Elastic $\pi - N$ scattering in the I= 3/2 channel

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The Cyprus Institute RESEARCH·TECHNOLOGY·INNOVATION
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aSToRC
Extended Twisted Mass Collaboration

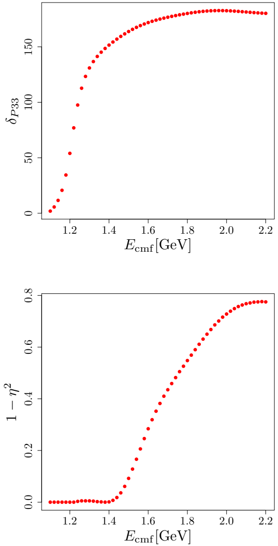
Baryon resonances, $\pi - N$ scattering

Δ resonance

- Lightest baryon resonance and spin $3/2$ particle
- Well isolated from other resonances
- Decays almost exclusively to πN ($\Delta \rightarrow N\gamma$ less then 1% of the total decay width)

Phenomenological importance

- Presence of Δ in compact neutron stars
- Onset of Δ could generate thermodynamical instabilities A.Raduta Phys.Lett.B 814 (2021)

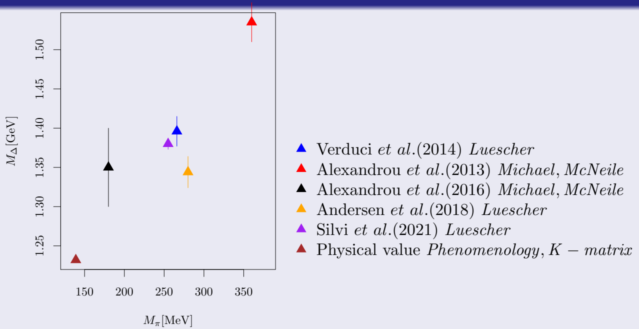


http://gwdac.phys.gwu.edu/

The Δ resonance

This work:Physical point $M_\pi = 139\mathrm{MeV}$

Lattice QCD results



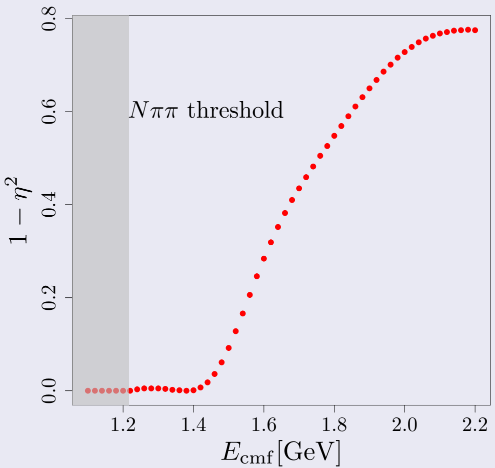
Talk at the conference in the same topic:

- Precise $I = 3/2$ and $I = 0$ meson-baryon scattering amplitudes from an $N_f = 2+1$ CLS ensemble at $m_\pi = 200\mathrm{MeV}$ by C. Morningstar Jul 28, 2021, 9:00 PM

The Δ resonance

Challange: $N\pi\pi$ threshold is very low

At the physical point $m_N + m_\pi < m_\Delta \longrightarrow \Delta$ is unstable



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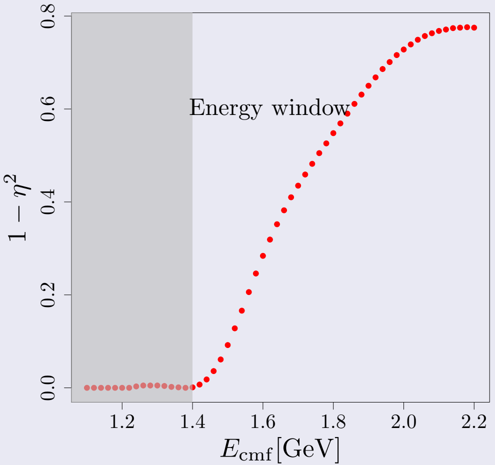


Resonances in LQCD

Luescher-method

- Two particle energy levels in a finite box with size L
- Volume dependence of the energy shift related to scattering observables at $L = \infty$

$$\det\left(\mathcal{M}^{\vec{P}}_{J\ell\mu,J'\ell'\mu'} - \delta_{JJ'}\delta_{\ell\ell'}\delta_{\mu\mu'}\cot\delta_{J\ell}\right) = 0$$

- Determinant is taken in angular momentum space

Parameters

- Configurations: 2+1+1 Twisted mass Clover,
  - $M_\pi = 139\mathrm{MeV}, \quad a = 0.08\mathrm{fm}$
  - $L = 5.1\mathrm{fm}, \quad M_\pi \cdot L = 3.6, \quad N_s = 64, \quad N_t = 128$
- Measurements: 400 configurations, 64 source position each with Gauss-smearing at source,sink.

