The Mixing of η_c and Pseudoscalar Glueball

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Motivation

- Glueball may exist according to QCD
- Lattice Study of glueball
  - C. J. Morningstar and M. J. Peardon, Phys. Rev. D **56** (1997), 4043-4061,[arXiv:hep-lat/9704011 [hep-lat]]
  - Y. Chen, A. Alexandru, S. J. Dong, T. Draper, I. Horvath, F. X. Lee, K. F. Liu, N. Mathur, C. Morningstar and M. Peardon, *et al.* Phys. Rev. D **73** (2006), 014516 doi:10.1103/PhysRevD.73.014516
  - E. Gregory, A. Irving, B. Lucini, C. McNeile, A. Rago, C. Richards and E. Rinaldi, JHEP **10** (2012), 170 doi:10.1007/JHEP10(2012)170
  - W. Sun et al (CLQCD), Chin. Phys. C (in press), arXiv:1702.08174(hep-lat)
- Large total decay width of η_c, $\Gamma_{\eta_c} = 32.0(7)$Mev, it should be surpressed by OZI rule
- Previous study
  - $M_G \sim M_{\eta_c}$
  - Phemenogical Study of Mixing

    Y.-D. Tsai, H.-n. Li, and Q. Zhao, Phys. Rev. D 85, 034002 (2012), arXiv:1110.6235 [hep-ph]

    W. Qin, Q. Zhao, and X.-H. Zhong, Phys. Rev. D 97, 096002 (2018), arXiv:1712.02550 [hep-ph]

Mixing On the Lattice

- $|G_i\rangle \to$ the i-th pure glueball state
  $|(c\bar{c})_i\rangle \to$ the i-th pure $c\bar{c}$ state
  $|\eta_i\rangle \to$ the i-th η_c state dominated by $c\bar{c}$
  $|g_i\rangle \to$ the i-th glueball state dominated by pure glueball
  $\mathcal{O}_G \to$ the pure glueball operator
  $\mathcal{O}_{c\bar{c}} \to$ the pure $c\bar{c}$ operator
- Suppose
  - the mixing exist
  - mixing is slight
  - only consider the mixing of close state

$$H = \begin{pmatrix} m_{G_1} & x_1 \\ x_1 & m_{(cc)_1} \end{pmatrix} \oplus \begin{pmatrix} m_{G_2} & x_2 \\ x_2 & m_{(cc)_2} \end{pmatrix} \oplus \cdots \quad (1)$$

  where $x_i \to 0$. $H|\eta_i\rangle = m_{\eta_i}|\eta_i\rangle$, $H|g_i\rangle = m_{g_i}|g_i\rangle$.
- The mixed eigenstates are related to unmixed eigenstates via mixing angle

$$\begin{bmatrix} |g_i\rangle \\ |\eta_i\rangle \end{bmatrix} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i \\ \sin\theta_i & \cos\theta_i \end{bmatrix} \begin{bmatrix} |G_i\rangle \\ |(c\bar{c})_i\rangle \end{bmatrix} \quad (2)$$

$$\sin\theta_i \approx \pm\frac{x_i}{\Delta_i}, \quad where \quad \Delta_i = m_{(cc)_i} - m_{G_i} \quad (3)$$

- Suppose operators we constructed on the lattice are pure glueball operator and pure $c\bar{c}$ operator, $\mathcal{O}_G$ and $\mathcal{O}_{c\bar{c}}$. $\langle\mathcal{O}_{c\bar{c}}(t)\mathcal{O}^{\dagger}_{c\bar{c}}(0)\rangle$, $\langle\mathcal{O}_G(t)\mathcal{O}^{\dagger}_G(0)\rangle$, $\langle\mathcal{O}_G(t)\mathcal{O}^{\dagger}_{c\bar{c}}(0)\rangle$, $\langle\mathcal{O}_{c\bar{c}}(t)\mathcal{O}^{\dagger}_G(0)\rangle$ should be nonzero.
-

