QED corrections to QCD quantities using massive photons

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DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

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Outline

Isospin breaking on the Lattice

Isospin

- approximate symmetry of SM
- strong isospin broken by $\frac{m_u - m_d}{\Lambda_{\mathrm{QCD}}} \lesssim 1\%$
- weak isospin broken by α_{FS}: $\alpha_{FS} \approx 1/137 \lesssim 1\%$

$\Rightarrow$ **Must include isospin breaking** to reach sub-percent precision

sIB can be systematically included through insertions of scalar density

[1110.6294]

QED is harder (PBC & Gauss law)

- QED_{TL}
  $\Rightarrow$ Violates reflection positivity
- QED_{L} [0804.2044]
  $\Rightarrow$ non-local
  $\Rightarrow$ power-like FSE
- QED_{C^*} [1509.01636]
  $\Rightarrow$ Partial breaking of flavour symmetry
- QED_{M} [1507.08916]
  $\Rightarrow$ local
  $\Rightarrow$ exponential FSE
  $\Rightarrow$ Control $m_\gamma \to 0$ limit?

QED$_M$ [1507.08916]

$$\mathcal{L}_{\mathrm{QED}_M} = \frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2\xi} (\partial_\mu A_\mu)^2 + \frac{1}{2} m_\gamma^2 A_\mu^2 + \overline{\psi}(\gamma_\mu D_\mu + m)\psi$$

- Add a mass term for the photon
  $\Rightarrow$ Provides a mass gap
  $\Rightarrow$ FSE are exponential if infrared cut-offs sufficiently large

$$M_\pi L \gg 1, \quad m_\gamma L \gg 1$$

- Correct order limits required

$$\text{QCD+QED observables} = \lim_{m_\gamma \to 0} \lim_{L \to \infty} \neq \lim_{L \to \infty} \lim_{m_\gamma \to 0}$$

- Zero-mode re-emerges as $m_\gamma \to 0$ (previous talk by A. Shindler)

GOAL of this talk

$\Rightarrow$ Demonstrate that we can control the required limits

$\Rightarrow$ Determine parameter space $(m_\gamma / M_\pi,\ m_\gamma L)$ of validity of QED$_M$

Set-up of our calculation

QCD: Mixed action approach

Sea Pure QCD ensembles with $N_f = 2+1+1$ provided by MILC and CalLat
HISQ sea fermions [0610092]
Gradient flowed to remove residual chiral symmetry breaking and noise [1701.07559]

Val Möbius domain wall fermions [0409118]:
- $\Rightarrow$ Chirally symmetric
- $\Rightarrow$ Simpler renormalisation
- $\Rightarrow$ $O(a)$ improved

- Tested and established by CalLat [1805.12130, 2011.12166]

Two ensembles

- **a12m310** and **a12m310XL**
- $a \sim 0.12\,\mathrm{fm}$
- $M_\pi \sim 310\,\mathrm{MeV}$
- Only differ in volume: $M_\pi L \sim 4.5$ and $M_\pi L \sim 9.0$

QED action

- Landau gauge (i.e. $\xi \to 0$)
- 6 photon masses:

$$\frac{m_\gamma}{M_\pi} \in \left\{\frac{1}{8}, \frac{1}{4}, \frac{1}{3}, \frac{5}{12}, \frac{1}{2}, \frac{2}{3}\right\}$$

to determine *breaking point*

Correlation functions and zero-mode [1507.08916]

Correlation functions for hadron $\mathbb{P}$ has functional form

$$C_{\mathbb{P}}^{SY}(t;m_\gamma) = \sum_i \frac{Z_i^S Z_i^Y}{2M_i} e^{-M_i t - xt^2} (+\text{backwards contribution})$$

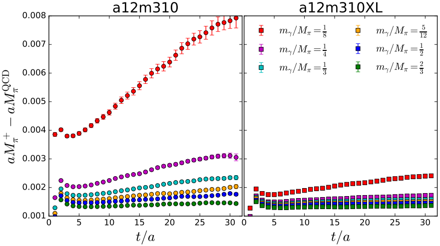
where $Z_i^Y \equiv \langle 0 | \, \mathbb{P}_i^Y \rangle$ for an operator interpolating to the quantum numbers $\mathbb{P}$ and with a smearing $Y \in \{P, S\}$.
Zero-mode only present if $\mathbb{P}$ has non-zero charge Q,

$$x = \frac{4\pi\alpha_{FS} Q^2}{2m_\gamma^2 L^3 T}.$$

does not add any free parameters.

