

# INVESTIGATING EXOTIC HEAVY-LIGHT TETRAQUARKS WITH $2 + 1$ FLAVOUR LATTICE QCD

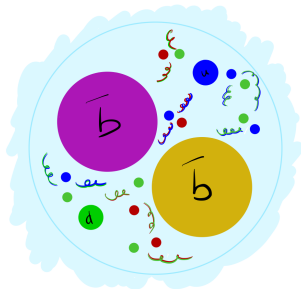
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w/ R. J. Hudspith, A. Francis,  
R. Lewis, K. Maltman



27 July 2021  
(or 28 July 2021!)



## SCHEMATIC MODEL OF BARYONS AND MESONS

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Received 4 January 1964

The existence of tetraquarks  
(and other exotic states) has  
long been suspected!

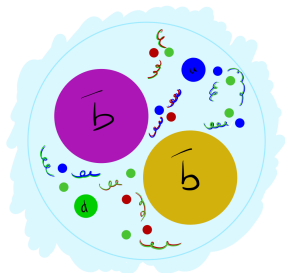
A simpler and more elegant scheme can be constructed if we allow non-integral values for the charges. We can dispense entirely with the basic baryon  $\Lambda$  if we assign to the triplet  $t$  the following properties: spin  $\frac{1}{2}$ ,  $z = -\frac{1}{3}$ , and baryon number  $\frac{1}{3}$ . We then refer to the members  $u^{\frac{2}{3}}$ ,  $d^{-\frac{1}{3}}$ , and  $s^{-\frac{1}{3}}$  of the triplet as "quarks"  $q$  and the members of the anti-triplet as anti-quarks  $\bar{q}$ . Baryons can now be constructed from quarks by using the combinations  $(qqq)$ ,  $(qqq\bar{q})$ , etc., while mesons are made out of  $(q\bar{q})$ ,  $(qq\bar{q}\bar{q})$ , etc. It is assumed that the lowest baryon configuration  $(qqq)$  gives just the representations **1**, **8**, and **10** that have been observed, while the lowest meson configuration  $(q\bar{q})$  similarly gives just **1** and **8**.

# Diquarks

- ★ We are interested in:
  - ▶ light diquarks in a colour  $\bar{3}_c$ , flavour  $\bar{3}_f$  and spin 0 configuration
    - “good light diquark”
  - ▶ heavy diquarks in a colour  $3_c$ , relative s-wave configuration

The term “good diquark” is of Jaffe’s invention, for a nice review: [\[hep-ph/0409065\]](#)

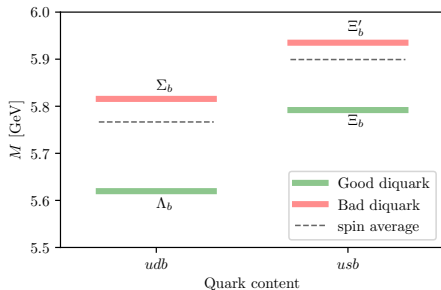
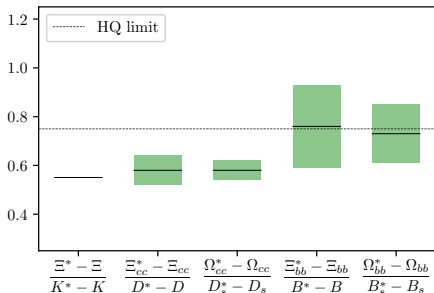
- ★ Degenerate heavy quarks:  $J^P = 1^+$
- ★ Otherwise: we have access to  $J^P = 0^+$  or  $J^P = 1^+$
- ★ Light diquark  $\implies I = 0$  or  $I = 1/2$



# Information from baryons and mesons

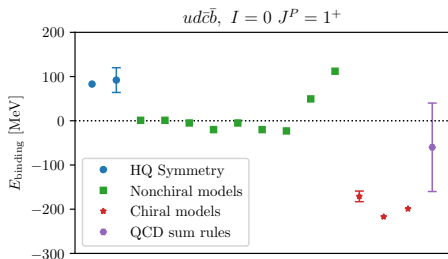
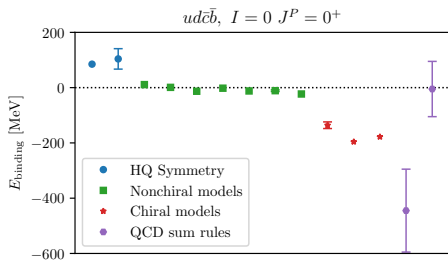
- ★ Ordinary baryon and meson spectra can provide constraints for models
- ★  $3_c \bar{Q}\bar{Q}$  serves as near-static colour source, like a single  $Q$  in a baryon (plus attractive colour Coulomb interaction)