$$
\begin{aligned}
C_{GC}(t) &= \langle\mathcal{O}_G(t)\mathcal{O}^{\dagger}_{c\bar{c}}(0)\rangle \\
&= \langle\mathcal{O}_G(t)\sum_i\{|g_i\rangle\langle g_i| + |\eta_i\rangle\langle\eta_i|\}\mathcal{O}_G(0)\rangle \\
&= \sum_{n,m\neq 0}\sqrt{Z_{G_n}}\sqrt{Z_{(c\bar{c})_m}}\langle G_n|e^{-Ht}\sum_i\{|g_i\rangle\langle g_i| + |\eta_i\rangle\langle\eta_i|\}|(c\bar{c})_m\rangle \\
&= \sum_i\sqrt{Z_{G_i}}\sqrt{Z_{(c\bar{c})_i}}\cos\theta_i\sin\theta_i\{e^{-m_{g_i}t} - e^{-m_{\eta_i}t}\}
\end{aligned} \tag{4}
$$

where we suppose these states with big mass difference don't mix. Consider boundary condition

$$
\begin{aligned}
C_{GC}(t) = \sum_i\sqrt{Z_{G_i}}\sqrt{Z_{(c\bar{c})_i}}\cos\theta_i\sin\theta_i\times \\
\left\{\left(e^{-m_{g_i}t}\pm e^{-m_{g_i}(T-t)}\right) - \left(e^{-m_{\eta_i}t}\pm e^{-m_{\eta_i}(T-t)}\right)\right\}
\end{aligned} \tag{5}
$$

- similar to C_{GC},

$$C_{CC}(t) \approx \sum_i Z_{(c\bar{c})_i}\{e^{-m_{\eta_i}t} + e^{-m_{\eta_i}(T-t)}\} \tag{6}$$

$$C_{GG}(t) \approx \sum_i Z_{G_i}\{e^{-m_{g_i}t} + e^{-m_{g_i}(T-t)}\} \tag{7}$$

- Because of degenerate charm sea quarks, $O_{c\bar{c}} = \frac{1}{\sqrt{2}}(\bar{c}_1\Gamma c_1 + \bar{c}_2\Gamma c_2)$.

$$\langle O_{c\bar{c}}(t)O^\dagger_{c\bar{c}}(0)\rangle = -\langle \mathrm{Tr}(G(t)\Gamma G(0)\Gamma)\rangle + 2\langle \mathrm{Tr}(G(t)\Gamma)\mathrm{Tr}(G(0)\Gamma)\rangle \tag{8}$$

$$\langle O_G(t)O^\dagger_{c\bar{c}}(t)\rangle = -\sqrt{2}\langle \mathrm{Tr}(O_G(t))\mathrm{Tr}(G(0)\Gamma)\rangle \tag{9}$$

Glueball → Big statistics
Disconnected part → Distillation method

Numerical Details

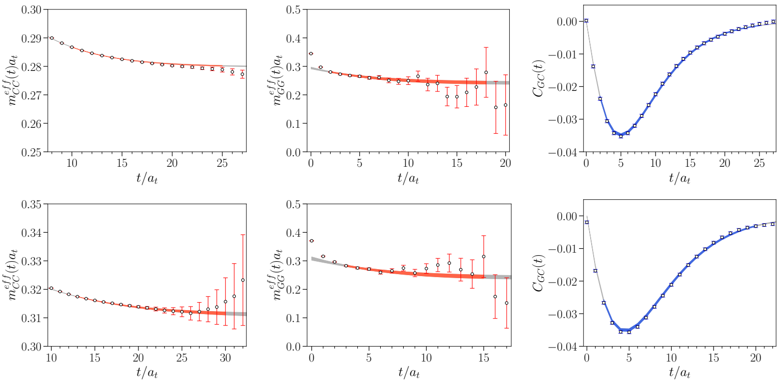
- $N_f = 2$ gauge ensembles with degenerate charm sea quarks

ensemble	$L^3 \times T$	β	a_s(fm)	ξ	N_{cfg}	$m_{J/\psi}$(MeV)
I	$16^3 \times 128$	2.8	0.1026	5	~ 7000	2743
II	$16^3 \times 128$	2.8	0.1026	5	~ 6000	3068

- Charm sea quark are important because mixing can only happen via disconnected diagram
- We use the tadpole improved anisotropic clover fermion action and tadpole improved Morningstar-Peardon gauge action W. Sun, L. C. Gui, Y. Chen, M. Gong, C. Liu, Y. B. Liu, Z. Liu, J. P. Ma and J. B. Zhang, Chin. Phys. C **42** (2018) no.9, 093103
- With distillation method for $(c\bar{c})$
- Two states simultaneous fit
- Use two pseudoscalar $c\bar{c}$
  $O_{(c\bar{c})} = \bar{c}\gamma_5 c$ and $O_{(c\bar{c})} = \bar{c}\gamma_5\gamma_4 c$

