Evidence of Glueball at Physical Point

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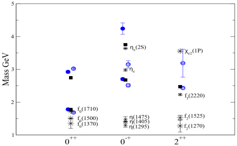
Motivation

	m_π / MeV	m_{0++} / MeV	m_{2++} / MeV	m_{0-+} / MeV
$N_f = 2$[1]	938	1417(30)	2363(39)	2573(55)
	650	1498(58)	2384(67)	2585(65)
$N_f = 3$[2]	360	1795(60)	2620(50)	-
quenched[3]	-	1710(50)(80)	2390(30)(120)	2560(35)(120)
quenched[4]	-	1730(50)(80)	2400(25)(120)	2590(40)(130)



1. What's the results at physical point?

[1] Sun et al., "Glueball spectrum from $N_f = 2$ lattice QCD study on anisotropic lattices".

[2] Gregory et al., "Towards the glueball spectrum from unquenched lattice QCD".

[3] Chen et al., "Glueball spectrum and matrix elements on anisotropic lattices".

[4] Morningstar and Peardon, "The Glueball spectrum from an anisotropic lattice study".

Configuration set

1. $N_f = 2 + 1$ dynamical confiugrations generated by RBC/UKQCD collaboration;
2. Accessed through the agreement between χQCD Collaboration
3. Large volumn, physical pion mass

$L^3 \times T$	a / fm	m_π / MeV	La / fm	$N_{\rm conf}$
$48^3 \times 96$	$0.1141(2)$	~ 139	~ 5.5	364
$64^3 \times 128$	$0.0836(2)$	~ 139	~ 5.3	300

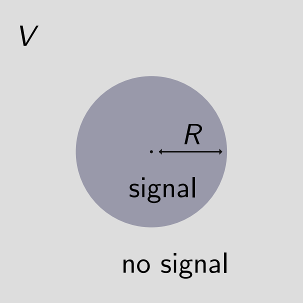
Cluster Decomposition Error Reduction

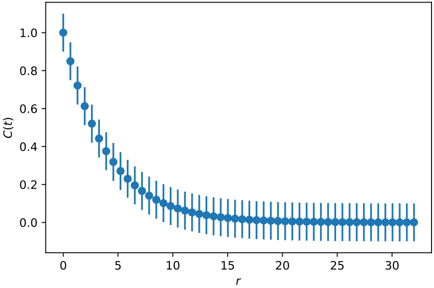
[5] Correlation funcion in Euclidean space: translation invariance and mass gap implies that

$$|\langle 0|T\mathcal{O}(x)\mathcal{O}(y)|0\rangle|^2 = Ar^{-\frac{3}{2}}e^{-mr}, \quad r = |x-y| \tag{1}$$

Since

$$\int_0^R \mathrm{d}r r^2 A r^{-\frac{3}{2}} e^{-mr} = \frac{\sqrt{\pi}\mathrm{erf}(\sqrt{mR})}{m^{\frac{3}{2}}} - \frac{\sqrt{R}e^{-mR}}{m} \tag{2}$$





[5] Liu, Liang, and Yang, "Variance Reduction and Cluster Decomposition".

Typically in numberic calculation the summation over source and sink coordinates is performed, the signal sacturate by $r < R \approx \frac{8}{m}$

$$C(t) = \frac{1}{V^2} \sum_{\vec{x}} \sum_{\vec{y}} \langle 0 | T \mathcal{O}_\alpha(\vec{x}, t) \mathcal{O}_\beta(\vec{y}, 0) | 0 \rangle$$

$$= \frac{1}{V^2} \sum_{\vec{r}} \sum_{\vec{x}} \langle 0 | T \mathcal{O}_\alpha(\vec{x} + \vec{r}, t) \mathcal{O}_\beta(\vec{x}, 0) | 0 \rangle$$

$$C(R, t) = \sum_{|\vec{r}| \leqslant R} \sum_{\vec{x}} \langle 0 | T \mathcal{O}_\alpha(\vec{x} + \vec{r}, t) \mathcal{O}_\beta(\vec{x}, 0) | 0 \rangle$$