Two-point correlation functions ($\Delta-\Delta, \Delta-\pi N, \pi N-\pi N$)

Two-point function

$$C_{\mathcal{O}\bar{\mathcal{O}}}(\vec{p},t) = \langle \bar{\mathcal{O}}(\vec{p},t)\mathcal{O}(\vec{p},0)\rangle$$

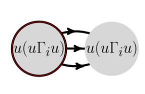
- Total momentum $|\vec{p}| \leq 3\frac{2\pi}{L}$
- Contractions expensive: Done on GPU

Interpolating operators source/sink

- $\mathcal{O}^i_{\Delta^{++}} = (u C\gamma_i u)\, u$
- $\mathcal{O}_{\pi^+ N^+} = \bar{d} i\gamma_5 u \cdot (u C\gamma_5 d) u$

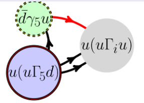
$\Delta-\Delta$

- Baryon contraction



$\Delta-\pi N$

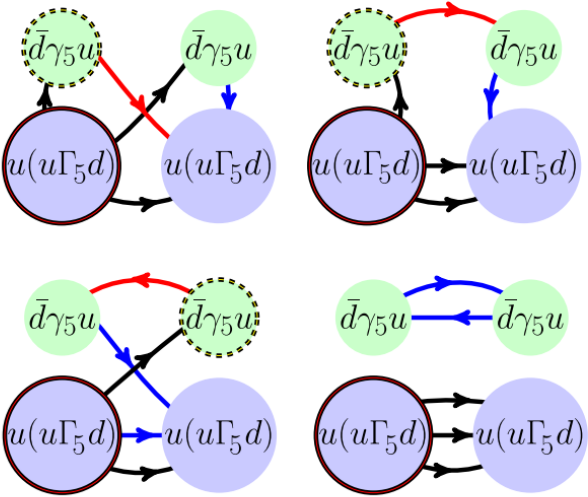
- Sequential propagator



$\pi N-\pi N$ more difficult sink-sink

$\pi N - \pi N$

- Sink to sink with fully time-diluted stochastic propagators
- Cutting the whole diagram by the stochastic piece to factors
- Expensive factor calculation on the GPU



Consequences of finite volume: Projections

- Instead of spin we have the degrees of freedom:
  - irrep, irrep row(μ), $\#$ occurances

Irreps in this work

$\vec{p}_{\text{tot}}$, irrep name	ℓ	N_{dim}
$\vec{p}=(0,0,0), \mathrm{G1_u}$	s	8x8
$\vec{p}=(0,0,0), \mathrm{Hg}$	p, f	9x9
$\vec{p}=(0,0,1), \mathrm{G1}$	s,p,d	24x24
$\vec{p}=(0,0,1), \mathrm{G2}$	p,d	18x18
$\vec{p}=(1,1,0), \mathrm{(2)G}$	s,p,d	30x30
$\vec{p}=(1,1,1), \mathrm{(3)G}$	s,p,d	16x16
$\vec{p}=(1,1,1), \mathrm{F1}$	p,d	6x6
$\vec{p}=(1,1,1), \mathrm{F2}$	p,d	6x6