$\bar{c}\gamma_5 c$

$$\begin{aligned} C_{GG}(t) &= \sum_i Z_{G_i}\left(e^{-m_{g_i}t}+e^{-m_{g_i}(T-t)}\right) \\ C_{CC}(t) &= \sum_i Z_{(cc)_i}\left(e^{-m_{\eta_i}t}-e^{-m_{\eta_i}(T-t)}\right) \\ C_{GC}(t) &= -\sum_{i=1}^{2}\sqrt{Z_{G_i}Z_{(cc)_i}}\cos\theta_i\sin\theta_i\left(e^{-m_{g_i}t}-e^{-m_{g_i}(T-t)}-(e^{-m_{\eta_i}t}-e^{-m_{\eta_i}(T-t)})\right) \end{aligned} \quad (10)$$



$\bar{c}\gamma_5\gamma_4 c$

- $\partial_\mu J_5^\mu(x) = 2m_c\bar{c}(x)\gamma_5 c(x) + q(x), \;\; q(x) = \frac{g^2}{16\pi^2}\epsilon^{\alpha\beta\rho\sigma}G^a_{\alpha\beta}G^a_{\rho\sigma}$
  $\langle 0|J_5^\mu(x)|G_i, p\rangle = if_{G_i}p^\mu e^{-ip\cdot x}$
  $\langle 0|\partial_\mu J_5^\mu|G_i\rangle \approx \langle 0|q(x)|G_i\rangle$
  $\langle 0|\partial_\mu J_5^\mu(0)|G_i, \mathbf{p}=0\rangle = m^2_{G_i}f_{G_i}$
- $\mathcal{O}_{\gamma_5\gamma_4}$ can couple to glueballs:

$$\langle 0|\mathcal{O}_{\gamma_5\gamma_4}|G_i, \mathbf{p}=0\rangle \propto \frac{1}{m_{G_i}}\langle 0|q(0)|G_i\rangle$$

The correlator $C_{GC}(t)$ should be modified as

$$C_{GC}(t) = \sqrt{Z_{G_1}}\langle 0|\mathcal{O}_{\gamma_5\gamma_4}|G_1\rangle \cos^2\theta_1 e^{-m_{g_1}} - \sum_{i=1}^{2}\sqrt{Z_{G_i}Z_{(\gamma_5\gamma_4),i}} \times \cos\theta_i \sin\theta_i \left(e^{-m_{g_i}t} + e^{-m_{g_i}(T-t)} - (e^{-m_{\eta_i}t} + e^{-m_{\eta_i}(T-t)})\right)$$

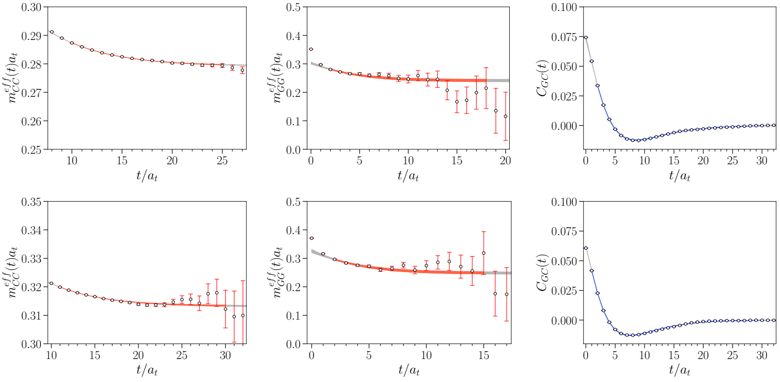


Figure: $\Gamma_{cc} = \gamma_5\gamma_4$. The effective mass of $C_{GG}(t)$ and $C_{CC}(t)$ and the correlation function of $C_{GC}(t)$.

