Gradient-flow scale setting with $N_f = 2 + 1 + 1$ Wilson-clover twisted-mass fermions

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Extended Twisted Mass Collaboration

Scale-setting in the high precision era

- Lattice QCD has entered the high-precision era: sub-1% errors desired for many physical quantities
- Renormalisation and scale-setting often limiting factors
- Need high precision scaling quantities with controlled systematic errors and overall errors of few per-mille

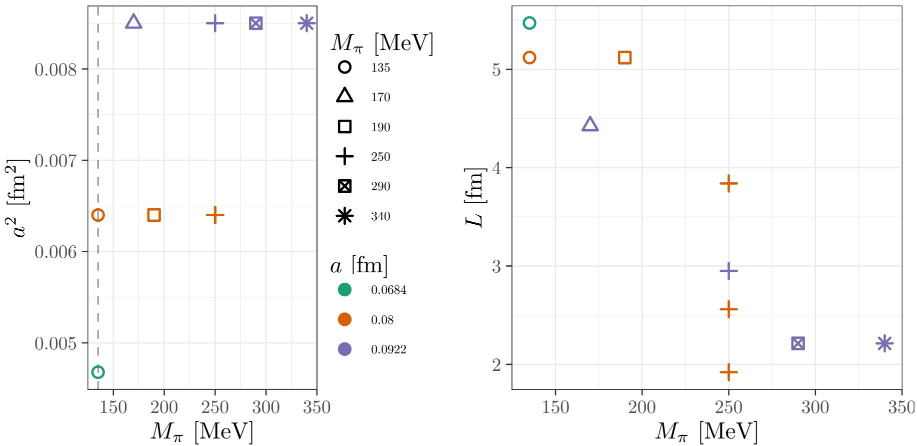
⇒ Try gradient flow scales to achieve this

This work done as part of the calculation of f_K/f_π and the determination of the quark masses in [arXiv:2104.06747] and [arXiv:2104:13408].

Caveats

- ongoing project, results "publication-ready" but will evolve
- use $f_\pi^{\rm iso}$ to set the scale, $\frac{M_\pi^{\rm iso}}{f_\pi^{\rm iso}}$ to fix the physical light quark mass point
- no QED / strong IB effects taken into account
  - in the future: include QED and strong IB effects, use M_Ω & other quantities to set the scale

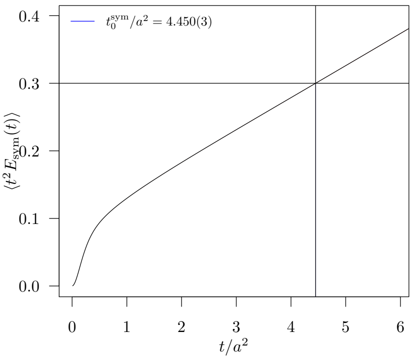
$N_f = 2+1+1$ Wilson-clover twisted-mass ensemble landscape



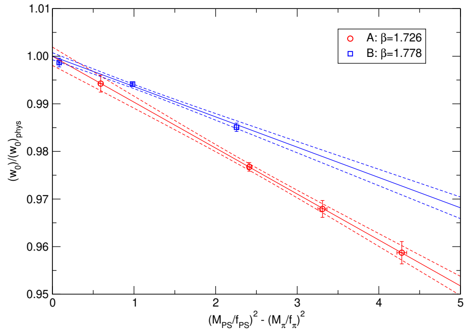
- $9+1$ ensembles (smallest one only to check FSE)
- $M_\pi : 135 - 350$ MeV, $a \in [0.0684, 0.0800, 0.0922]$ fm, $L : 2 - 5.5$ fm
- renormalised sea strange & charm quark masses: constant (physical) to within few %

ETMC gradient-flow scale-setting

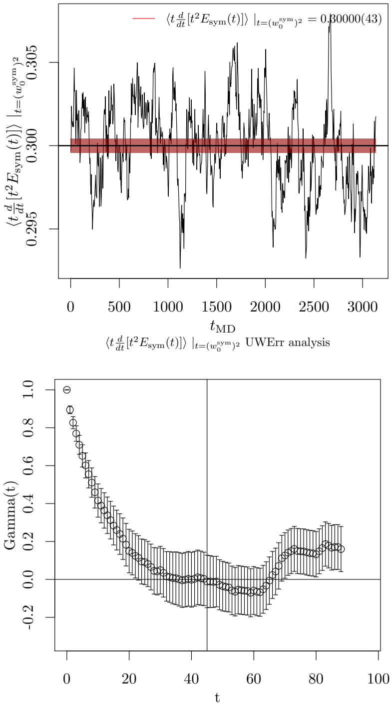
Use tmLQCD and follow [M. Lüscher, JHEP08(2010)071], [S. Borsányi et al., JHEP09(2012)010] and [O. Bär & M. Golterman, Phys.Rev.D 89(2014)3 and 89(2014)9].