Hg irrep $\vec{p}_{\text{tot}}=(0,0,0), p_N=1, p_\pi=1, \mu=0$

- Occurance a

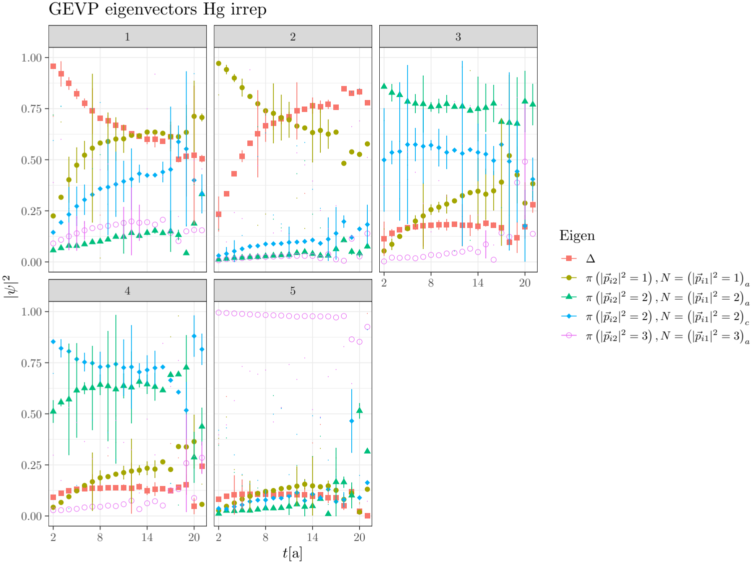
$$0.5\left(N_{-1,0,0}(0)\pi_{1,0,0}-iN_{0,-1,0}(0)\pi_{0,1,0}\right.$$
$$\left.+iN_{0,1,0}(0)\pi_{0,-1,0}-N_{1,0,0}(0)\pi_{-1,0,0}\right)$$

- Occurance b

$$0.5\left(N_{-1,0,0}(3)\pi_{1,0,0}-N_{0,-1,0}(3)\pi_{0,1,0}\right.$$
$$\left.-N_{0,1,0}(3)\pi_{0,-1,0}+N_{1,0,0}(3)\pi_{-1,0,0}\right)$$

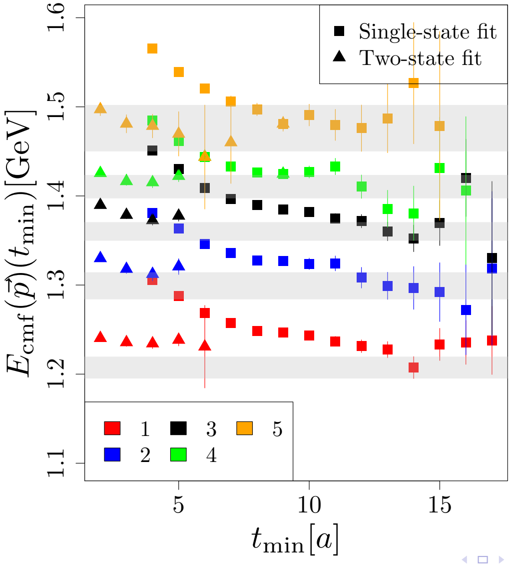
Behind the scenes:GEVP

Why we need the expensive $\pi - N$ two-hadron correlation function?

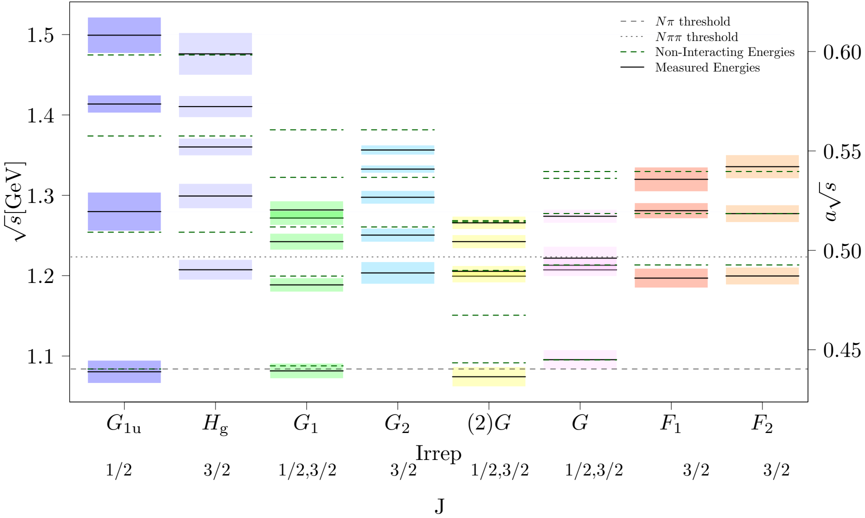


Spectrum,stability plot

Comparison of single and double exponential fits as a function of $t_{\min}$



Spectrum summary (Preliminary)

