Topic on Pion Spectrum

Low Energy Constant Fitting
And
Mixed-Action Effect

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CONTENT

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1. Motivation

1.1 Quark mass dependence of m_{PS}, f_{PS}

$SU(2) NLO$ formulae

$$M_\pi^2 = M^2\{1 - \frac{1}{2}x \ln\frac{\Lambda_3^2}{M^2} + \mathcal{O}(x^2)\}$$

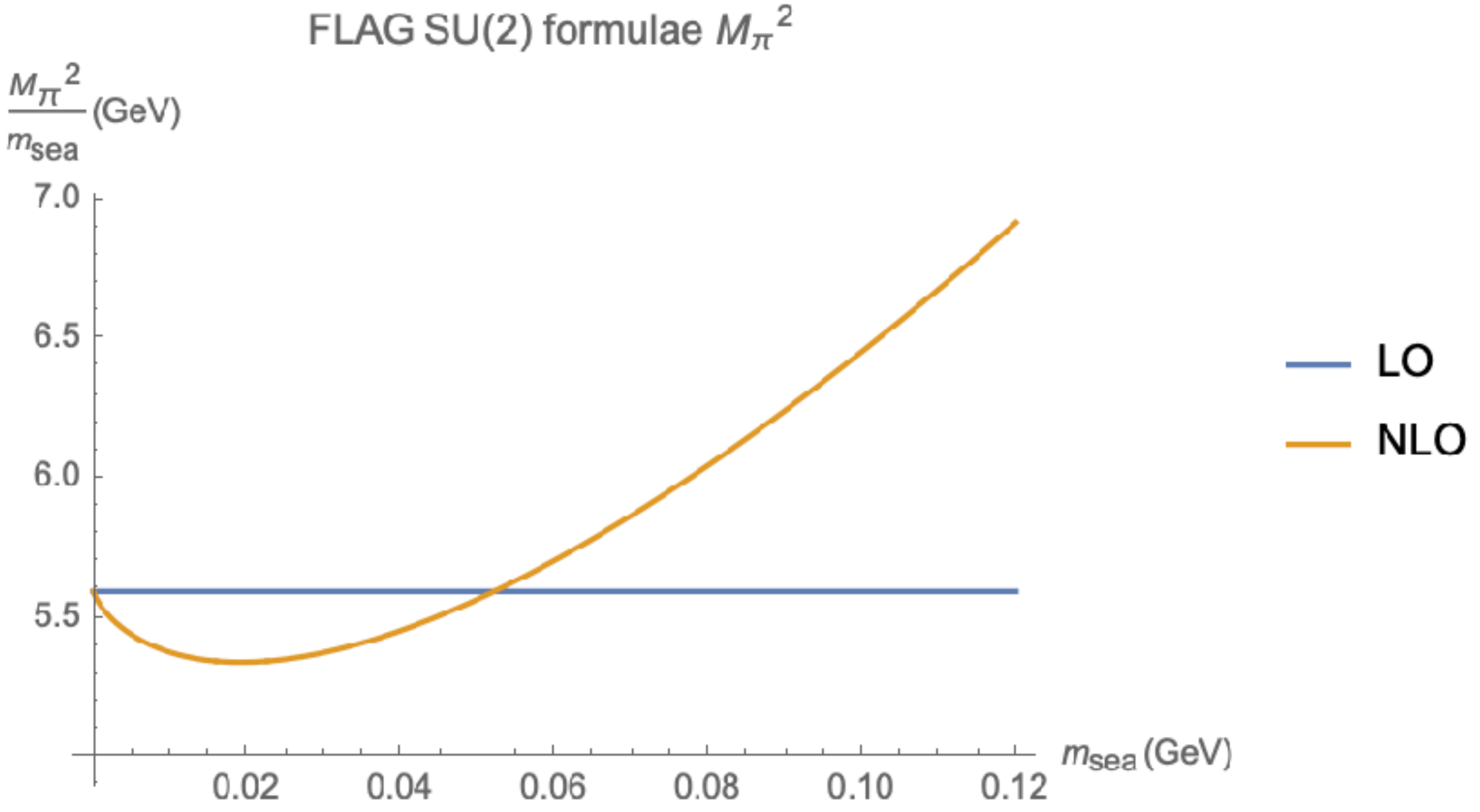
$$F_\pi = F\{1 + x\ln\frac{\Lambda_4^2}{M^2} + \mathcal{O}(x^2)\}$$