Results

ensemble	Γ	m_{η_1}(MeV)	m_{g_1}(MeV)	θ_1	x_1(MeV)
I	γ_5	2691(2)	2317(51)	7.7(1.1)°	50(10)
	$\gamma_5\gamma_4$	2685(1)	2317(43)	6.8(8)°	44(7)
	avg.	2686(1)	2317(46)	7.1(9)°	46(8)
II	γ_5	2987(9)	2308(63)	4.9(6)°	58(9)
	$\gamma_5\gamma_4$	3013(3)	2385(40)	4.2(3)°	46(4)
	avg.	3010(4)	2363(47)	4.3(4)°	49(6)

- Results of γ_5 and $\gamma_5\gamma_4$ are Compatible within errors
- x_1 is insensitive to m_{η_c}
- mixing angle is different for emsemble I and ensemble II : mass difference
- Nonzero mixing angle

X(2370)

- $X(2370)$ observed by BESIII

  M. Ablikim et al. (BESIII), Phys. Rev. Lett. 106, 072002 (2011), arXiv:1012.3510 [hep-ex]

  M. Ablikim et al. (BESIII), Eur. Phys. J. C 80, 746 (2020), arXiv:1912.11253 [hep-ex]

  $\mathrm{J}/\psi \to \gamma \mathrm{X}(2370) \to \gamma\pi^+\pi^-\eta'$

$$\begin{aligned} M_{X(2370)} &= 2341.6 \pm 6.5(\text{stat.}) \pm 5.7(\text{syst.})\ \text{MeV} \\ \Gamma_{X(2370)} &= 117 \pm 10(\text{stat.}) \pm 8(\text{syst.})\ \text{MeV} \end{aligned}$$

  $\mathrm{J}/\psi \to \gamma \mathrm{X}(2370) \to \gamma \mathrm{K}\bar{\mathrm{K}}\eta'$

$$\begin{aligned} M_{X(2370)} &= 2376.3 \pm 8.7(\text{stat.})^{+3.2}_{-4.3}(\text{syst.})\ \text{MeV} \\ \Gamma_{X(2370)} &= 83 \pm 17(\text{stat.})^{+44}_{-6}(\text{syst.})\ \text{MeV} \end{aligned}$$

  $\mathrm{Br}(J/\psi \to \gamma X \to \gamma\eta' K^+K^-) = (1.79 \pm 0.23(\text{stat.}) \pm 0.65(\text{syst.})) \times 10^{-5}$
  $\mathrm{Br}(J/\psi \to \gamma X \to \gamma\eta' K_S K_S) = (1.18 \pm 0.32(\text{stat.})) \pm 0.39(\text{syst.})) \times 10^{-5}$

- Lattice Calculation L.-C.Gui et al. Phys. Rev. D 100,054511 (2019),arxiv:1906.03666[hep-lat]

  $\mathrm{Br}(J/\psi \to \gamma G) = 2.31(80) \times 10^{-4}$

Suppose X(2370) is dominated by pseudoscalar glueball. Then we can may have

$$\begin{aligned} \sin\theta &\approx \frac{x_1}{m_{\eta_c} - m_{X(2370)}} \approx 0.080(10) \\ \Delta m_{\eta_c} &\approx \frac{x_1^2}{m_{\eta_c} - m_{X(2370)}} \approx 3.9(9)\ \text{MeV} \end{aligned} \quad (11)$$

Γ_{η_c}

$$\frac{|\mathcal{M}(X \to \mathrm{LH})|}{|\mathcal{M}(c\bar{c} \to \mathrm{LH})|} \approx \left(\frac{M_X \Gamma_X}{M_{\eta_c}\Gamma_{c\bar{c}}}\right)^{1/2}$$

$$\begin{aligned}\frac{\Gamma_{\eta_c}}{\Gamma_{c\bar{c}}} &\approx \left|\cos\theta + \sin\theta \frac{|\mathcal{M}(X \to \mathrm{LH})|}{|\mathcal{M}(c\bar{c} \to \mathrm{LH})|}\right|^2 \\ &\approx 1 + 2\sin\theta \left(\frac{M_X \Gamma_X}{M_{\eta_c}\Gamma_{\eta_c}}\right)^{1/2} \left(\frac{\Gamma_{\eta_c}}{\Gamma_{c\bar{c}}}\right)^{1/2}\end{aligned}$$

Take $\Gamma_X \approx 100\text{Mev}$ and $\Gamma_{\eta_c} \approx 32.0\text{Mev}$, we have

$$\frac{\Gamma_{\eta_c}}{\Gamma_{cc}} = 1.29(4) \qquad\qquad \Gamma_{cc} \approx 24.9(8)\text{Mev}$$

Summary

- We performed the first calculation of the mixing of glueball and pseudoscalar charmonium with dynamical ensembles
- For pseudoscalar, the mixing of $(c\bar{c})$ meson and glueball do exist
- Possible explanation of the large total decay width of η_c
- This work was submitted to arxiv,arXiv.org:2107.12749

Thanks for your attention!