Numbers from PDG & [1409.0497]



- ★ Baryon spectrum suggests “good” light diquarks result in strong attraction.
- ★ Lighter quark mass  $\rightarrow$  stronger attraction

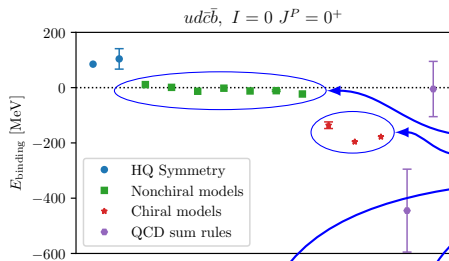
# Example of model and other predictions



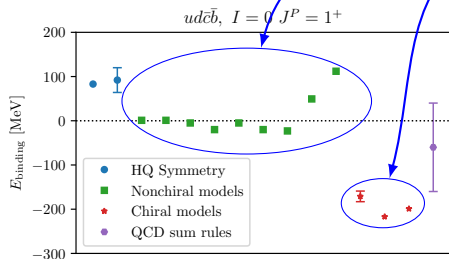
Examples of various  $ud\bar{c}\bar{b}$  masses relative to the lowest two-meson threshold in the  $I = 0$ ,  $J^P = 0^+$  and  $I = 0$ ,  $J^P = 1^+$  channels.

We discuss model results completely for all channels in R.J. Hudspith, BC, A. Francis, R. Lewis and K. Maltman *Phys. Rev. D* 102, 114506 (2020), [2006.14294].

# Example of model and other predictions



All fit to the ordinary meson/baryon spectrum; clear tetraquark difference



Examples of various  $ud\bar{c}\bar{b}$  masses relative to the lowest two-meson threshold in the  $I = 0, J^P = 0^+$  and  $I = 0, J^P = 1^+$  channels.

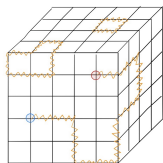
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# SOME LATTICE DETAILS

# Ensembles

$N_f = 2 + 1$  Wilson-clover ensembles – includes and extends PACS-CS ensembles

$L^3 \times T$	$m_\pi$ [MeV]	$N_{\text{cnfg}}$	$a^{-1}$ [GeV]
$32^3 \times 64$	700	399	2.194(10)
	575	400	
	415	400	
	299	800	
	182*	121	
$48^3 \times 64$	192	122	
	165	88	



- ★ Coulomb gauge-fixed wall sources
- ★  $c$  quarks: Relativistic Heavy Quark action
- ★  $b$  quarks: Nonrelativistic QCD (NRQCD) action

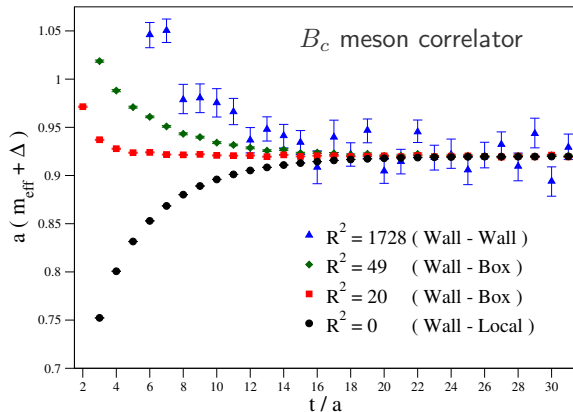


# Recent update: Box-Sinks

R. J. Hudspith, BC, A. Francis, R. L Lewis, K. Maltman [2006.14294]

Improvement: box-sinks for better overlap with ground states.