- evolve gauge field in t using Wilson plaq. $S_{\rm W}$
  - $\dot{V}_t(x,\mu) = -g_0^2 \left[\partial_{x,\mu} S_{\rm W}(V_t)\right] V_t(x,\mu)$ with
  - $V_t(x,\mu)|_{t=0} = U(x,\mu)$
- clover discretisation of $E = \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu}$
- set gradient flow scale by requiring
  - $t = t_0/a^2$ when $t^2 \langle E(t) \rangle = 0.3$
  - $t = w_0^2/a^2$ when $t \frac{d}{dt}\left[t^2 \langle E(t)\rangle\right] = 0.3$
- extrapolation to phys. point using 2-param fit at each β
  - $w_0/w_0^{\rm phys} = 1 + c \cdot \left[\left(\frac{M_{\rm PS}}{f_{\rm PS}}\right)^2 - \left(\frac{M_\pi^{\rm iso}}{f_\pi^{\rm iso}}\right)^2 \right]$
    - ★ $M_{\rm PS}$ and $f_{\rm PS}$ suitably FSE-corrected
    - ★ linear Ansatz works well for w_0 (and t_0)
- use $w_0^{\rm phys}$ as scaling quantity in subsequent analysis



Autocorrelations and sampling quality



- Typical sampling quality across ensemble landscape (here $80^3 \cdot 160$ lattice at phys. quark masses, finest lattice spacing)
- Error analysis using Gamma method [U. Wolff, Comput.Phys.Commun. 156 (2004)]
  - For some ensembles we *might* require exponential tails (but effect should be minimal)
- Statistical errors on $(w_0/a)^{\rm phys}$ between 0.75 and 1.9 per-mille

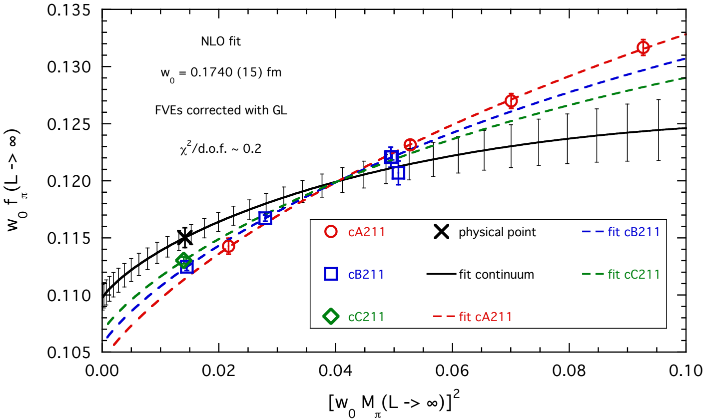
β	$(w_0/a)^{\rm phys}$ [fm]	$(\sqrt{t_0}/a)^{\rm phys}$ [fm]
1.726	1.8352(35)	1.5660(22)
1.778	2.1299(16)	1.80396(68)
1.836	2.5045(27)	2.1094(8)

- drop "phys" label in the following: $w_0^{\rm phys} \to w_0$

Scale setting with f_π

- NLO χ-PT fit of $w_0 f_\pi$ with Gasser-Leutwyler FSE-corrections in $f_{\rm PS}$ and $M_{\rm PS}$

$$w_0 f_\pi(L \to \infty) = w_0 f \left[1 - 2\xi \log(\xi) + 2A_1\xi + A_2\xi^2 + \frac{a^2}{w_0^2}(D_0 + D_1\xi)\right]$$



- $\xi = \frac{M_{\rm PS}^2(L\to\infty)}{(4\pi f)^2}$
- $A_2 = 0$
  - $\chi^2/\text{d.o.f.} \sim 0.2$
- $D_1 \cdot a^2 M_{\rm PS}^2$ term important to describe the data well
- systematics: allow for A_2 and D_1 independently

$w_0 = 0.17390(157)(030)$ fm $\qquad f = 124.0(6)(7)$ MeV $\qquad \bar{\ell}_4 = 3.44(27)(36)$

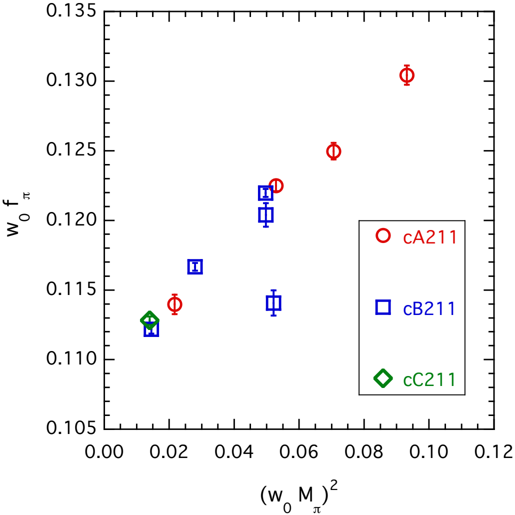
9.2 per-mille error on w_0 too large!

