phys. Rev. D 101 (2020) 074514.



◆ : cubic transformation of ★

■ : Other transformation of ★

◆ and ■ have same r



Exact partial wave decomposition on the lattice

N Λ and N Σ potentials

● I=1/2 (N Λ -N Σ)

➤ N Λ -N Σ coupled potential

- 1S0 central potential
- 3S1-3D1 central & tensor potential

➤ effective N Λ potential

- 1S0 central potential
- **3S1-3D1 central & tensor potential**

● I=3/2 (N Σ)

- 1S0 central potential
- 3S1-3D1 central & tensor potential

threshold

$$E_{\Lambda N \pi}$$

$$E_{\Lambda N \pi}$$

$$E_{\Lambda N}$$

$$E_{\Lambda N}$$

$$E_{\Sigma N \pi}$$

$$E_{\Sigma N \pi}$$

example:

$$E_{\Lambda N \pi} = m_{\Lambda} + m_N + m_{\pi} \simeq 2210[\text{MeV}]$$

effective NA central&tensor potential

central

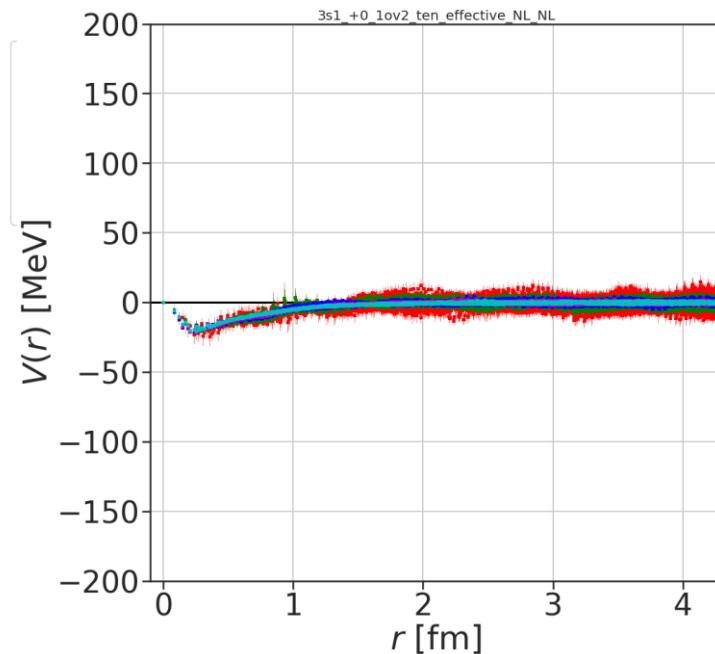
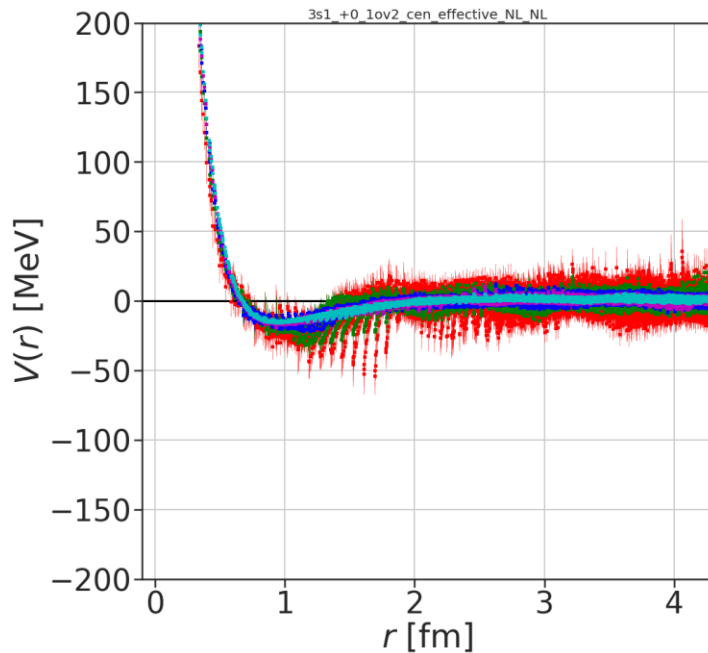
tensor