$$S^B(x, t) = \frac{1}{N} \sum_{r^2 \leq R^2} S(x + r, t)$$



# TETRAQUARKS ON THE LATTICE

# Fitting our tetraquarks

Construct correlators,  $C_{\mathcal{O}_1\mathcal{O}_2}(t) = \sum_n \frac{\langle 0|\mathcal{O}_1|n\rangle\langle n|\mathcal{O}_2|0\rangle}{2E_n} e^{-E_n t}$  from:

$$\begin{aligned}D(\Gamma_1, \Gamma_2) &= (\psi_a^T C \Gamma_1 \phi_b)(\bar{\theta}_a C \Gamma_2 \bar{\omega}_b^T), \\E(\Gamma_1, \Gamma_2) &= (\psi_a^T C \Gamma_1 \phi_b)(\bar{\theta}_a C \Gamma_2 \bar{\omega}_b^T - \bar{\theta}_b C \Gamma_2 \bar{\omega}_a^T), \\M(\Gamma_1, \Gamma_2) &= (\bar{\theta} \Gamma_1 \psi)(\bar{\omega} \Gamma_2 \phi), & N(\Gamma_1, \Gamma_2) &= (\bar{\theta} \Gamma_1 \phi)(\bar{\omega} \Gamma_2 \psi), \\O(\Gamma_1, \Gamma_2) &= (\bar{\omega} \Gamma_1 \psi)(\bar{\theta} \Gamma_2 \phi), & P(\Gamma_1, \Gamma_2) &= (\bar{\omega} \Gamma_1 \phi)(\bar{\theta} \Gamma_2 \psi).\end{aligned}$$

We want to solve a GEVP to get energy levels, “principle correlators”:

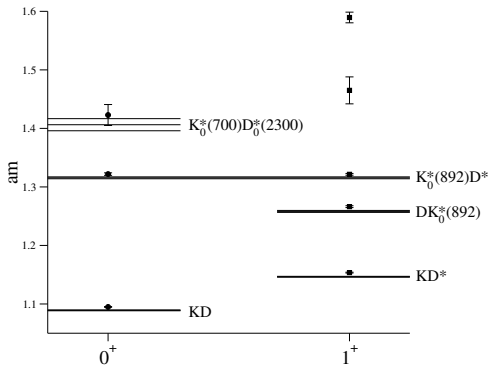
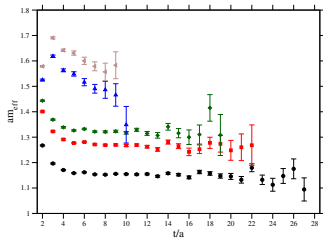
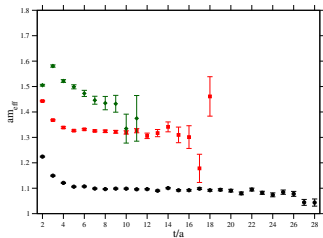
$$C_i(t) = \sum_{j,k} V_{ij}(\tau)^\dagger C_{jk}(t) V_{ki}(\tau)$$

where  $V$  is made from columns of the eigenvector solution to:

$$C_{ij}(t)v_j(t) = \lambda_i C_{ij}(t+t_0)v_j(t).$$

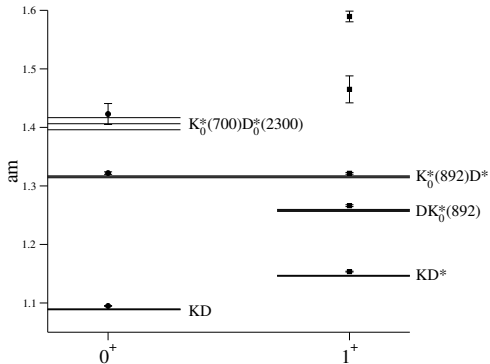
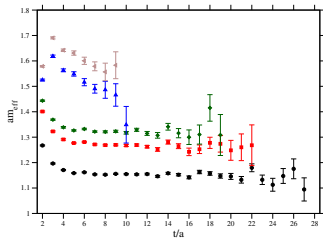
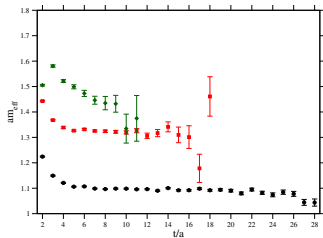
# $ud\bar{s}\bar{c}$ tetraquarks