$$= \sum_{r \leqslant R} K(\vec{r}, t)$$

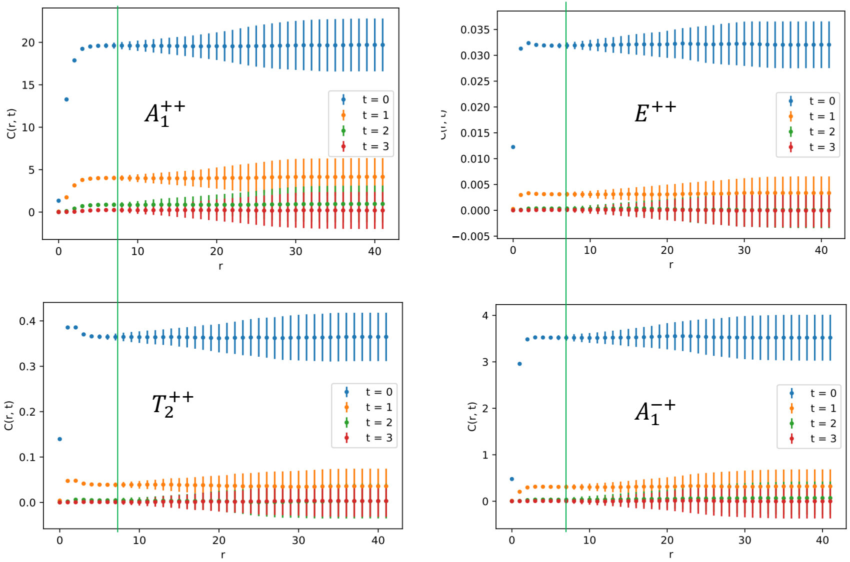
where

$$K(\vec{r}, t) = \frac{1}{V} \sum_{\vec{k}} e^{-i\vec{k}\cdot\vec{r}} \tilde{\mathcal{O}}_\alpha(-\vec{k}, t) \mathcal{O}_\beta(\vec{k}, 0)$$

$$, \tilde{\mathcal{O}}_\alpha(\vec{k}, t) = \sum_{\vec{x}} e^{-i\vec{k}\cdot\vec{x}} \mathcal{O}(\vec{x}, t).$$

calculated using FFT.

$\mathcal{O}_G$ - $\mathcal{O}_G$ correlation functions for different cut off R



We can choose $R = 7a$ in this case.

Glueball AA-operators

[6] Operators constructed from Wilson loop $\mathcal{O}_G$;
[7] Operators constructed from gauge field

$$\mathcal{O}_{AA}^{l,s}(r,\vec{x},t) = \frac{1}{N_r}\sum_{|\vec{r}|=r}\sum_{\vec{x}} C_{ij}^{m} Y_{lm}(\hat{r}) A_i(\vec{x}+\vec{r},t) A_j(\vec{x},0) \tag{3}$$

where

1. r_s: radius coordinate of the Bethe-Salpeter wave function operator;
2. s, l: total spin and orbital momentum of the two gauge fields;
3. N_r: the multiplicity of $\vec{r}$ with $|\vec{r}| = r$.

AA-operators are gauge dependent, need fixing using Coulomb gauge.

[6] Chen et al., "Glueball spectrum and matrix elements on anisotropic lattices".
[7] Liang et al., "Wave functions of $SU(3)$ pure gauge glueballs on the lattice".

Extracting AA-operators from gauge links

$$A_\mu \propto \ln U_\mu$$
$$U_\mu = P\mathrm{diag}(\lambda_1, \lambda_2, \lambda_3)P^{-1}$$
$$A_\mu \propto P\mathrm{diag}(\ln\lambda_1, \ln\lambda_2, \ln\lambda_3)P^{-1}$$

AA-operators for A_1^{++}, A_1^{-+}, E^{++}, T_2^{++}

A_1^{++}	$\frac{1}{\sqrt{3}}(A_1A_1 + A_2A_2 + A_3A_3)$
E^{++}	$\frac{1}{\sqrt{2}}(A_1A_1 - A_2A_2)$
	$\frac{1}{\sqrt{6}}(A_1A_1 + A_2A_2 - 2A_3A_3)$
T_2^{++}	$\frac{1}{\sqrt{2}}(A_1A_2 + A_2A_1)$
	$\frac{1}{\sqrt{2}}(A_2A_3 + A_3A_2)$
	$\frac{1}{\sqrt{2}}(A_1A_3 + A_3A_1)$
A_1^{-+}	$\epsilon_{ijk}A_iA_j\hat{r}_k$

Bethe-Salpeter wave functions

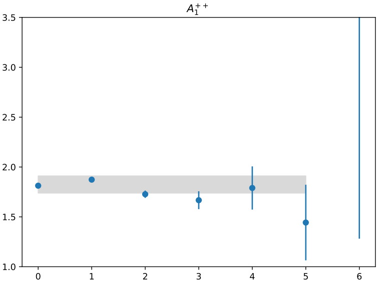
1. Wave function extraction
   Optimized glueball operator: $\mathcal{O}_{\mathrm{G}}^{(n)} = \sum_\alpha c_\alpha^{(n)} \mathcal{O}_\alpha$
   $\mathcal{O}_{\mathrm{AA}}$-$\mathcal{O}_{\mathrm{G}}$ correlation function:

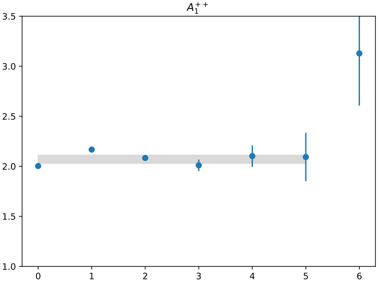
$$\begin{aligned}\langle 0|\mathcal{O}_{\mathrm{AA}}(r,t)\mathcal{O}_{\mathrm{G}}^{(n)}|0\rangle &= \sum_n \langle 0|\mathcal{O}_{\mathrm{AA}}(r,0)|n\rangle \langle n|\mathcal{O}_{\mathrm{G}}^{(n)}|0\rangle\, e^{-m_n t} \\ &= Z_0\Phi_0(r)e^{-m_0 t} + Z_1\Phi_1(r)e^{-m_1 t} + \cdots\end{aligned}$$