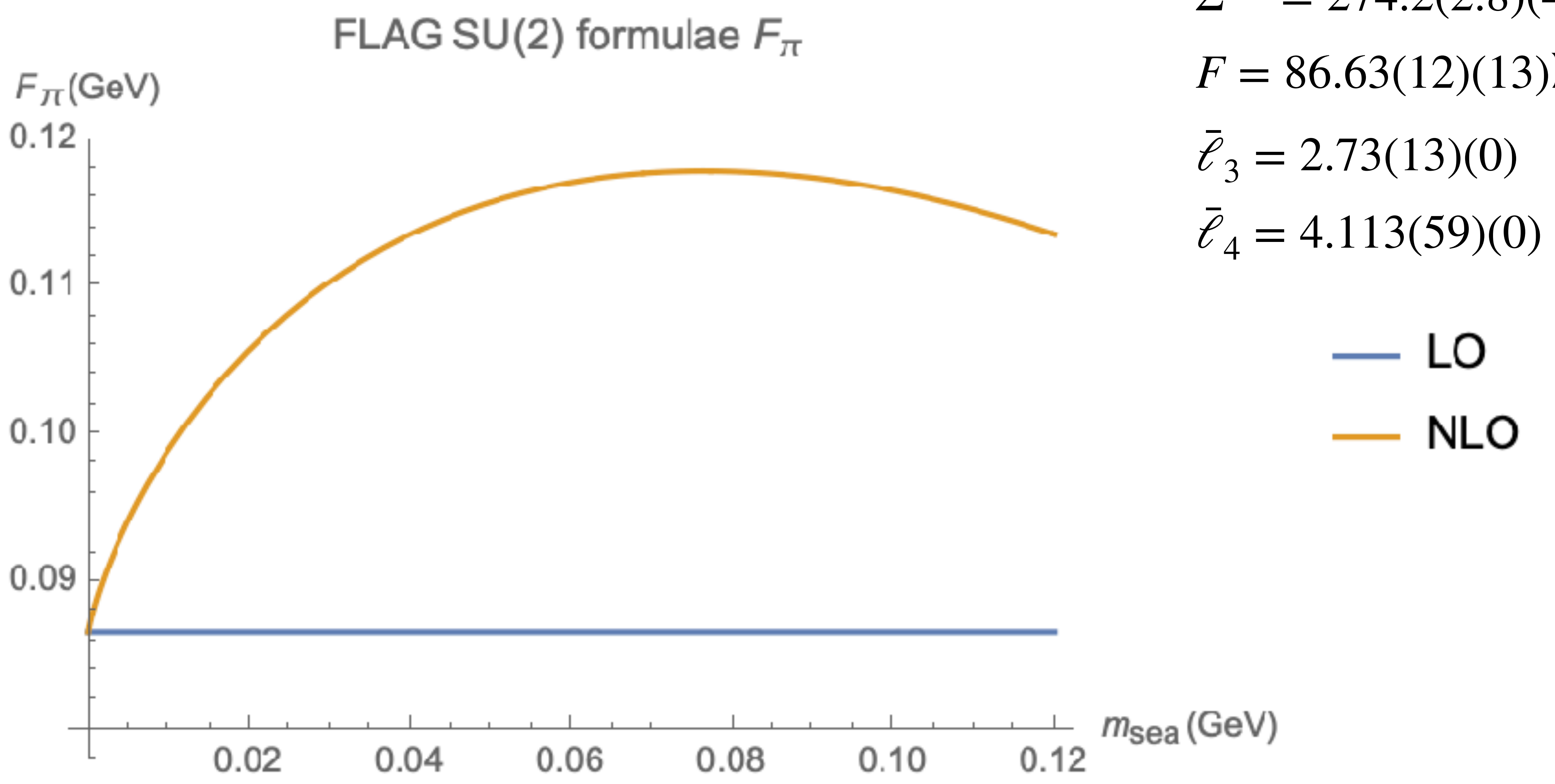
Where

$$x = \frac{M^2}{(4\pi F)^2}$$

$$M^2 = \frac{2\Sigma m}{F^2} \propto m$$

$$\bar{\ell}_n = \ln\frac{\Lambda_n^2}{M^2_{\pi,\mathrm{phys}}}$$





$\Sigma^{1/3} = 274.2(2.8)(4.0)\mathrm{MeV}$

$F = 86.63(12)(13)\mathrm{MeV}$

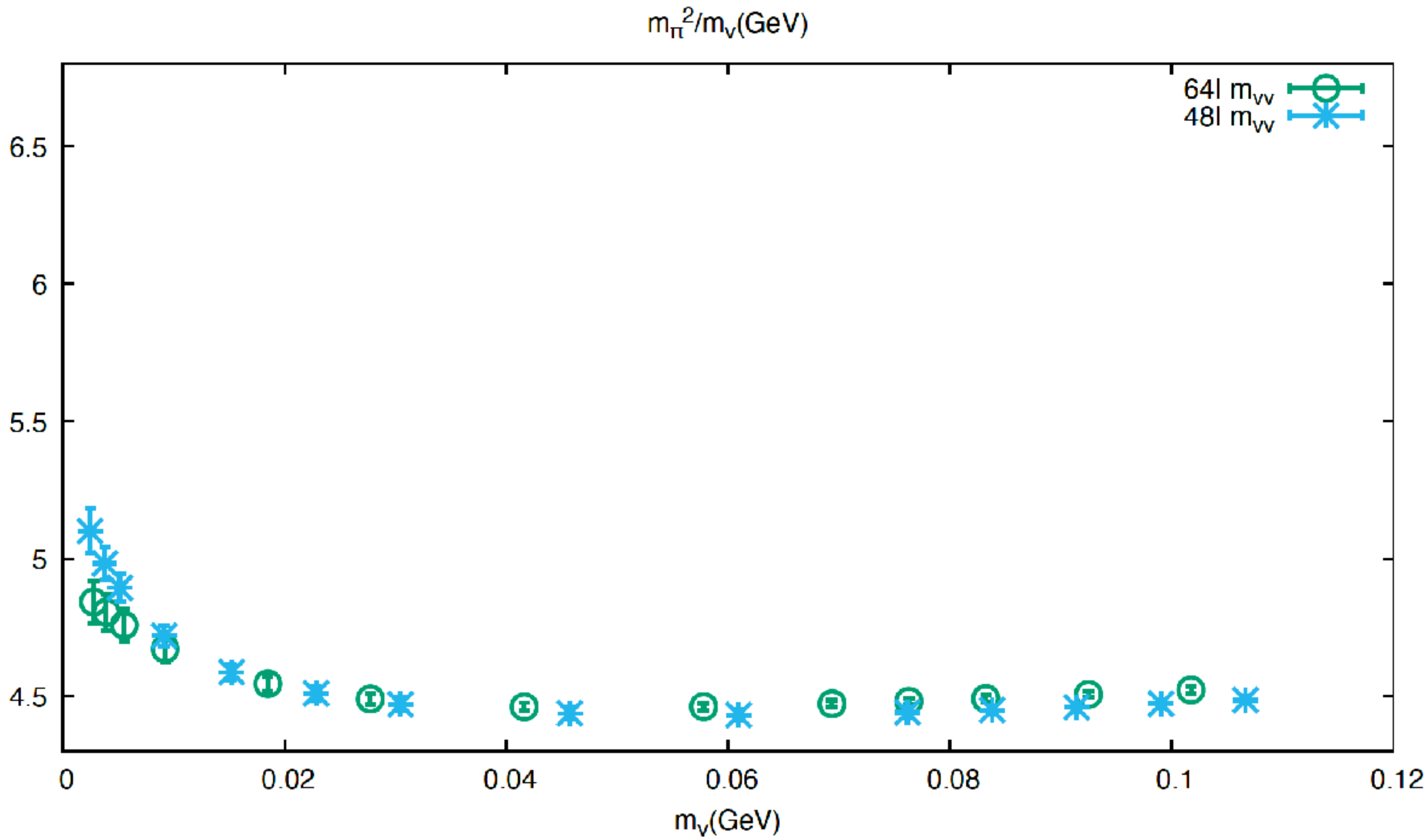
$\bar{\ell}_3 = 2.73(13)(0)$

$\bar{\ell}_4 = 4.113(59)(0)$

1. Motivation

Val_Val pion mass: $m_{\pi,vv}$
Sea_Sea pion mass: $m_{\pi,ss}$
Val_Sea pion mass: $m_{\pi,vs}$
Sea quark mass: m_s
Val quark mass: m_v