R. J. Hudspith, BC, A. Francis, R. Lewis, K. Maltman [2006.14294]



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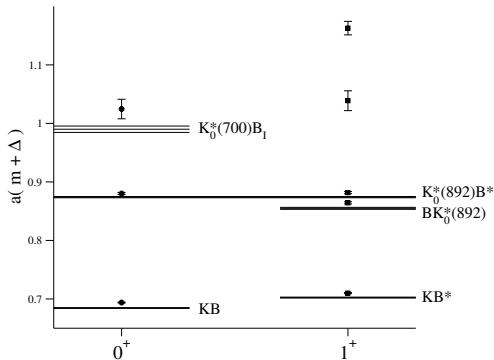
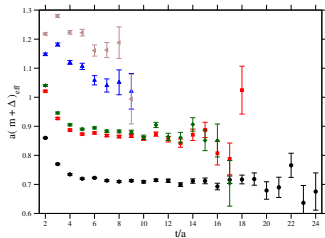
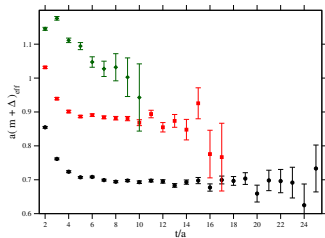
R. J. Hudspith, BC, A. Francis, R. Lewis, K. Maltman [2006.14294]



★ No evidence of deep binding in  $0^+$  or  $1^+$  channels

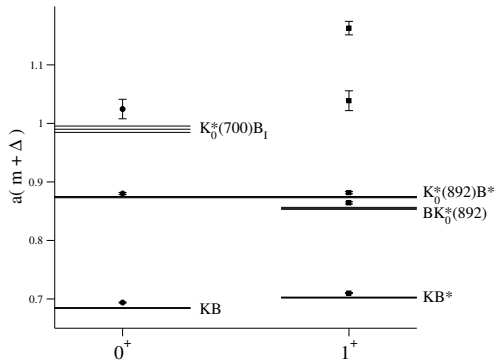
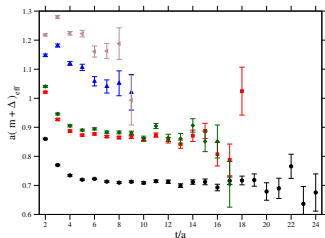
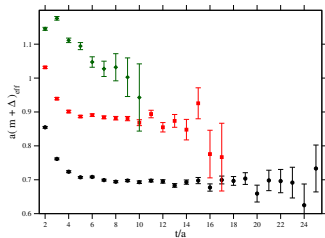
# $ud\bar{s}\bar{b}$ tetraquarks

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# $ud\bar{s}\bar{b}$ tetraquarks