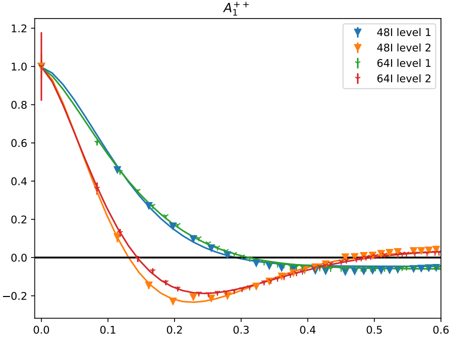
2. Mass spectrum optimization
   Find r_0 such that $\Phi_1(r_0) = 0$

$$\langle 0|\mathcal{O}_{\mathrm{AA}}(r_0,t)\mathcal{O}_{\mathrm{G}}^{(n)}|0\rangle = Z_0\Phi_0(r)e^{-m_0 t} + (n \geqslant 2\,\text{contributions})$$

Scalar A_1^{++} operator







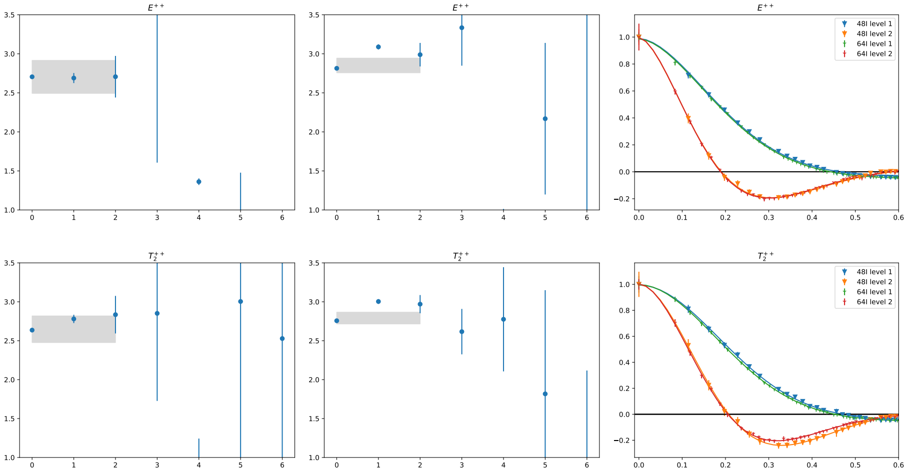
$$\Phi_0(r) = A\, e^{-(r/r_0)^\alpha}$$

$$\Phi_1(r) = A\,(1 - \beta r^\alpha)\, e^{-(r/r_0)^\alpha}$$

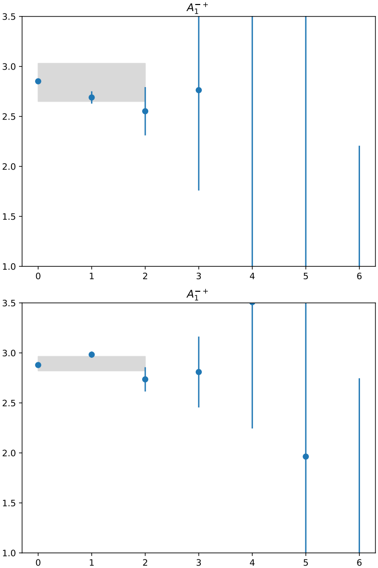
$$m(48I) = 1.82 \pm 0.09$$

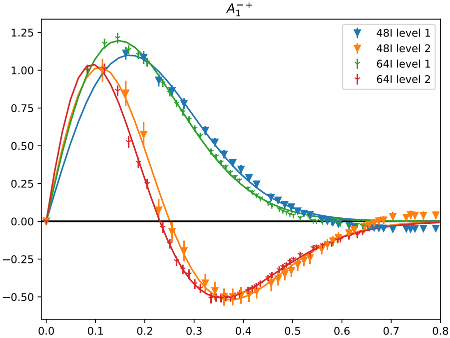
$$m(64I) = 1.96 \pm 0.08$$

Tensor E^{++}, T_2^{++} operators



Pseudo-scalar A_1^{-+} operators





$$\Phi_0(r) = Are^{-(r/r_0)^\alpha}$$

$$\Phi_1(r) = A(1-\beta r^\alpha)re^{-(r/r_0)^\alpha}$$

Summary

1. CDER significantly improved the signal
2. Bethe-Salpeter wave function in agreement with quenched results
3. Evidence of Glueballs?

Thanks for you attention!