S. Simula's improved quantity X_π

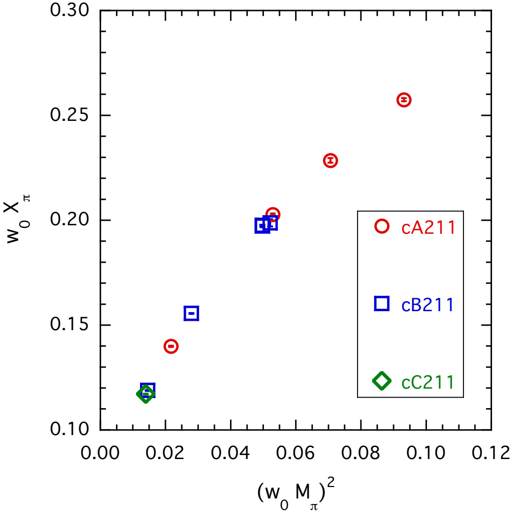
$$X_\pi = (f_\pi M_\pi^4)^{(1/5)}$$

- much less affected by lattice artefacts
- stat. errors smaller by factor of ~ 2 or more

$w_0 f_\pi$



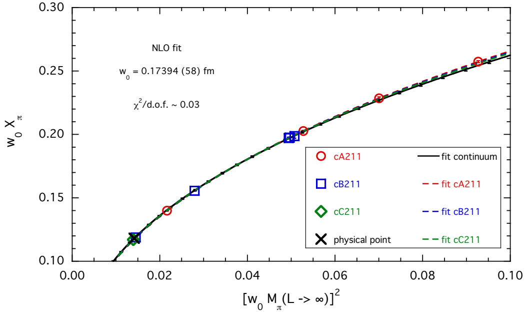
$w_0 X_\pi$



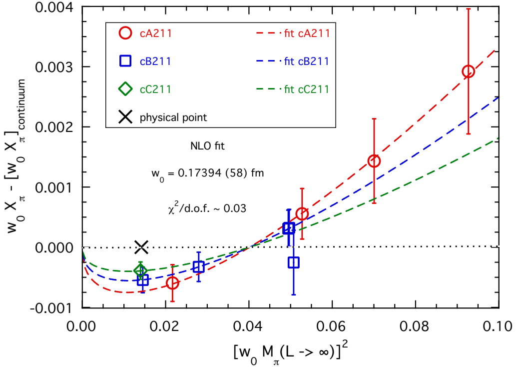
● $\beta = 1.726$ ■ $\beta = 1.778$ ◆ $\beta = 1.836$

Residual lattice artefacts in $w_0 X_\pi$

$$w_0 X_\pi = w_0 \left(f_\pi M_\pi^4 \right)^{(1/5)}$$



$w_0 X_\pi - [w_0 X_\pi]_{\mathrm{continuum}}$



$$w_0 X_\pi = (w_0 f) \left\{ (4\pi)^4 \xi^2 \left[1 - 2\xi \log(\xi) + 2A_1 \xi + A_2' \xi^2 + a^2 \left(D_0' + D_1' \xi \right) \right] \right\}^{1/5}$$
$$\cdot \left(1 + F_{FVE}\, \xi^2 e^{-M_\pi L} / (M_\pi L)^{3/2} \right)$$

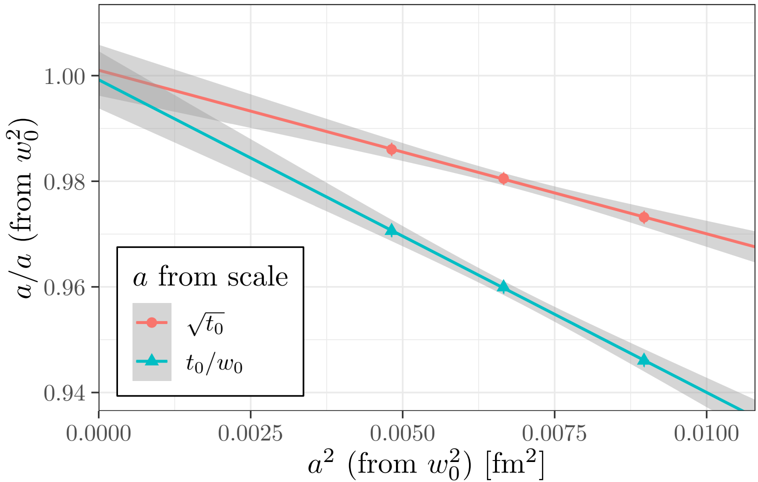
- $\chi^2/\mathrm{d.o.f.} \sim 0.03$, $\mathcal{O}(a^2)$- and $\mathcal{O}(a^2 M_{\mathrm{PS}}^2)$-effects at few-per-mille level
- FSE start at NNLO $\mathcal{O}(\xi^2)$, residual effects at the level of stat. err.

