Hypercubic symmetry restauration of Boriçi - Creutz fermions: costs and effects

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The 38th International Symposium on Lattice Field Theory

In memoriam Artan BORIÇI
(19.04.1965 - 25.03.2021)

- In LQCD community known for the BC fermions
- More than 60 publications
- More than 50 international conferences & workshops
- Created the first group of LQCD in Albania
- PhD advisor of R.OSMANAJ & D. XHAKO
- Krylov subspace methods in lattice QCD
- Chiral fermions on the lattice

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Boriçi - Creutz fermions

*[Michael Creutz JHEP04(2008)017] & [Artan Boriçi Phys. Rev. D **78**, 074504 (2008)]*

- Minimally doubled fermions
  - Preserve exactly the chiral symmetry
  - Strictly local
  - 2 flavours (species of opposite chirality)

- The Dirac operator for these fermions in the momentum space:

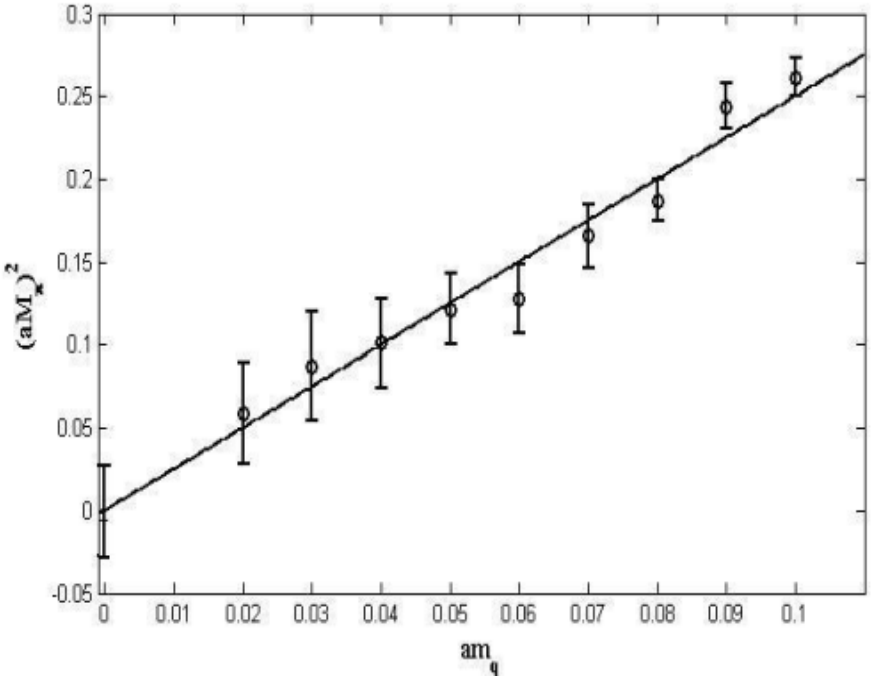
$$D(p) = \sum_{\mu} i\gamma_{\mu} \sin p_{\mu} + \sum_{\mu} i\gamma'_{\mu} \cos p_{\mu} - 2i\Gamma \qquad \gamma'_{\mu} = \Gamma\gamma_{\mu}\Gamma \quad \Gamma = \sum_{\mu} \gamma_{\mu}/2$$