Example of zero-mode treatment: $M_{\pi^+} - M_{\pi^{\rm QCD}}$

$e^{-xt^2} \Rightarrow$ linear term in $aM_{\rm eff}\ldots$



Zero-mode present

Example of zero-mode treatment: $M_{\pi^+} - M_{\pi^{\rm QCD}}$

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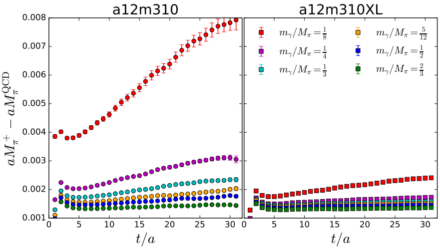
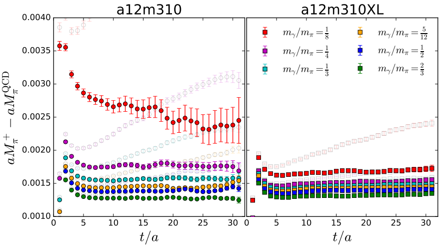


Zero-mode present

..can be accurately removed



Zero-mode removed

Next steps:

1. Correlation function fits
2. Need to remove Finite Size Effects
3. Need to take limit $m_\gamma \to 0$

Fitting strategies: splittings and ratios

Write QCD+QED quantities in terms of $\delta X = X^{\mathrm{QED+QCD}} - X^{\mathrm{QCD}}$:

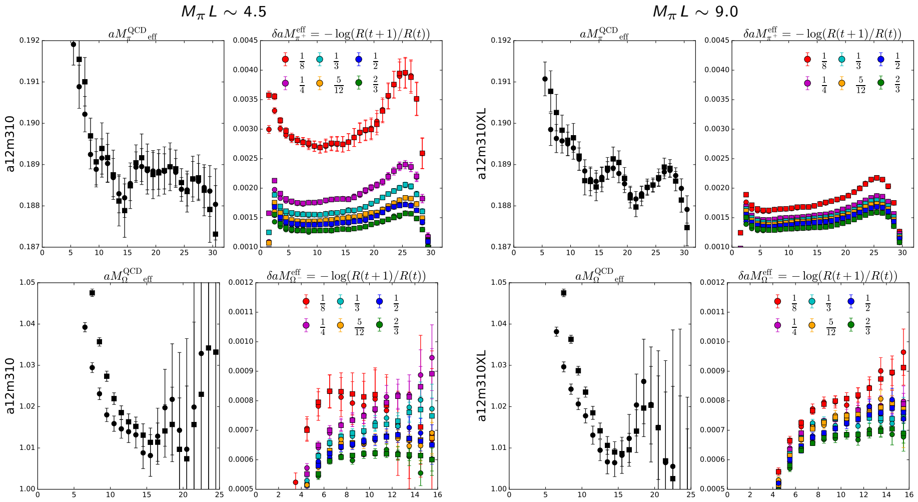
$$C^{\mathrm{QCD+QED}}(t; m_\gamma) = \sum_i (A_i + \delta A_i) \left(e^{-(M_i + \delta M_i)t - xt^2} \right)$$

and consider $R(t; m_\gamma) \equiv e^{xt^2} C^{\mathrm{QCD+QED}}(t) / C^{\mathrm{QCD}}(t)$

$$R(t; m_\gamma) = \frac{\sum_i (A_i + \delta A_i) e^{-(M_i + \delta M_i)t}}{\sum_i A_i e^{-M_i t}} \approx \left(1 + \frac{\delta A_0}{A_0} \right) e^{-\delta M_0 t}$$

$\Rightarrow$ **Directly access mass-splittings δM_i and amplitude-splittings δA_i.**

A first look at the data



- m_γ-dependence ↑ as $M_\pi L$ ↓
- Effect strongest for π^+

Fitting strategies: correlated differences vs combined fits

$$\chi^2 = \sum_i \sum_j (y_i - f(t_i))(\mathrm{cov}^{-1})_{ij}(y_j - f(t_j))$$

Individual fits

1. Fit $C^{\mathrm{QCD}}(t)$: $\Rightarrow M_i^{\mathrm{QCD}}, A_i^{\mathrm{QCD}}$
2. Fit $C^{\mathrm{QED}}(t)$: x$\Rightarrow M_i^{\mathrm{QED}}, A_i^{\mathrm{QED}}$
3. δM_i and δA_i from correlated differences of fit results.