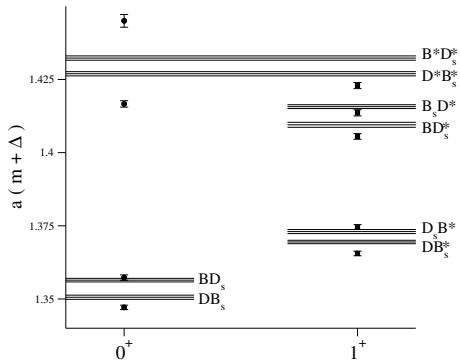
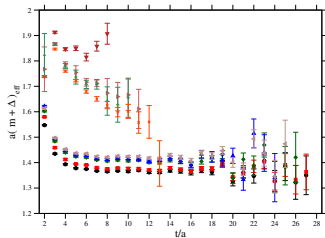
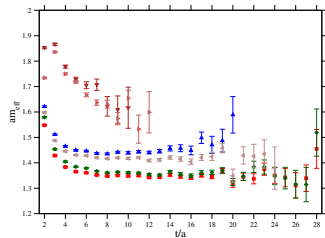
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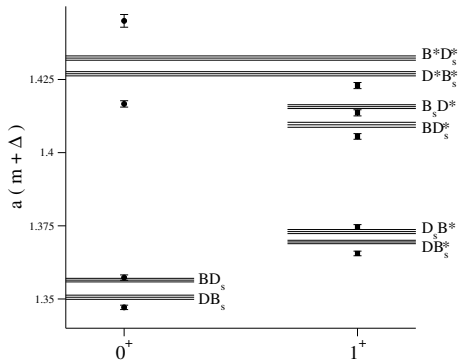
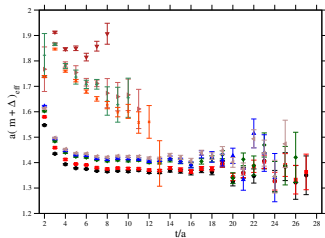
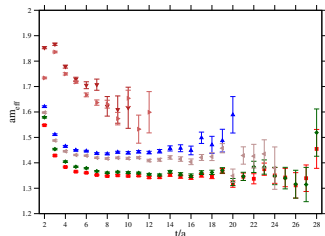
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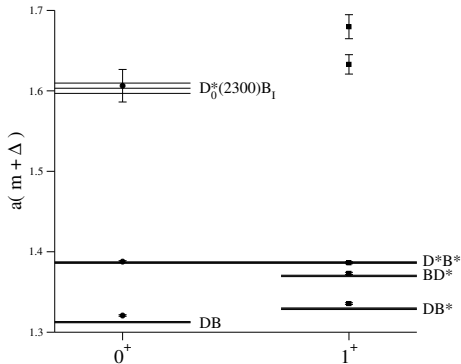
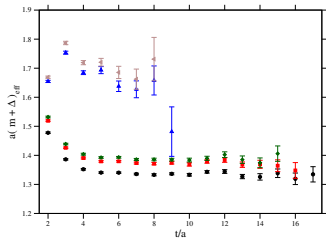
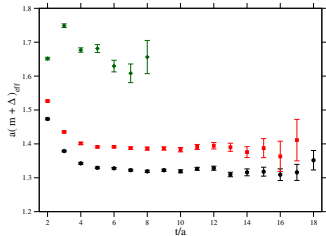


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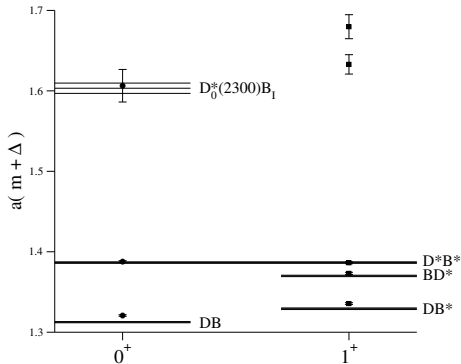
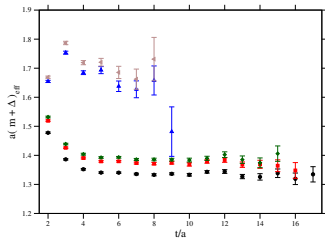
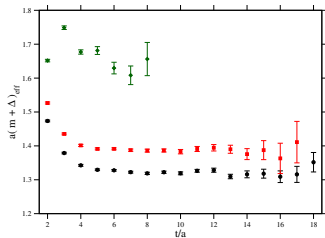


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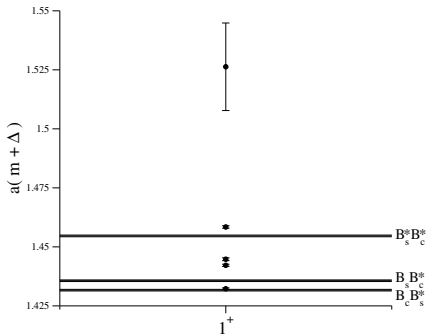
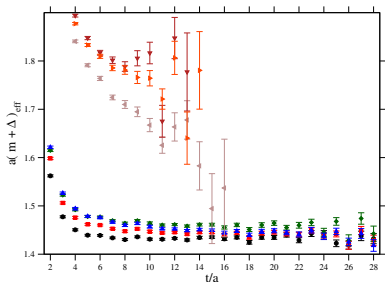


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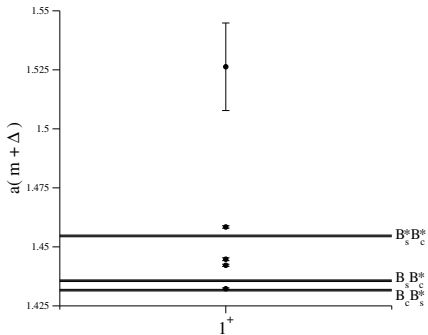
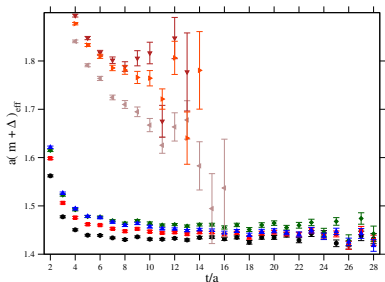