1.2 Mixed-action effect



Overlap Fermion Sea

↓

Overlap Valence+Other Sea

↓

Correlator formed by Valence+Sea can suffer from discretization error

↓

Discretization error may be related to the lattice spacing

1. Motivation

Val_Val pion mass: $m_{\pi,vv}$

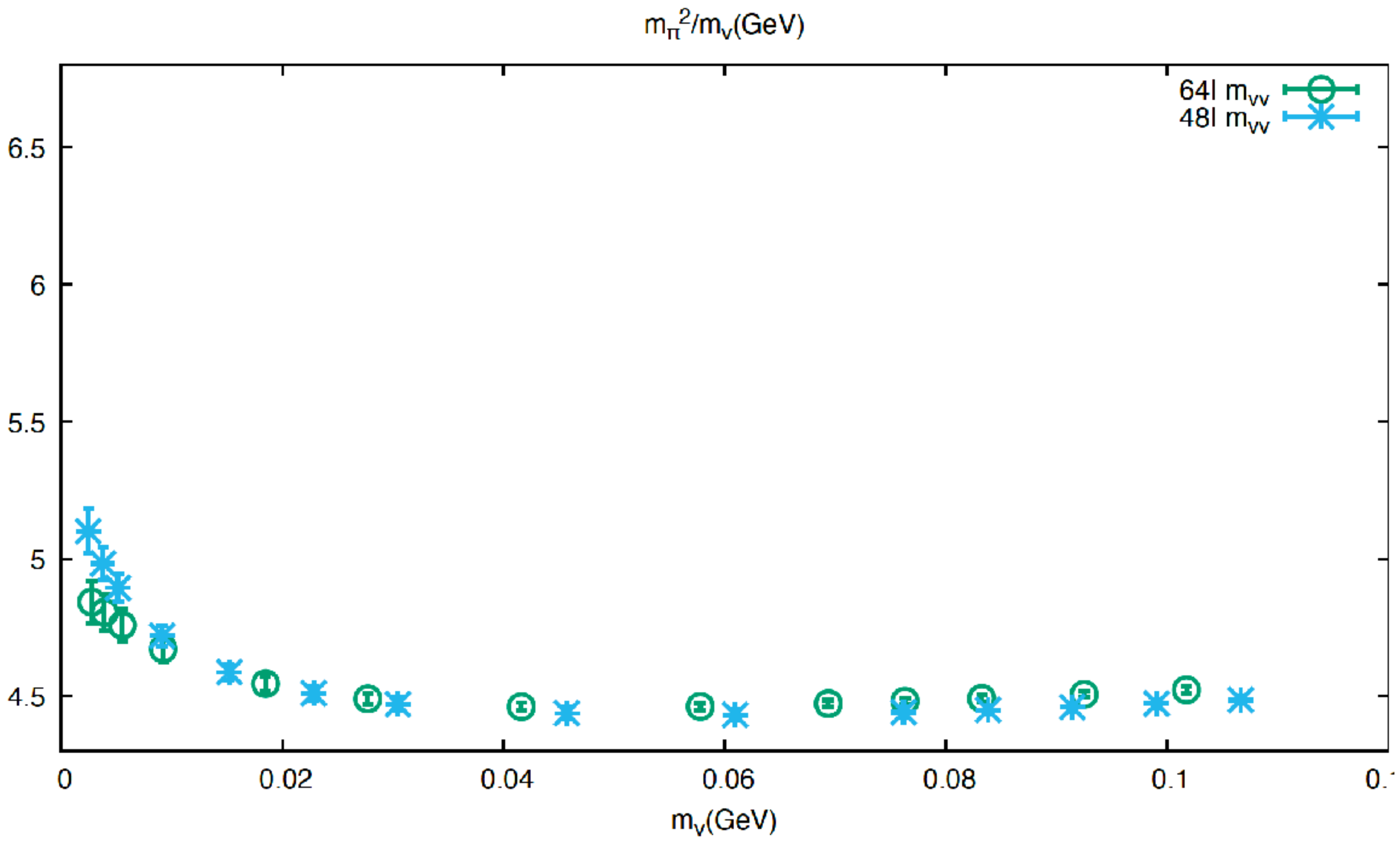
Sea_Sea pion mass: $m_{\pi,ss}$

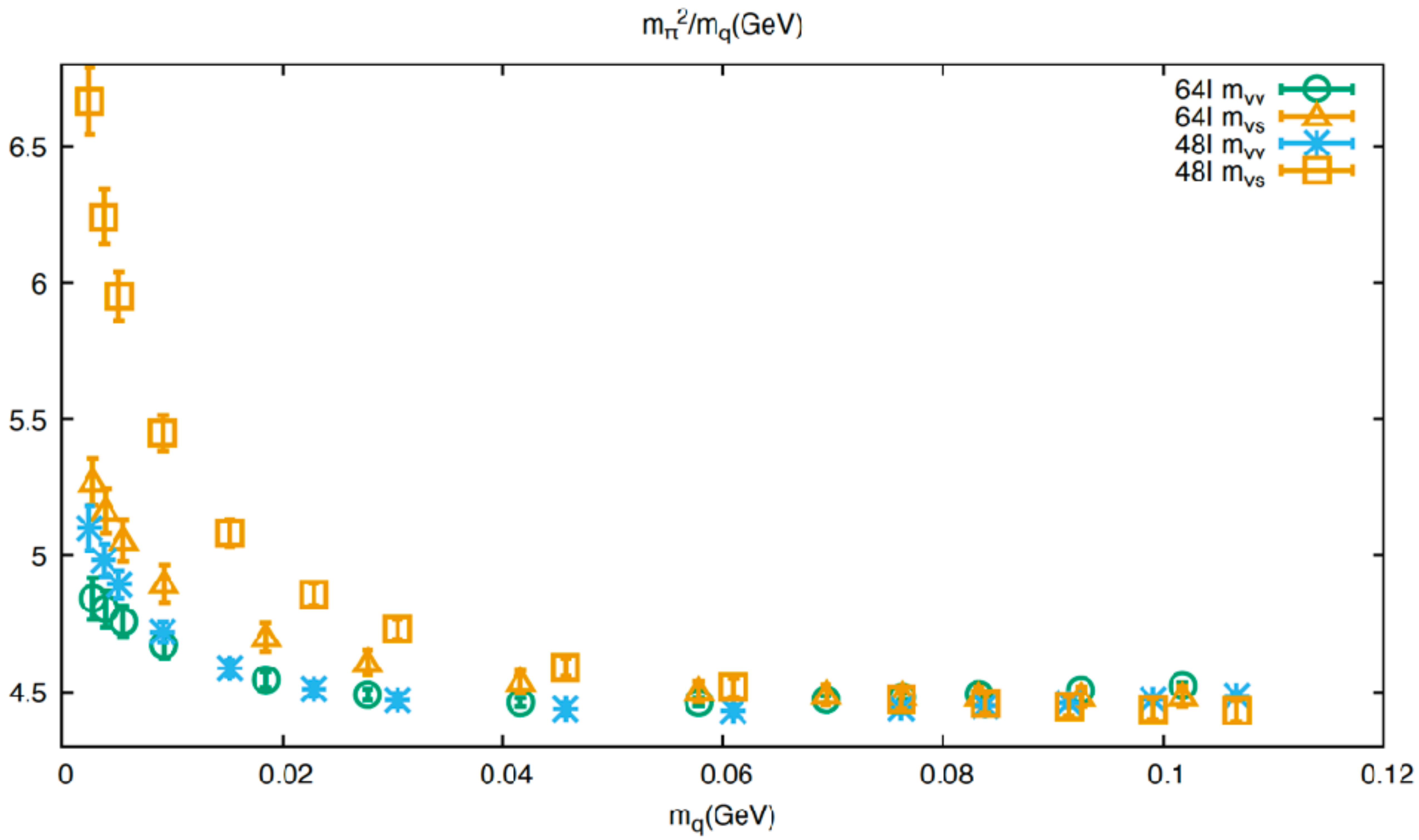
Val_Sea pion mass: $m_{\pi,vs}$

Sea quark mass: m_s

Val quark mass: m_v

1.2 Mixed-action effect





1. Motivation

Val_Val pion mass: $m_{\pi,vv}$
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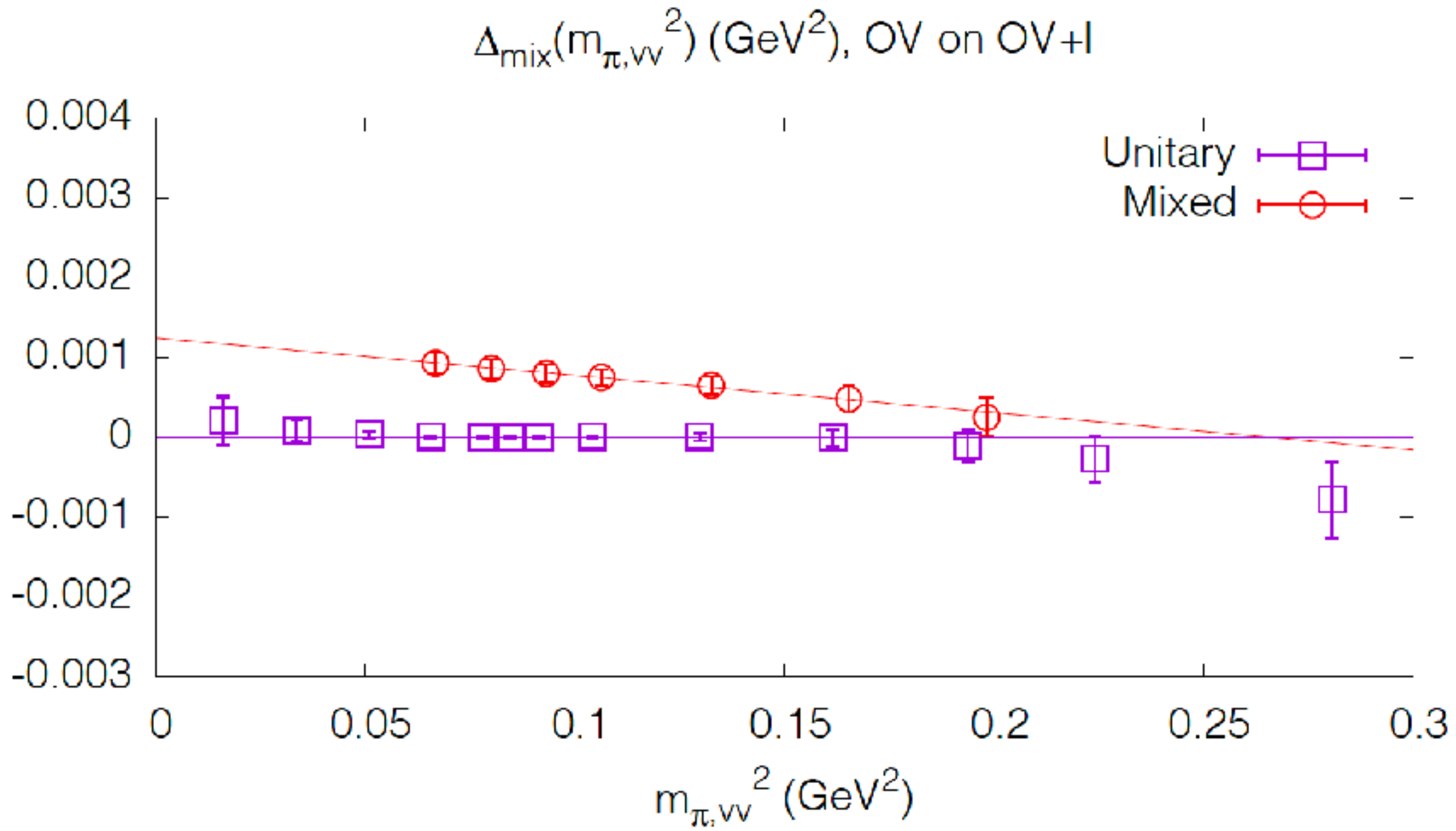
Sea quark mass: m_s
Val quark mass: m_v

1.2 Mixed-action effect

$$MAPQ\chi PT \qquad \Delta_{mix} = m_{\pi,vs}^2 - \frac{m_{\pi,vv}^2 + m_{\pi,ss}^2}{2}$$

Action	Symbol	$L^3 \times T$	a(fm)	$m_{\pi,ss}$(MeV)
OV+I	JLQCD	24, 48	0.112	290

Valence	Sea	$\Delta_{mix}(\text{GeV}^4 a^2)$	a(fm)	$m_{\pi,ss}$(GeV)	
Overlap	Clover	0.35(14)	0.09	0.19	arXiv: 0710.4769
Overlap	Clover	0.55(23)	0.09	0.30	
DW	Staggered	0.25(01)	0.13		arXiv: 0705.0572
DW	Staggered	0.21(02)	0.12		arXiv: 0803.0129
DW	Staggered	0.17(04)	0.09		
Overlap	DW	0.03(01)	0.11/0.08	0.30~0.40	arXiv: 1204.6256



$$\Delta_{mix}(m_{\pi,vv}, m_{\pi,ss}, a) = \Delta_{mix}^{(0)}(m_{\pi,ss}, a) + \Delta_{mix}^{(1)}(m_{\pi,ss}, a) m_{\pi,vv}^2$$