$$y_j = C(t_j)$$

$$f(t_j) = \sum_{i=0}^{1} A_i^2 \left(e^{-M_i t_j - x t_j^2}\right)$$

✗ No cross correlations

Combined fits

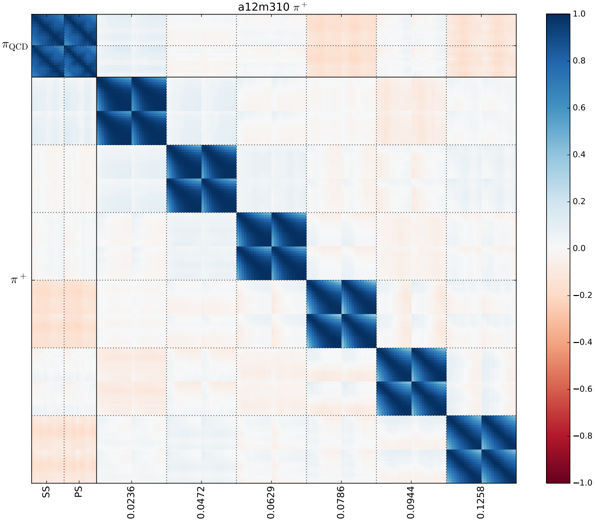
1. **Simultaneously fit** $C^{\mathrm{QCD}}(t)$ and $R \equiv C^{\mathrm{QED}}/C^{\mathrm{QCD}}$

$$y_j \begin{cases} C^{\mathrm{QCD}}(t_j) & j < N \\ R(t_j; m_\gamma) & j \geq N \end{cases}$$

$$f(t_j) \begin{cases} \sum_{i=0}^{1} A_i^2 \left(e^{-M_i t_j}\right) \\ \dfrac{\sum_{i=0}^{1} (A_i + \delta A_i)^2 \left(e^{-(M_i + \delta M_i) t_j - x t_j^2}\right)}{\sum_{i=0}^{1} A_i^2 \left(e^{-M_i t_j}\right)} \end{cases}$$

✓ Includes all correlations

Example for a correlation matrix for the combined fit

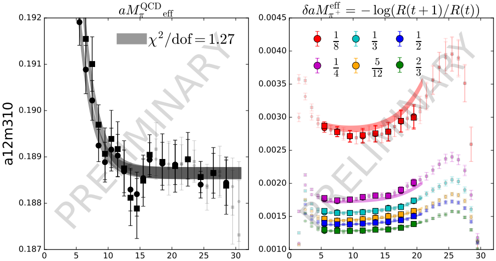


The blocks from top-left to bottom-right are

- $C^{SS}_{\rm QCD}(t)$
- $C^{SP}_{\rm QCD}(t)$
- $C^{SS}_{{\rm QED,m}^1_\gamma}/C^{SS}_{\rm QCD}(t)$
- $C^{SP}_{{\rm QED,m}^1_\gamma}/C^{SP}_{\rm QCD}(t)$
- $C^{SS}_{{\rm QED,m}^2_\gamma}/C^{SS}_{\rm QCD}(t)$
- $C^{SP}_{{\rm QED,m}^2_\gamma}/C^{SP}_{\rm QCD}(t)$
- ...

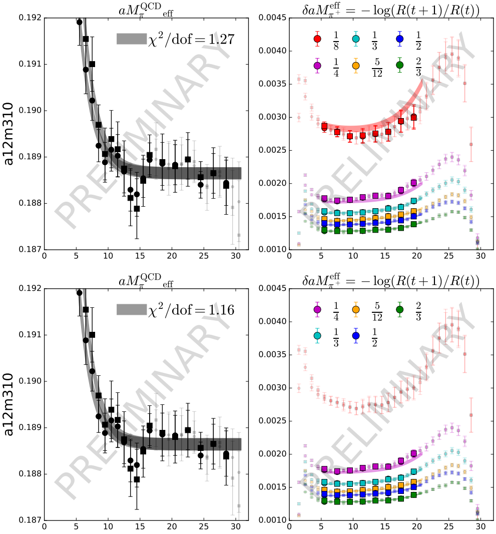
Significantly reduces correlations!