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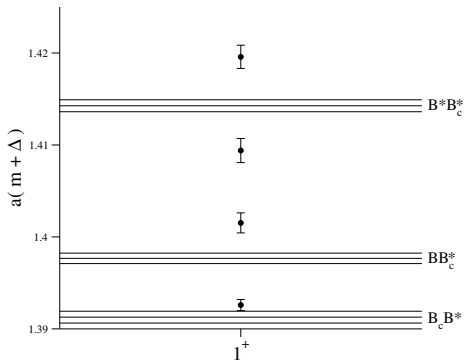
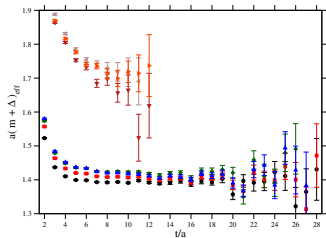


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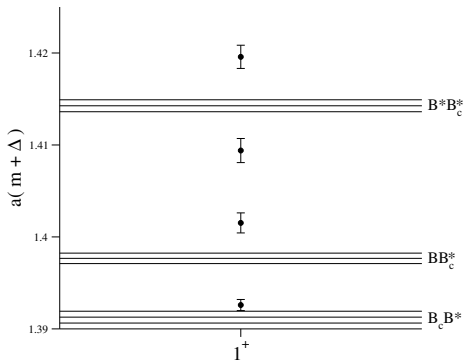
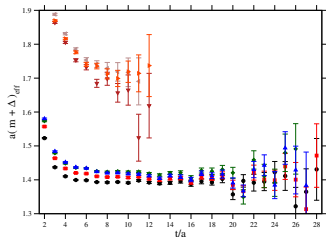


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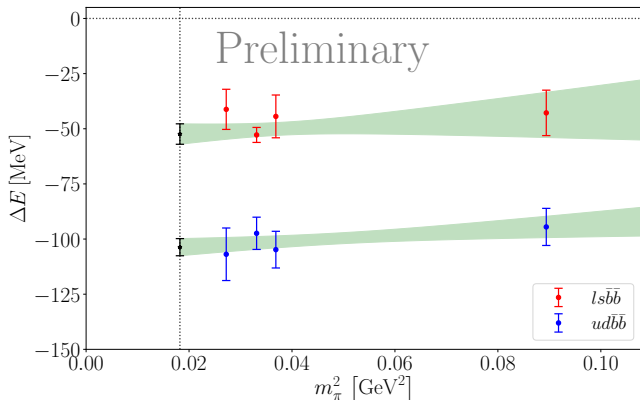


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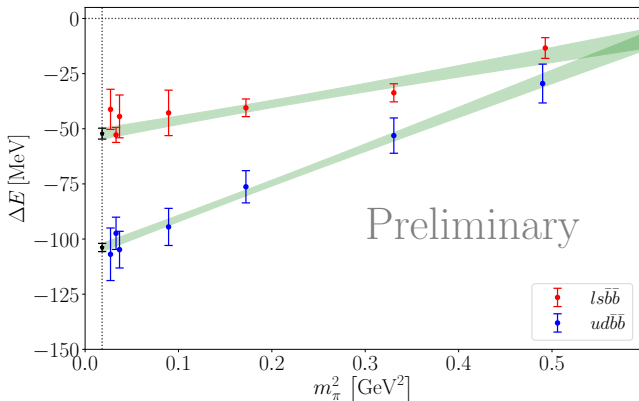
# Doubly-bottom tetraquarks



- ★  $I = 0, J^P = 1^+$   $ud\bar{b}\bar{b}$  and  $I = 1/2, J^P = 1^+$   $ls\bar{b}\bar{b}$  strong-interaction stable.
- ★ Consistent binding of  $ud\bar{b}\bar{b}$  found by lattice groups + preliminary  $ls\bar{b}\bar{b}$ .