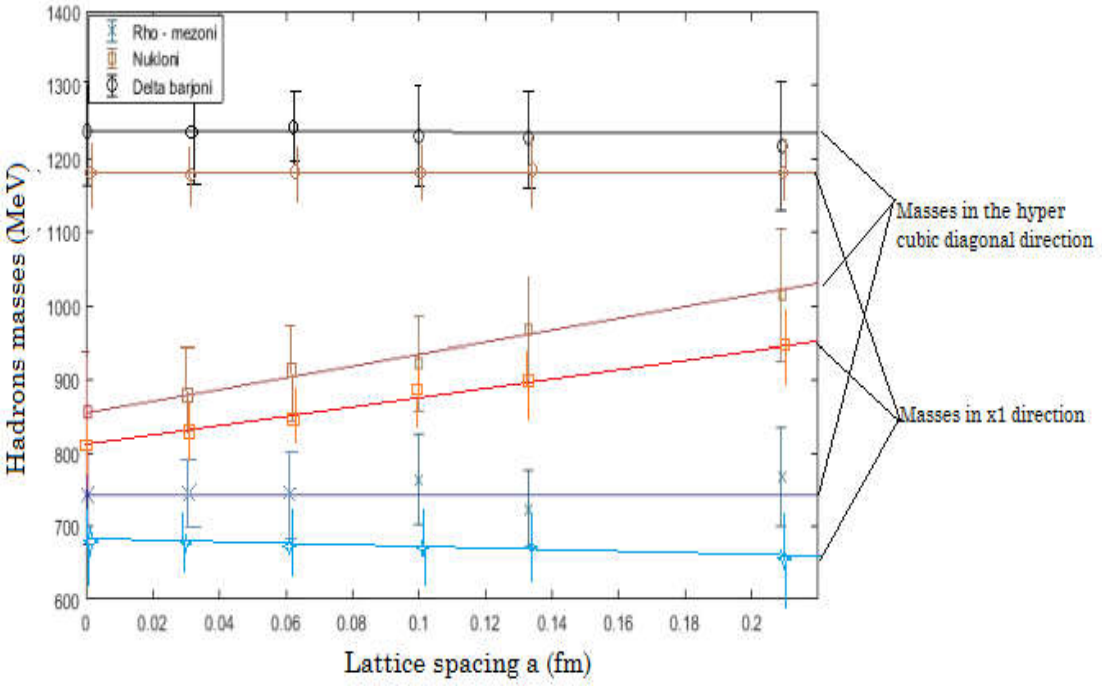
$$p_1 = (0,\ 0,\ 0,\ 0) \qquad p_2 = (\pi/2,\ \pi/2,\ \pi/2,\ \pi/2)$$

- **Break the hyper – cubic symmetry, a typical feature of them**

  There is a prefered direction in the euclidian spacetime, defined by the line that connect the two poles of the operator , which corrispond to the major hyper - cube diagonal

Effects of the broken hyper-cubic symmetry on the light hadrons spectrum





- Quenched approximation
- BC fermionic action
- Wilson gauge action
- 500 configurations

- Point source for quark propagators
- BiCGstab inverter
- Point splitting method
- FermiQCD

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c_4

Restauration of hypercubic broken symmetry of Boriçi - Creutz fermions

- Hypercubic symmetry has to be restored by adding counterterms

- The Dirac operator for these fermions in the momentum space with counterterms:

$$D_{BC}(p) = i \sum_{\mu}{}' \left[\gamma_\mu \sin p_\mu + \gamma'_\mu \cos p_\mu + c_4 \Gamma \sin p_\mu \right] + i(c_3 - 2)\Gamma$$

Perturbative calculations [Capitani et.al. JHEP 1009, 027]

- The counterterm $\mathbf{ic_3\Gamma}$ is used to restore it (we have not considered and set to zero c_4) ⟺ Study of the spontaneous chiral symmetry breaking (Chiral condensate creation)

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▪ Banks - Casher relation *(T. Banks, A, Casher, 1980),* provides a link between chiral condensate and the spectral density .

$$\lim_{\lambda\to 0}\lim_{m\to 0}\lim_{V\to\infty}\rho(\lambda,m)=\frac{\Sigma}{\pi} \qquad \Sigma=-\lim_{m\to 0}\lim_{V\to\infty}\left\langle\bar{\psi}\psi\right\rangle$$

(*If chiral symmetry is spontaneously broken by a non – zero value of the condensate the density of the quark modes in infinite volume does not vanish at the origin. A non – zero density conversely implies that the symmetry is broken)*

▪ Instead of the spectral density, the avarage number $\upsilon(M,m)$ of eigenmodes of the Dirac operator with eigenvalues $\alpha\le M^2$ turns out to be a more convenient quantity to consider. Since

$$\nu(\Lambda)=V\int_{-\Lambda}^{\Lambda}d\lambda\rho(\lambda,m)$$

the mode number ultimately carries the same information as the spectral density.
(L. Giusti, M. Luscher, 2009)

▪ Count modes using Gauss – Lanczos quadrature:

$$\Sigma_{eff}=\frac{\pi}{2}\frac{\nu(\Lambda)}{\Lambda V} \qquad \Lambda=\sqrt{M^2-m^2}$$