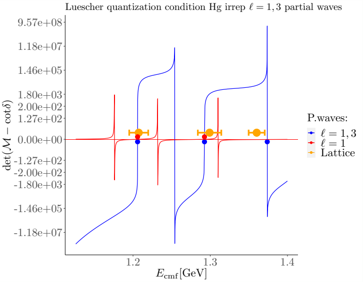


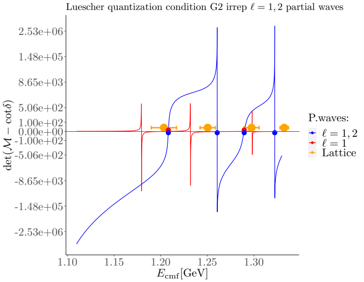
Phase-shifts, Partial-wave analysis,Energy level fit

Quantization conditions (QC) Göckeler et. al PRD 2012

- Phase shift parametrization:
  - $\ell = 0 \rightarrow \cot\delta_{\ell=0} = a_0 q_{\mathrm{cmf}}, \qquad \ell = 1 \rightarrow \tan\delta_{\ell=1} = \frac{\sqrt{s}\Gamma(\Gamma_R, s)}{{M_R}^2 - s}$
- We restrict ourselves to $\ell = 0, 1$ and check for $\ell \geq 2$

Example: Energy levels: roots of QC. Hg and G2 irreps

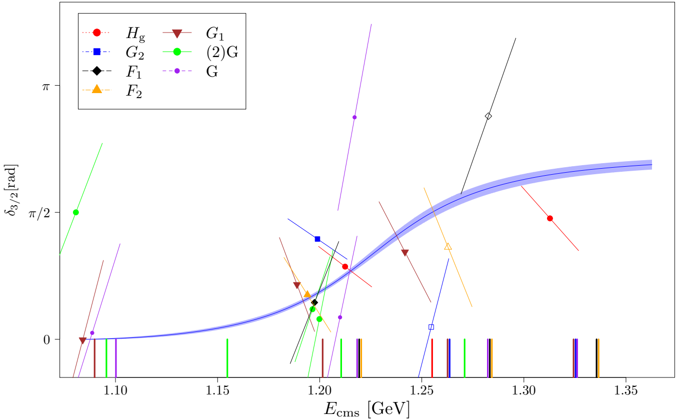




Luescher-analysis

Preliminary results:

- $M_R = 1255(25)\mathrm{MeV}, \Gamma_R = 140(120)\mathrm{MeV}, a_0 = -0.0016(6)\mathrm{MeV}^{-1}, \chi^2/\mathrm{dof} = 0.88$



Conclusion

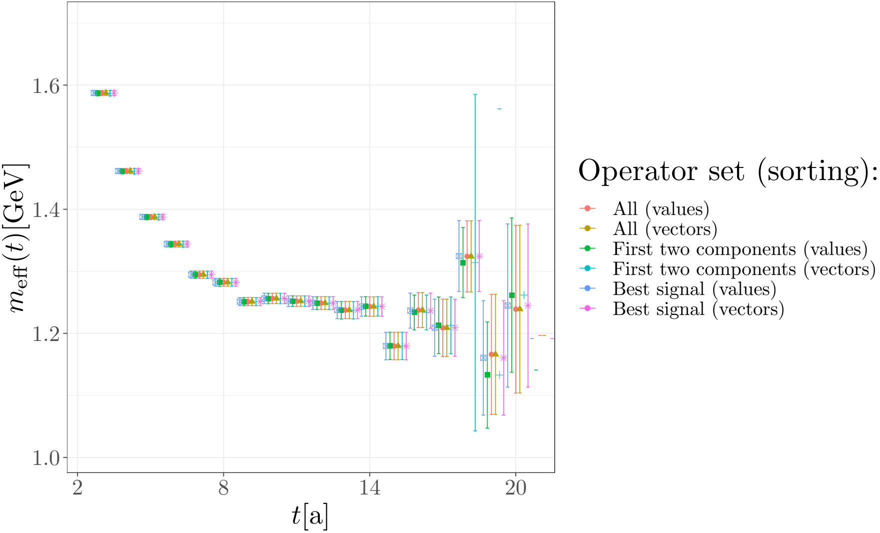
- We determine the parameters of the Δ resonance at the physical point
- We need to improve our determinations for the width
- Fit the scattering length using dedicated measurements
- Consider the $I = 1/2$ case as well, determining the σ-term