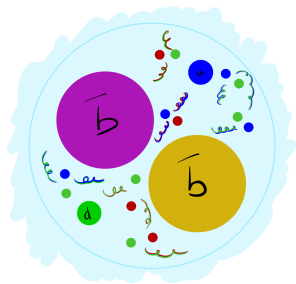
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# Summary

- ★  $I = 0, J^P = 1^+$   $ud\bar{b}\bar{b}$  state consistently found stable on lattice:  
 $\Delta E_{ud\bar{b}\bar{b}} \gtrsim 100$  MeV
- ★ Evidence for stable  $I = 1/2, J^P = 1^+$   $\ell s\bar{b}\bar{b}$ :  
 $\Delta E_{\ell s\bar{b}\bar{b}} \sim 50$  MeV
- ★ No evidence for deep binding in any other channel considered.
- ★ Chiral model deep binding predictions incompatible with lattice results; nonchiral colour magnetic spin-spin interaction models OK within current lattice errors.



Thank you!

THANK YOU

EXTRAS

# Operators

Type ( $\psi\phi\theta\omega$ )	$I(J)^P$	Diquark-Antidiquark	Dimeson
$udcb/udsb/udsc$	$0(1)^+$	$D(\gamma_5, \gamma_i), D(\gamma_t\gamma_5, \gamma_i\gamma_t)$ $E(\gamma_5, \gamma_i), E(\gamma_t\gamma_5, \gamma_i\gamma_t)$	$M(\gamma_5, \gamma_i) - N(\gamma_5, \gamma_i)$ $M(I, \gamma_i\gamma_5) - N(I, \gamma_i\gamma_5)$ $O(\gamma_5, \gamma_i) - P(\gamma_5, \gamma_i)$ $O(I, \gamma_i\gamma_5) - P(I, \gamma_i\gamma_5)$ $\epsilon_{ijk}M(\gamma_j, \gamma_k)$
	$0(0)^+$	$E(\gamma_5, \gamma_5), E(\gamma_t\gamma_5, \gamma_t\gamma_5)$	$M(\gamma_5, \gamma_5) - N(\gamma_5, \gamma_5)$ $M(I, I) - N(I, I)$ $M(\gamma_i, \gamma_i)$

Type ( $\psi\phi\theta\omega$ )	$I(J)^P$	Diquark-Antidiquark	Dimeson
$udbb$	$0(1)^+$	$D(\gamma_5, \gamma_i), D(\gamma_t\gamma_5, \gamma_i\gamma_t)$	$M(\gamma_5, \gamma_i) - N(\gamma_5, \gamma_i)$ $M(I, \gamma_i\gamma_5) - N(I, \gamma_i\gamma_5)$
$lsbb/ucbb/scbb$	$\frac{1}{2}(1)^+$	$D(\gamma_5, \gamma_i), D(\gamma_t\gamma_5, \gamma_i\gamma_t)$	$M(\gamma_5, \gamma_i), M(I, \gamma_i\gamma_5)$ $N(\gamma_5, \gamma_i), N(I, \gamma_i\gamma_5)$ $\epsilon_{ijk}M(\gamma_j, \gamma_k)$

Type ( $\psi\phi\theta\omega$ )	$I(J)^P$	Diquark-Antidiquark	Dimeson
<i>uscb</i>	$\frac{1}{2}(1)^+$	$D(\gamma_5, \gamma_i), D(\gamma_t\gamma_5, \gamma_i\gamma_t)$ $E(\gamma_5, \gamma_i), E(\gamma_t\gamma_5, \gamma_i\gamma_t)$	$M(\gamma_5, \gamma_i), M(I, \gamma_i\gamma_5)$ $N(\gamma_5, \gamma_i), N(I, \gamma_i\gamma_5)$ $O(\gamma_5, \gamma_i), O(I, \gamma_i\gamma_5)$ $\epsilon_{ijk}M(\gamma_j, \gamma_k)$
	$\frac{1}{2}(0)^+$	$E(\gamma_5, \gamma_5), E(\gamma_t\gamma_5, \gamma_t\gamma_5)$	$M(\gamma_5, \gamma_5), M(I, I)$ $N(\gamma_5, \gamma_5), N(I, I)$ $M(\gamma_i, \gamma_i)$

# NRQCD tuning