1. Low Energy Constant Fitting

$SU(3)NLO \quad PQ\chi PT$

1.3 Continuum fitting procedure

(1) Pseudoscalar Masses

Data points obtained from simulated quark→Low energy constants fitting→Continuous limit extrapolation

Main fitting function

$$M_{AB}^2 = \frac{\chi_A + \chi_B}{2}(1 + \delta_{tree}^M + \delta_{loop}^M)$$

Low energy constant to be fitted

$$\Sigma$$

$$f = f_\pi |_{m_u, m_d, m_s \to 0}$$

$$2L_6 - L_4$$

$$2L_8 - L_5$$

Other definitions

$$\chi_i = \frac{2\Sigma m_i}{f^2} \quad i = 1,2,3 \text{ sea quark (3 is strange)}, \quad i = A, B \text{ valence quark}$$

$$\delta_{tree}^M = \frac{8N}{f^2}\alpha_1 \bar{\chi} + \frac{4}{f^2}\alpha_2(\chi_A + \chi_B)$$

$$\chi_\pi + \chi_\eta = \chi_1 + \chi_2 + \chi_3 - \bar{\chi}$$

$$\delta_{loop}^M = \frac{1}{(4\pi f)^2 N} \sum_{i=A,B,\pi,\eta} (R_i \chi_i \log \chi_i)$$

$$\chi_\pi \chi_\eta = \chi_1 \chi_2 \chi_3 \overline{\chi^{-1}}$$

$$R_A = \frac{(\chi_A - \chi_1)(\chi_A - \chi_2)(\chi_A - \chi_3)}{(\chi_A - \chi_B)(\chi_A - \chi_\pi)(\chi_A - \chi_\eta)}$$

$$\bar{\chi} = \frac{1}{N}\sum_{i=1}^{3} \chi_i$$

$$R_\eta = \frac{(\chi_\eta - \chi_1)(\chi_\eta - \chi_2)(\chi_\eta - \chi_3)}{(\chi_\eta - \chi_A)(\chi_\eta - \chi_B)(\chi_\eta - \chi_\pi)}$$

$$\overline{\chi^{-1}} = \frac{1}{N}\sum_{i=1}^{3} \chi_i^{-1}$$

1. Low Energy Constant Fitting

1.3 Continuum fitting procedure

(2) Pseudoscalar Decay Constants

Data points obtained from simulated quark→Low energy constants fitting→Continuous limit extrapolation

Main fitting function

$$f_{AB} = f(1 + \delta^f_{tree} + \delta^f_{vs,loop} + \delta^f_{vv,loop})$$

Low energy constant to be fitted

$\Sigma \quad L_4 \quad L_5$

$f = f_\pi|_{m_u,m_d,m_s \to 0}$

Other definitions

$$\delta^f_{tree} = \frac{4N}{f^2}\bar{\chi}L_4 + \frac{2}{f^2}(\chi_A + \chi_B)L_5$$

$$\delta^f_{vs,loop} = -\sum_{i=1}^{3}\frac{1}{(4\pi f)^2}\frac{\chi_A+\chi_i}{8}\log(\frac{\chi_A+\chi_i}{2}) + (A \leftrightarrow B)$$

$$D_A = \frac{\prod_{i=1}^3(\chi_i - \chi_A)}{(\chi_\pi - \chi_A)(\chi_\eta - \chi_A)}$$

$$D_B = \frac{\prod_{i=1}^3(\chi_i - \chi_B)}{(\chi_\pi - \chi_B)(\chi_\eta - \chi_B)}$$

$$\delta^f_{vv,loop} = \frac{1}{(4\pi f)^2 N}\{-D_A - D_B + \frac{\log(\chi_A/\chi_B)}{(\chi_A - \chi_B)}[\chi_A D_A + \chi_B D_B + (\chi_A - \chi_B)^2] + (\chi_\pi R_\pi(\chi_B - \chi_A)[\frac{\log(\chi_\pi/\chi_A)}{\chi_A - \chi_\pi} - \frac{\log(\chi_\pi/\chi_B)}{\chi_B - \chi_\pi}] + (\pi \leftrightarrow \eta))\}$$