Correlator fit results - π^+ on a12m310



- Simultaneous correlated 2-state fits to two point functions and ratios work well ✓

Correlator fit results - π^+ on a12m310



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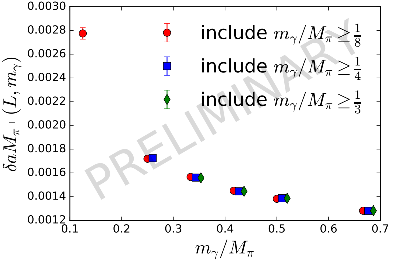
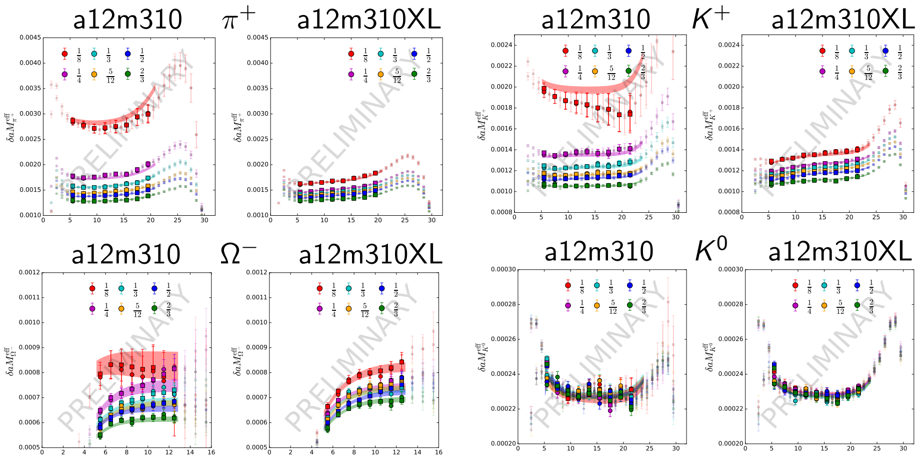


- Stable unter removal of lightest m_γ

Mass splittings - a12m310 vs a12m310XL



- Clear FSE in smallest m_γ on a12m310 (worst for pions)
- Neutral splitting order of magnitude smaller and $\approx m_\gamma$-independent.

Finite Volume corrections

Recall: Need to take the infinite volume limit before the $m_\gamma \to 0$ limit!

Guided by EFT[1507.08916, previous talk by A. Shindler]

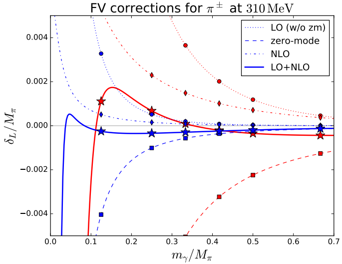
$$M(\infty) = M(L) - \delta_L M^{LO} - \delta_L M^{NLO}$$

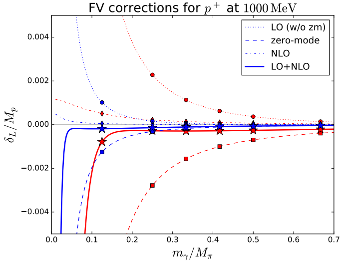
where

$$\frac{\delta_L M^{LO}}{M} = 2\pi\alpha Q^2 \frac{m_\gamma}{M}\left[\mathcal{I}_1(m_\gamma L) - \frac{1}{(m_\gamma L)^3}\right]$$

$$\frac{\delta_L M^{NLO}}{M} = \pi\alpha Q^2 \frac{m_\gamma^2}{M^2}\left[2\mathcal{I}_{1/2}(m_\gamma L) + \mathcal{I}_{3/2}(m_\gamma L)\right]$$

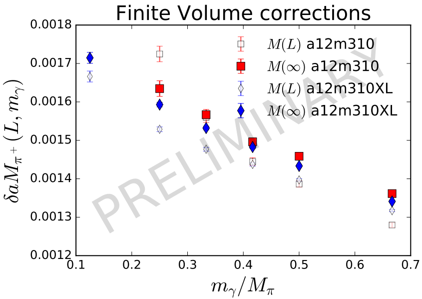
- ⇒ Only for charged hadrons ($\propto Q^2$)
- ⇒ Exponential FSE
- ⇒ Sizable for small $m_\gamma L$ and M:
  e.g. $m_\gamma/M_\pi = 1/8$ on a12m310 too small