Simulations

Lattices 48^4 / 64^4

Quenched approximation

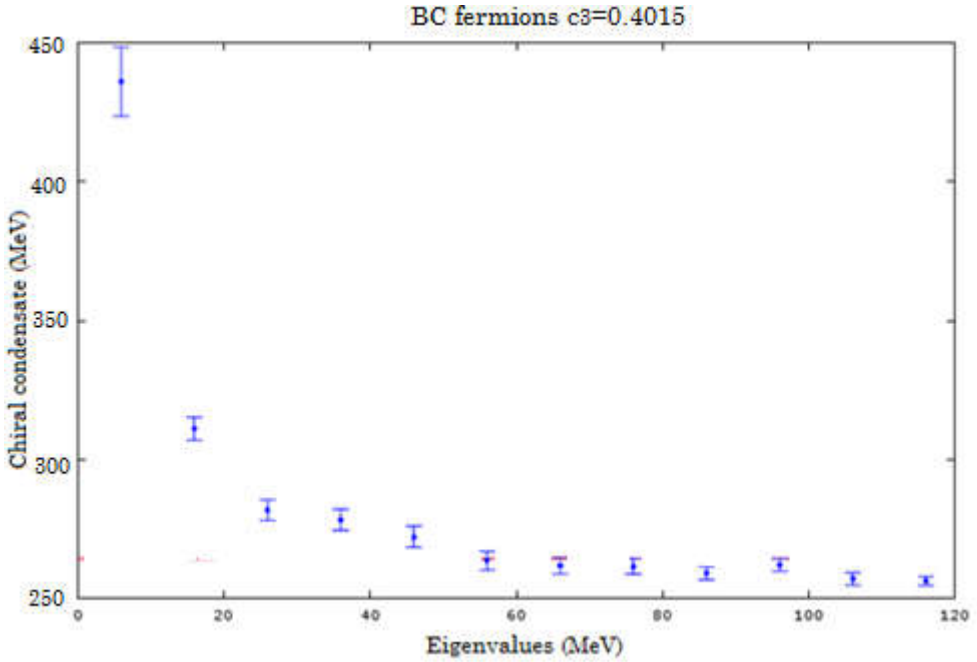
Wilson gauge action ($\beta = 6$, $\beta = 5.85$)

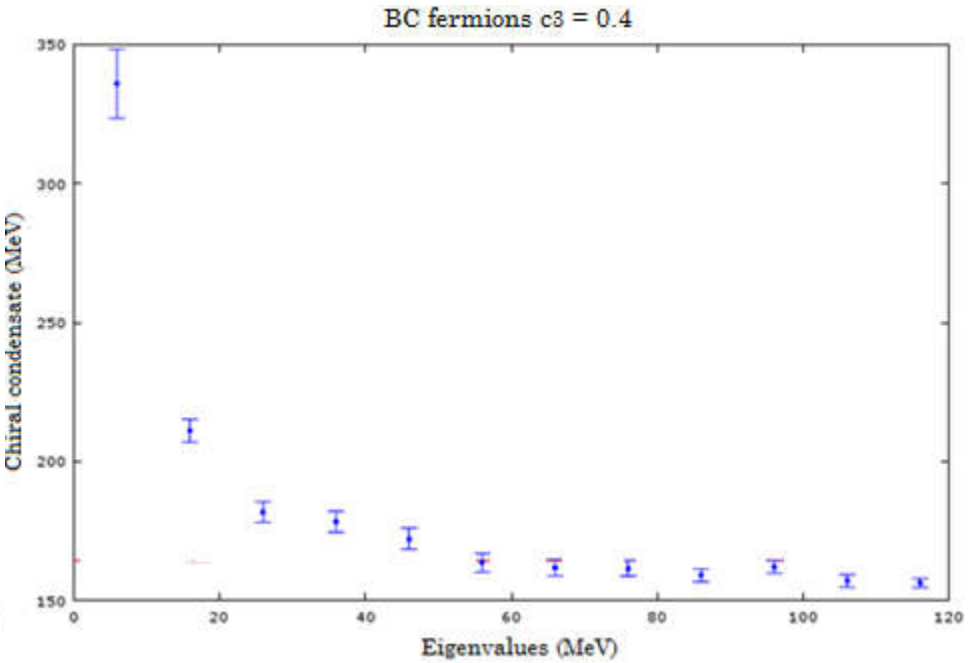
Boriçi – Creutz action → c_3 almost independent from a