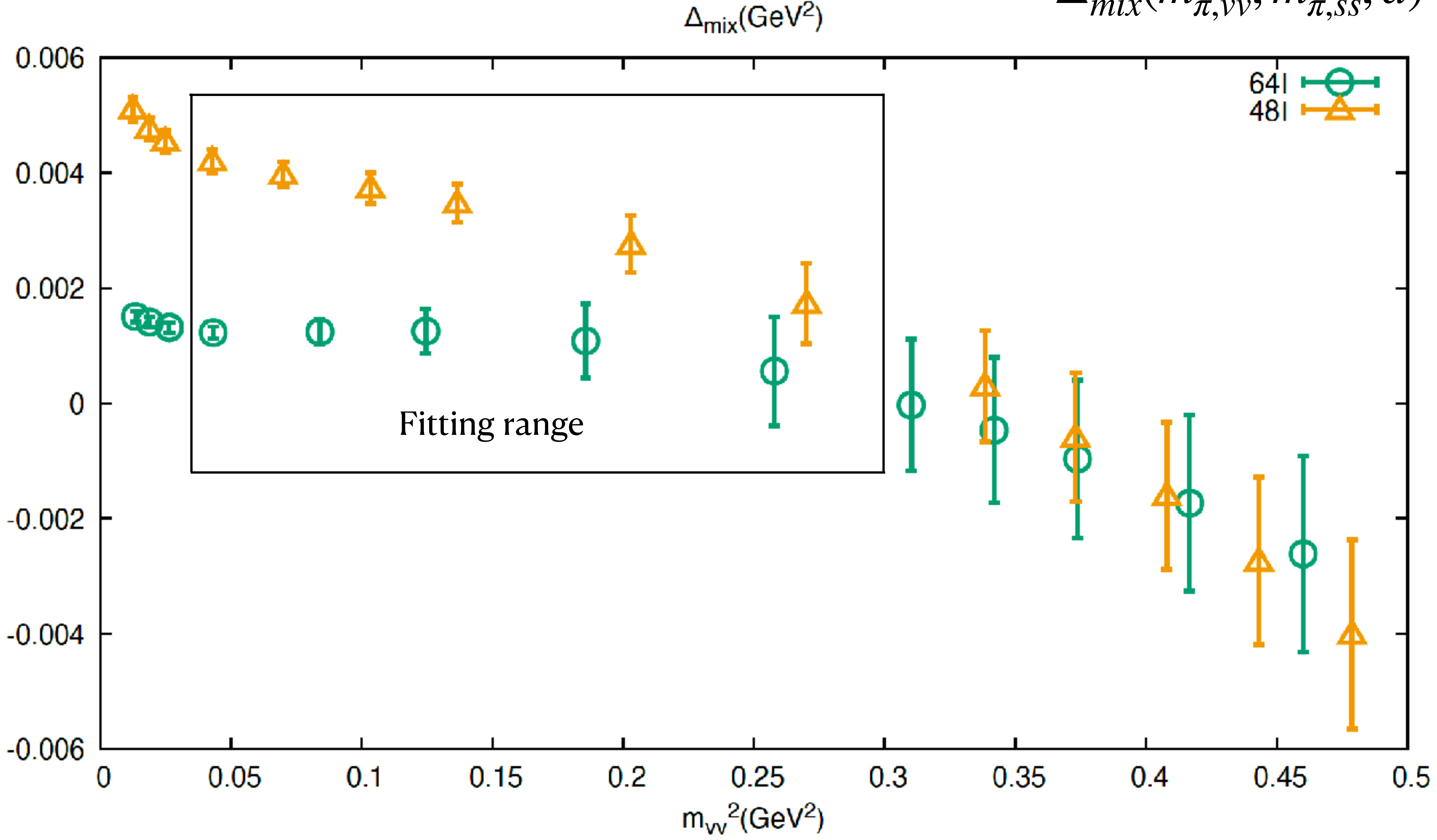
2. Numerical Setup

arXiv:hep-lat/1411.7017

Action	**Symbol**	$L^3 \times T$	$a(\text{fm})$	$m_{\pi,ss}(\text{MeV})$
DW+I	48I	$48^3 \times 96$	0.114	139
DW+I	64I	$64^3 \times 128$	0.084	139

3.1 Result: Mixed-Action Effect

$$\Delta_{mix}(m_{\pi,vv}, m_{\pi,ss}, a) = \Delta^{(0)}_{mix}(m_{\pi,ss}, a) + \Delta^{(1)}_{mix}(m_{\pi,ss}, a)m^2_{\pi,vv}$$