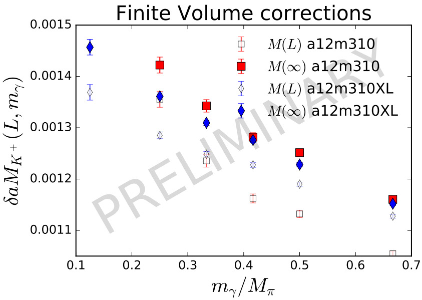




(a12m310, a12m310XL)

Taking the limit $L \to \infty$





- Finite volume corrections controlled for $m_\gamma/M_\pi \gtrsim 1/3$
- Monotonous behaviour with m_γ
- Little curvature

$\Rightarrow$ Need to extrapolate to $m_\gamma = 0$

Zero photon mass limit

Remains to take $m_\gamma \to 0$ limit - again guided by EFT [1507.08916]

$$M(0) = M(m_\gamma) - \Delta_\gamma M^{LO} - \Delta_\gamma^{NLO} + \mathcal{O}\left(\frac{m_\gamma^3}{M^2}\right)$$

$$\frac{\Delta_\gamma M^{LO}}{M} = -\frac{\alpha}{2} Q^2 \frac{m_\gamma}{M}$$

$$\frac{\Delta_\gamma M^{NLO}}{M} = \left(C\alpha - \frac{\alpha}{4\pi} Q^2\right) \frac{m_\gamma^2}{M^2}.$$

- Two free parameters: $M(m_\gamma = 0)$, C.
- Linear term has no free parameter and is absent for neutral hadrons

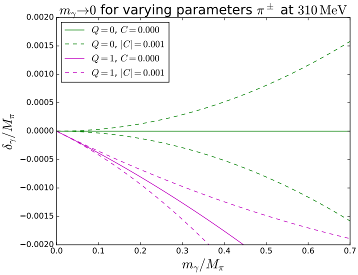
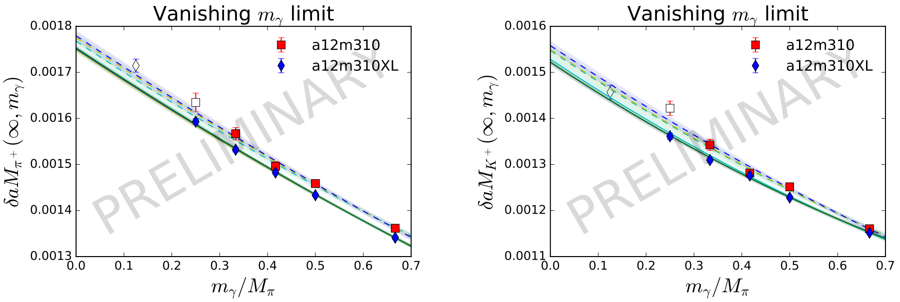


illustration of impact of parameters

Taking the limit $m_\gamma \to 0$



- Different coloured lines: different $m_\gamma^{\min}$
- Extrapolation very well controlled!
- Residual difference between two ensembles very small
  - Investigate systematic from correlator fit?
  - QCD FSE?

Broader plan beyond this feasibility study

Data generation

Have data for 3 more ensembles:

- a15m310L ($M_\pi L \sim 5.7$)
- a09m310 ($M_\pi L \sim 4.5$)
- a15m220 ($M_\pi L \sim 4.0$)

In 2021/22 allocation:

- a15m260 ($M_\pi L \sim 4.7$)
- a12m260 ($M_\pi L \sim 5.1$)
- a09m260 ($M_\pi L \sim 5.7$)
- a12m220XL ($M_\pi L \sim 6.4$)
- strong IB corrections
- reweight QED into sea [1202.6018]

Observables

- QCD+QED spectrum for mesons and baryon octet + decouplet
- quark masses
- Ω^- effect on scale setting
- $\Lambda - \Sigma$ mixing
- charged two hadron scattering

Next steps

- Continuum limit at $M_\pi \sim 310\,\mathrm{MeV}$
- Full chiral continuum limit

Conclusions

- QED_M provides **theoretically sound framework** to include QED
- QED_M **feasible on existing gauge configurations** with $M_\pi L \gtrsim 4$
- QED_M **feasible for**

$$\frac{1}{3} \lesssim \frac{m_\gamma}{M_\pi} \lesssim \frac{2}{3}$$

Conclusions

- Zero-mode treatment ✓
- Control infinite volume extrapolation ✓
- Control limit of zero-photon mass ✓

Outlook

- Continuum limit and chiral extrapolation for many observables
- Include sIB corrections and *un-quench* QED via reweighting

Draft for first paper in advanced state