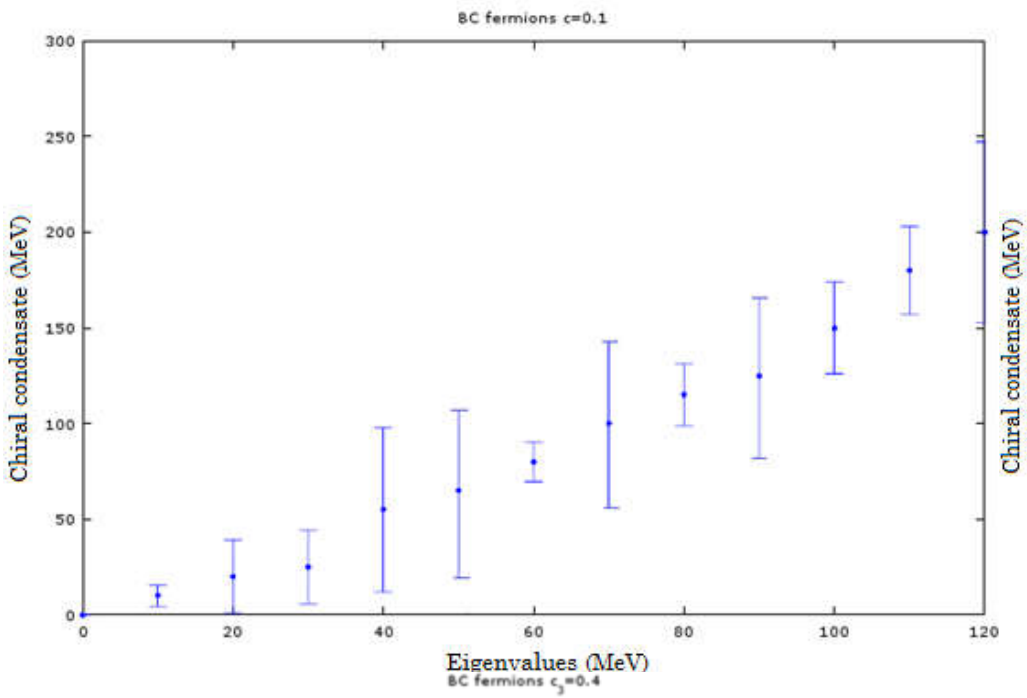
Zero quark mass (BC fermions are chiral fermions)

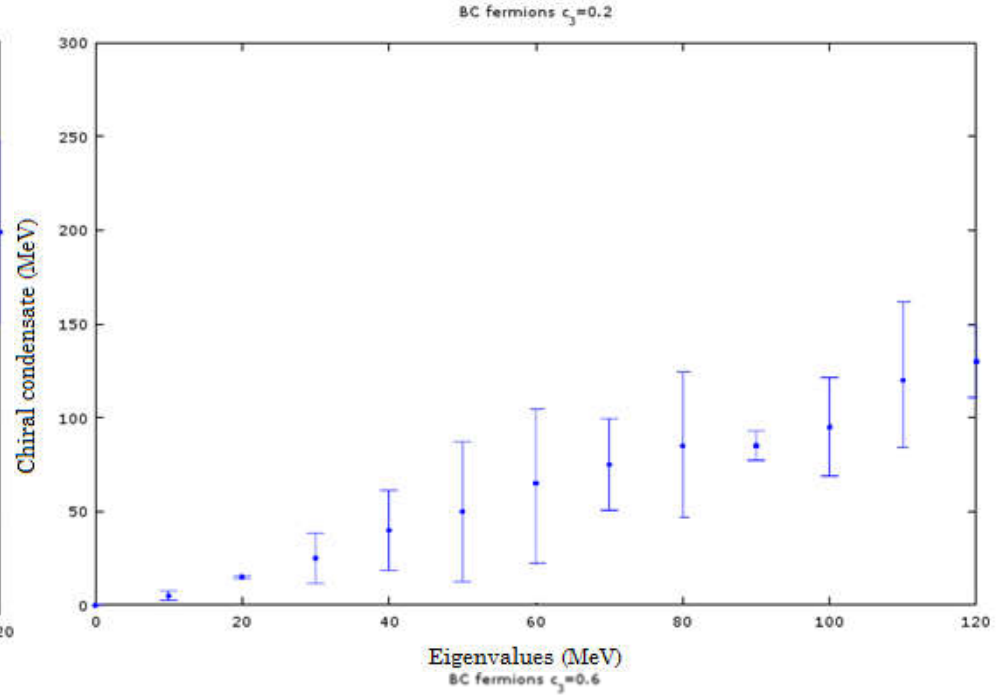
Different values of counterterms c3 (0.1, 0.2 0.35, 0.4, 0.6,0.8,1)