$$\Delta^{48I}_{mix} = 0.00504(11) - 0.0141(11)m^2_{vv}$$

$$\Delta^{64I}_{mix} = 0.00151(6) - 0.0052(13)m^2_{vv}$$

3.1 Result: Mixed-Action Effect

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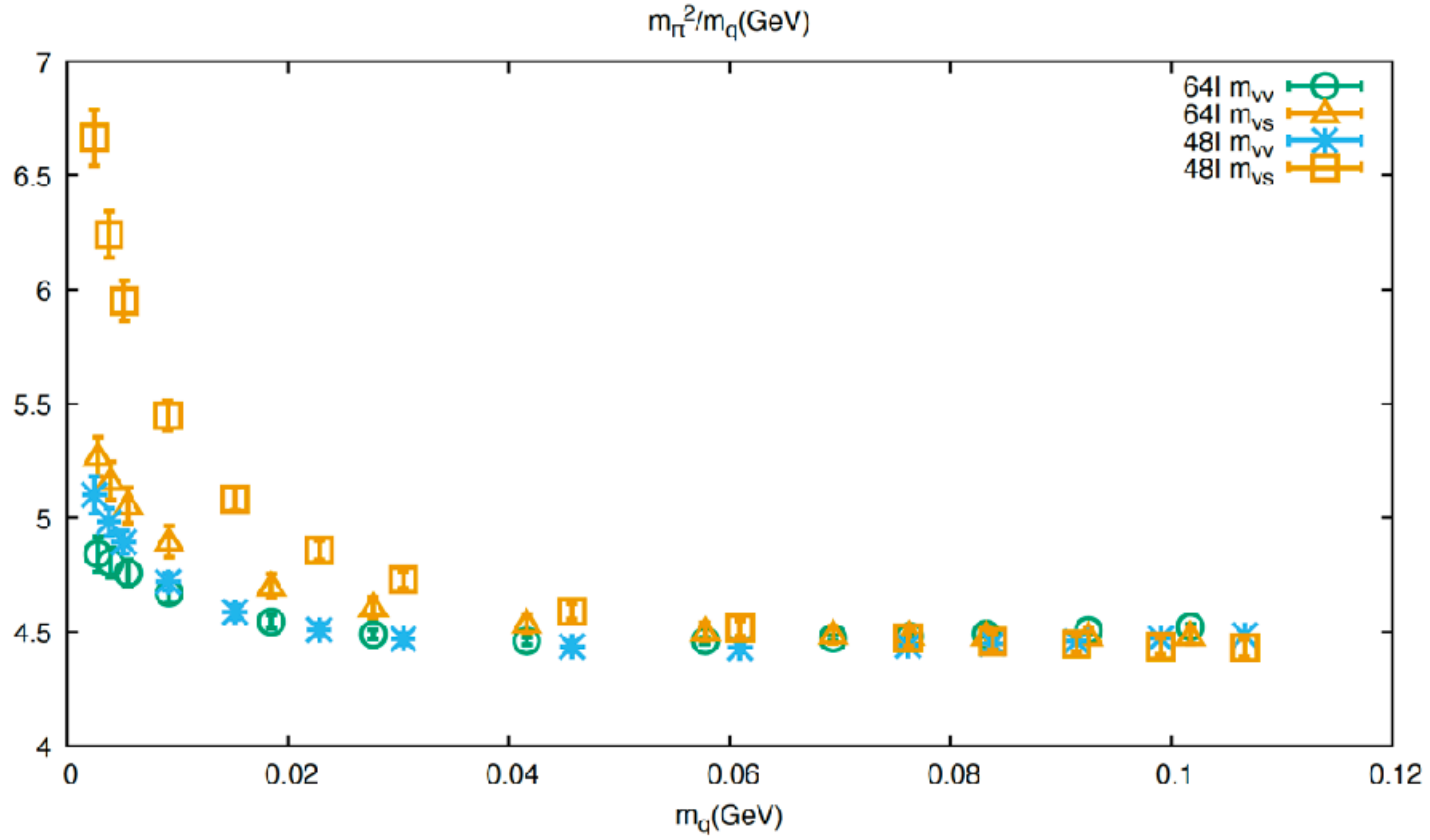
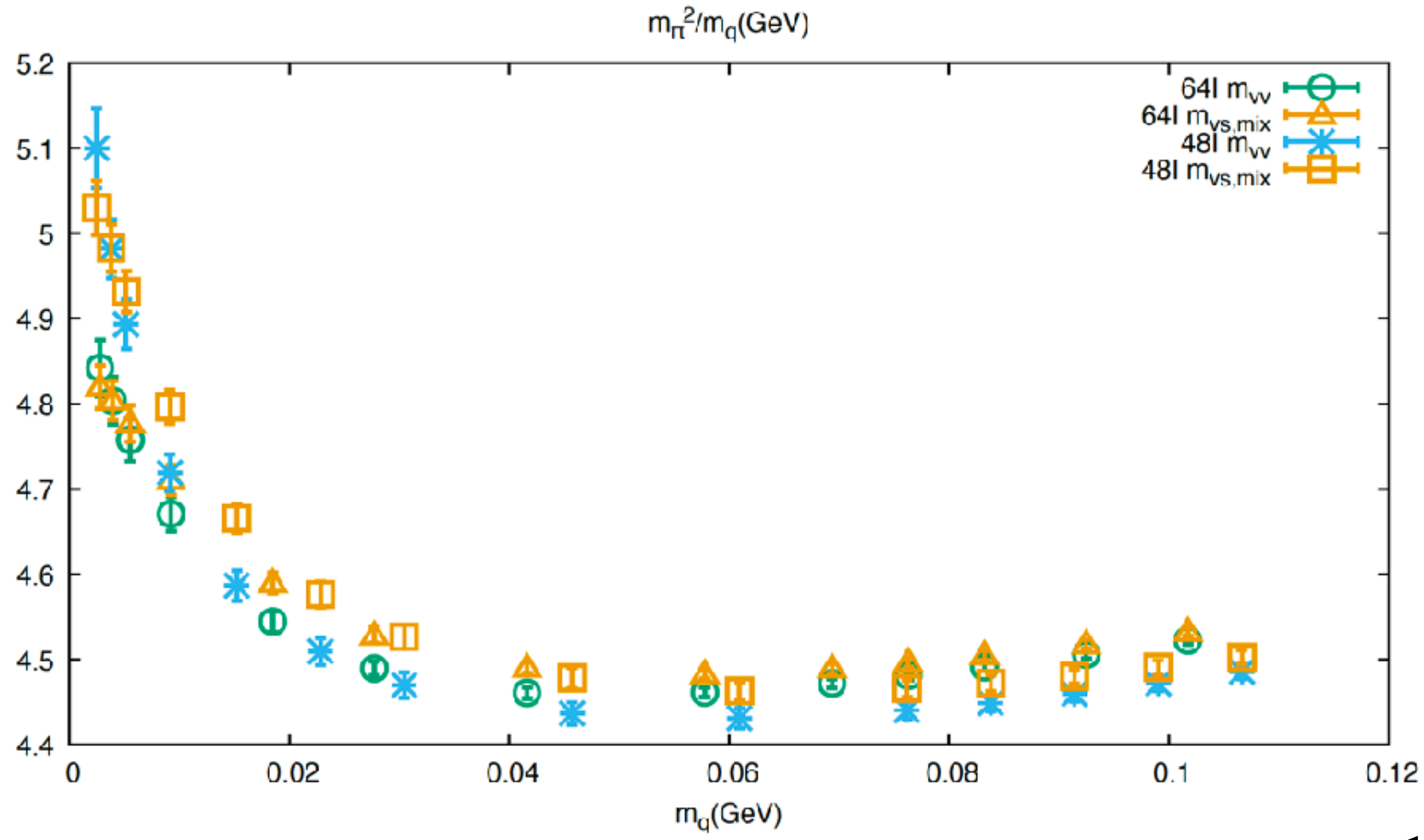
$$\Delta^{48I}_{mix} = 0.00504(11) - 0.0141(11) m^2_{vv}$$