Acknowledgement

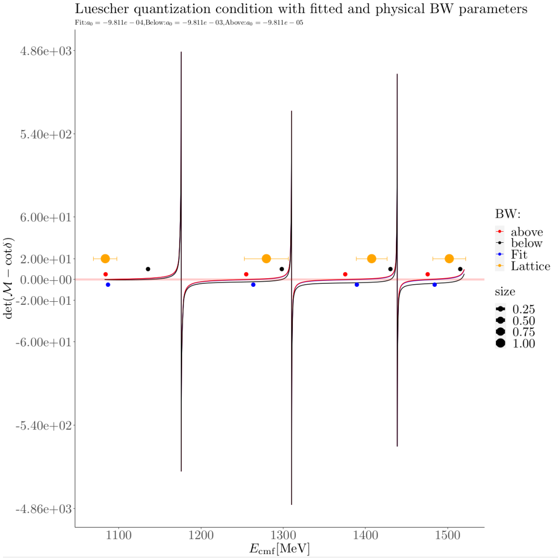
- The project is supported by PRACE, the measurements are doing on Piz-Daint cluster
- Thank you for your attention

Backup-slide, GEVP,Spectrum

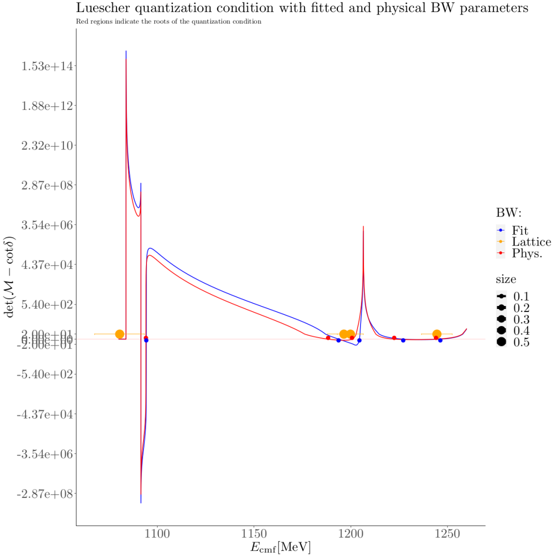
Example: Hg irrep 1. eigenvalue



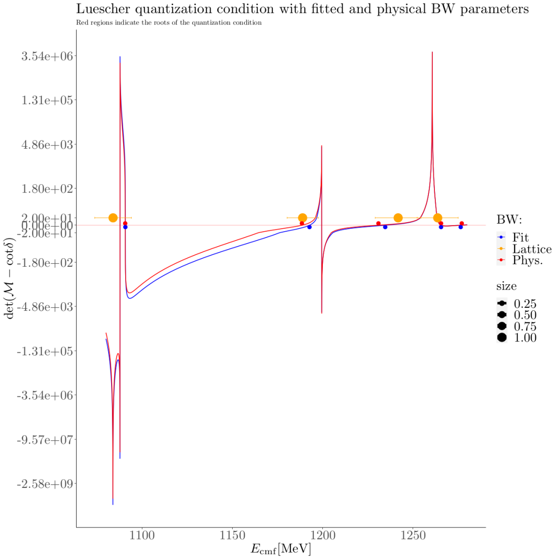
Quantization condition G1u



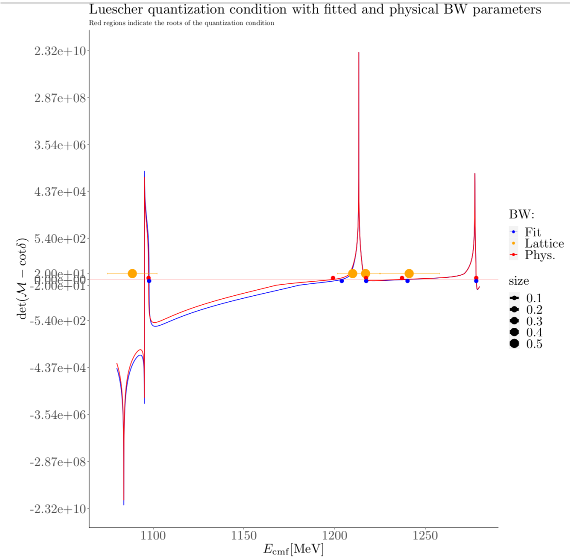
Quantization condition G1



Quantization condition 2G



Quantization condition 3G



Quantization condition F1,F2