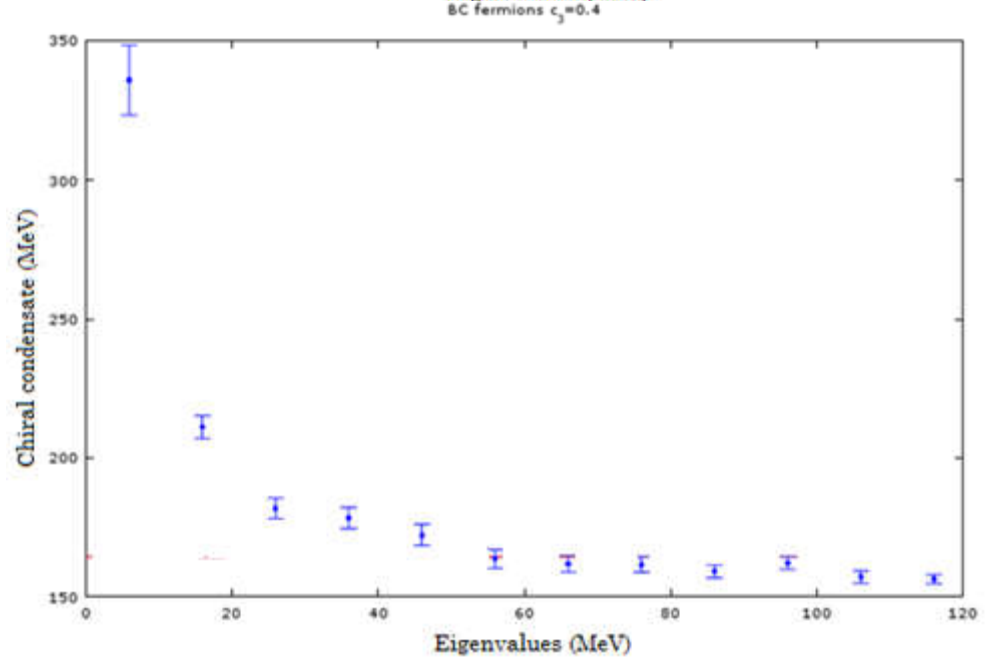
Gauss – Lanczos algorithm

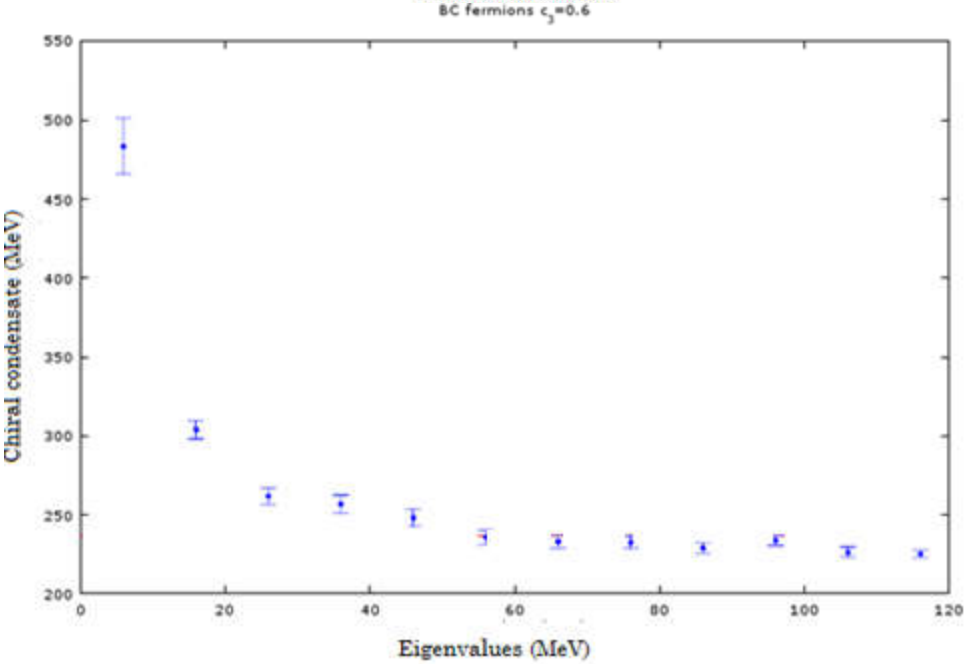












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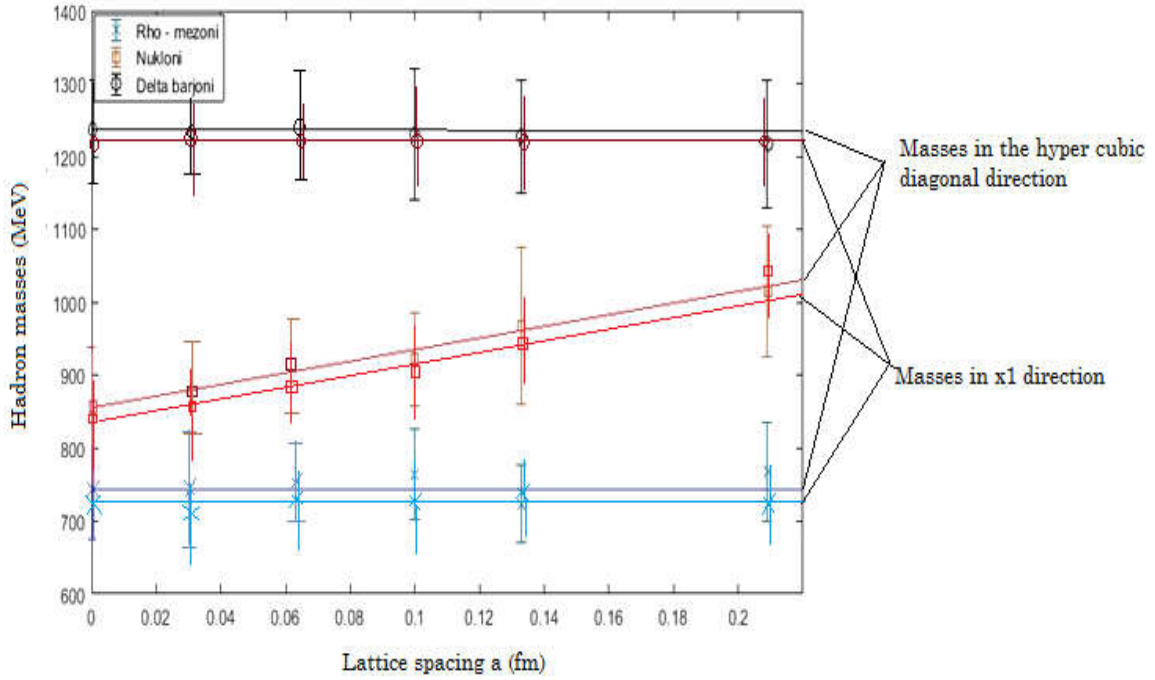
Light hadrons masses



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Hadron	Masses using the diagonal (MeV/c2)	Masses using the x1 direction(MeV/c2)
Rho - meson	741	690
Nucleon	849	805
Delta	1240	1187

BC without adding c_3

Hadron	Masses using the diagonal (MeV/c2)	Masses using the x1 direction(MeV/c2)
Rho - meson	741	730
Nucleon	860	852
Delta	1240	1236

Corrected BC by adding c_3

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Summary & Conclusions

- Boriçi - Creutz fermions (minimally doubled fermions) are known to preserve chiral symmetry for a degenerate quark doublet and are strictly local.
- These fermions are ideal for studying QCD with *u* and *d* quarks, can be helpful for Nf = 2 lattice simulations, relatively simpler and possibly faster than Ginsparg-Wilson fermions.
- The chiral condensate can be used as an order parameter for BC fermions, and help us to find the proper counter-term that restore partially the broken hyper-cubic symmetry.
- Tuning of counter-term coefficient c3 doesn't seem difficult and expensive, because apparently it is almost independent from the lattice spacing and coupling constant (further studies)
- Tuning c4 will require more care and work so as to properly restore the desired symmetry, because it depends on the lattice and the coupling constant we use on simulations. (further studies)
- Using BC fermions with the added counter-term c3, the difference between the light hadrons masses calculated in different directions decrease, so we can think that partially have restore the broken hyper - cubic symmetry.
- Anyway….BC fermions, seem to be not the most suitable fermions for spectroscopy in full QCD

THANK YOU FOR YOUR ATTENTION!