$$\Delta^{64I}_{mix} = 0.00151(6) - 0.0052(13) m^2_{vv}$$

$$m^2_{\pi,vs,mix} = m^2_{\pi,vs} - \Delta_{mix}$$





3.2 Result: Pseudoscalar Decay Constant

Local axial current

(1) $i\langle 0|A_\mu(x)|PS(p)\rangle = f_{PS} p_\mu e^{ip\cdot x}$

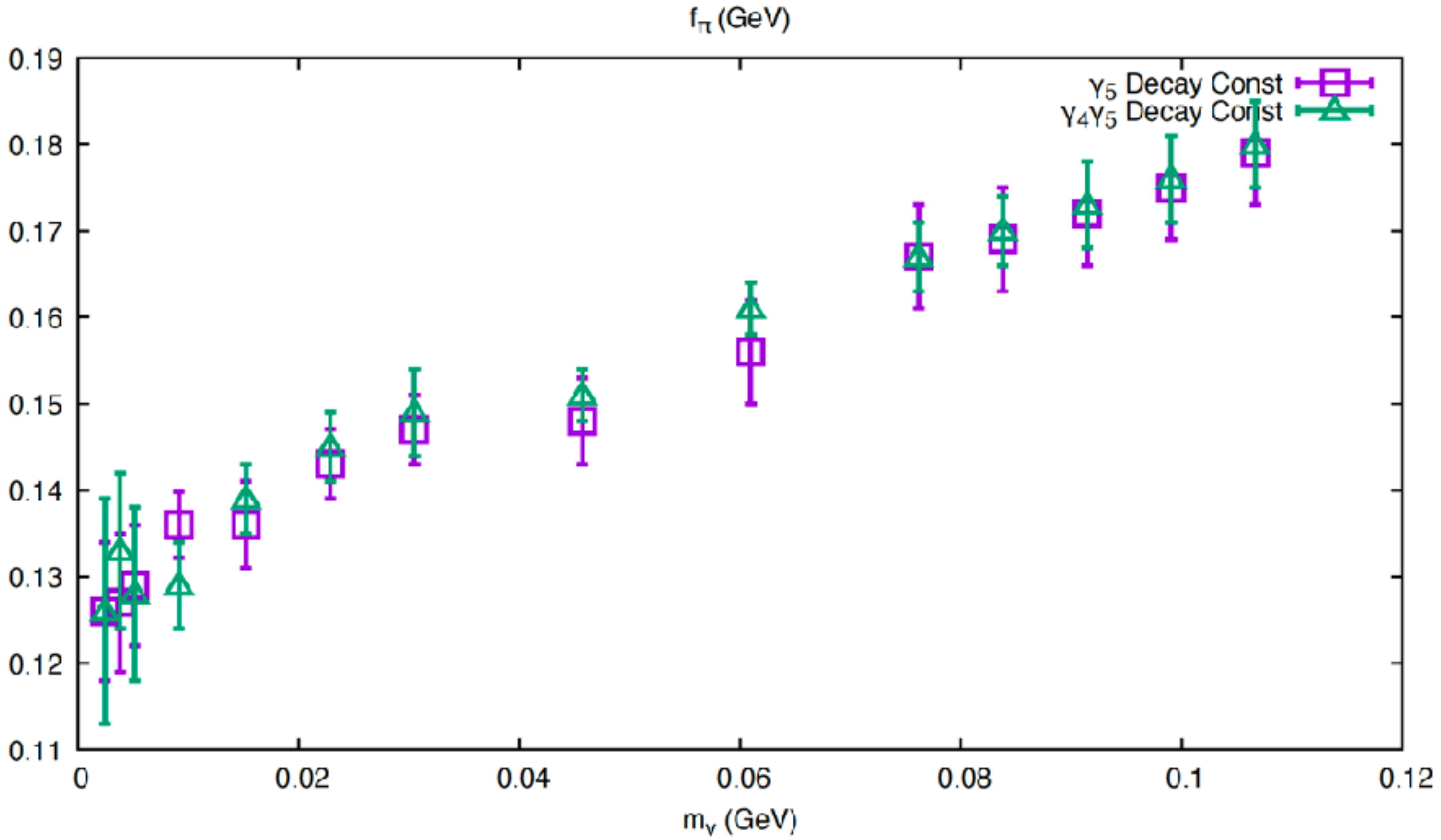
Meson Field

(2) $\partial A_\mu = i\frac{\partial}{\partial x_\mu}\bar{\psi}\gamma_\mu\gamma_5\psi$ $\rightarrow$

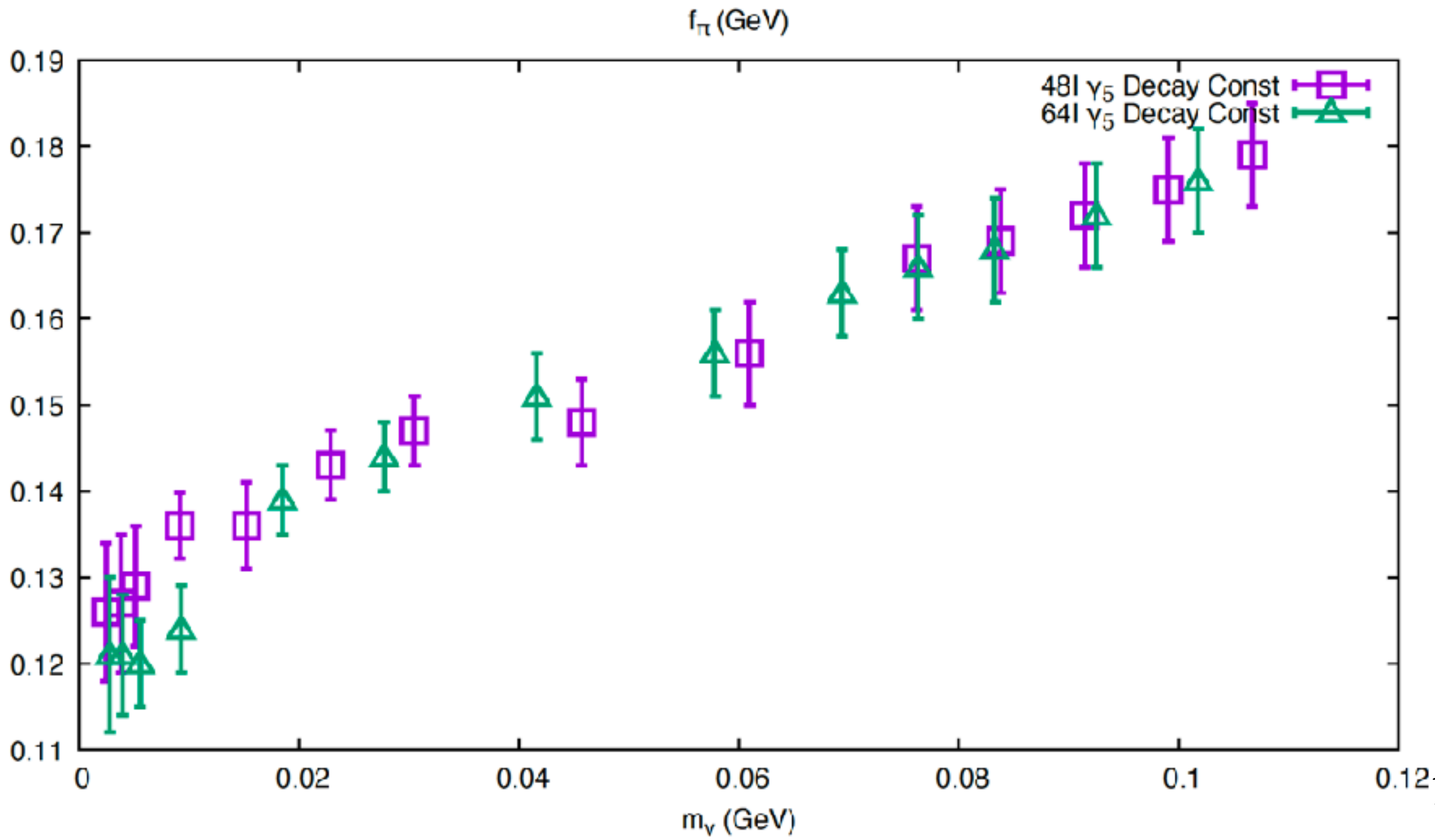
(3) $(m_{q_1}+m_{q_2})\langle 0|\bar{\psi}\gamma_5\psi|PS\rangle = M_{PS}^2 f_{PS}$

(4) $Z_A\langle 0|\bar{\psi}\gamma_4\gamma_5\psi|PS\rangle = M_{PS} f_{PS}$

$48I\, f_{\pi,vv}$



$48I\&64I\, f_{\pi,vv}$ $\quad N_{cfg} = 30$



4. Summary And Outlook

1. $m_{PS,vv}$ and $f_{PS,vv}$ have little difference under different ensembles with the same fermion action which can be ignored within the error.
2. $m_{PS,vs}$ get a large discretization error which may have a linear relationship with $m^2_{\pi,vv}$ and may be related to the lattice spacing.
3. Using $m^2_{\pi,vv}$, $f_{\pi,vv}$ and the subtracted $m^2_{\pi,vs,mix}$, we will complete the fitting in the near future.

THANKS FOR YOUR ATTENTION!