$3s1, l=1/2$

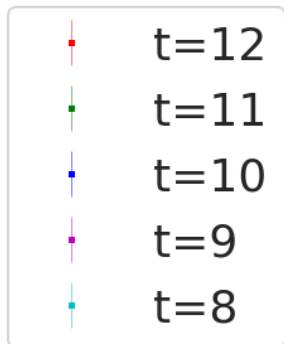
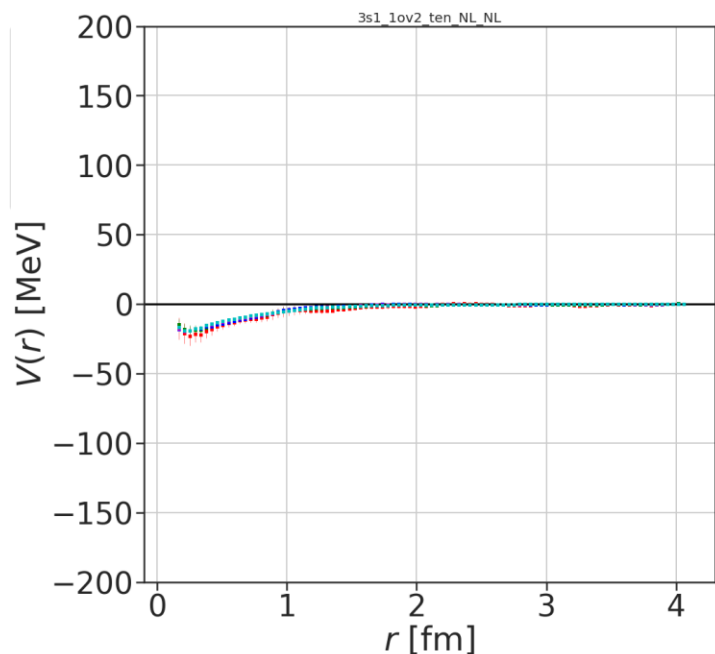
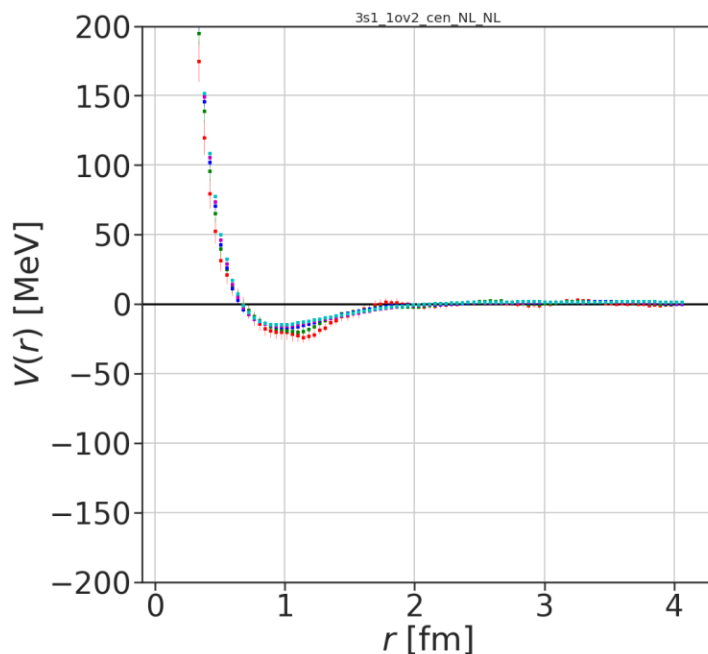
$t=8\sim 12$

binsize=18

w/o Misner
(A_1^+ projection)



w/ Misner



central

tensor

3s1, l=1/2

t=8~12

binsize=18

w/ Misner

**NA-N Σ coupled channel
central&tensor potential**

**effective NA
central&tensor potential**