$w_0 = 0.17383(57)(26)$ fm $\qquad f = 124.0(1.2)(0.7)$ MeV $\qquad \bar{\ell}_4 = 3.43(28)(36)$

~ 3.6 per-mille total error on w_0

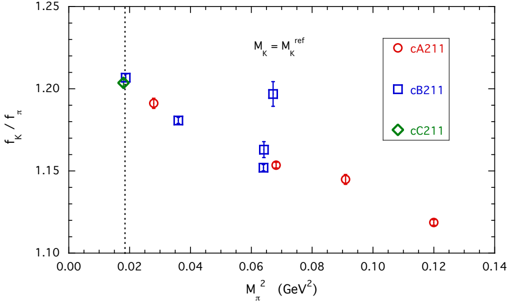
Relative scaling of w_0, t_0 and t_0/w_0

GF scale	$a(\beta=1.726)$ [fm]	$a(\beta=1.778)$ [fm]	$a(\beta=1.836)$ [fm]
w_0	0.09471 (39)	0.08161 (30)	0.06941 (26)
$\sqrt{t_0}$	0.09217 (41)	0.08002 (34)	0.06844 (29)
t_0/w_0	0.08960 (47)	0.07834 (41)	0.06737 (35)



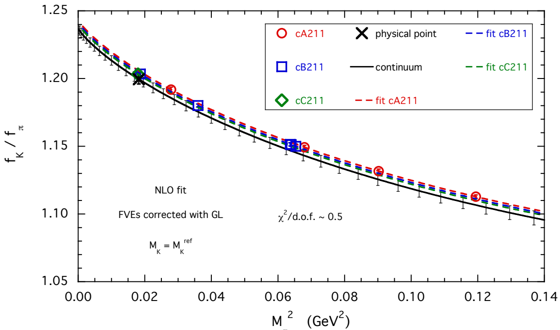
differences nicely compatible with a^2-scaling, but up to 5% at coarsest lattice spacing!

Determination of f_K/f_π in iso-symmetric QCD



- use input lattice spacing(s) from GF scale-setting in fixing valence m_s
- interpolate each $f_K/f_{\rm PS}$ to reference kaon mass $M_K^{\rm ref} \equiv \sqrt{(M_K^{\rm iso})^2 + \frac{M_{\rm PS}^2 - (M_\pi^{\rm iso})^2}{2}}$
- $\frac{f_K}{f_\pi}(L \to \infty) = \frac{f_K}{f_\pi}(L)\left[1 - \frac{5}{4}\overline{\xi}_\pi\ \widetilde{g}_1(M_\pi L)\right]$
- $\overline{\xi}_\pi = \frac{M_{\rm PS}^2(L)}{(4\pi f)^2}$

$$\frac{f_K}{f_\pi}(L \to \infty) = R_0\left[1 + \frac{5}{4}\xi\log(\xi) + R_1\xi + R_2\xi^2 + \frac{a^2}{w_0^2}\left(\widetilde{D}_0 + \widetilde{D}_1\xi\right)\right]$$

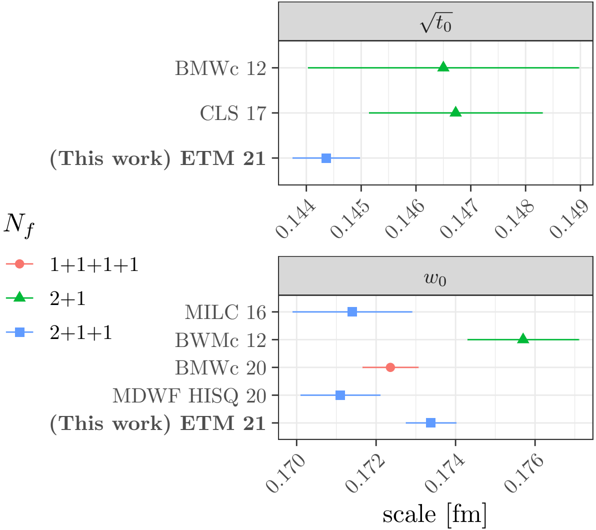


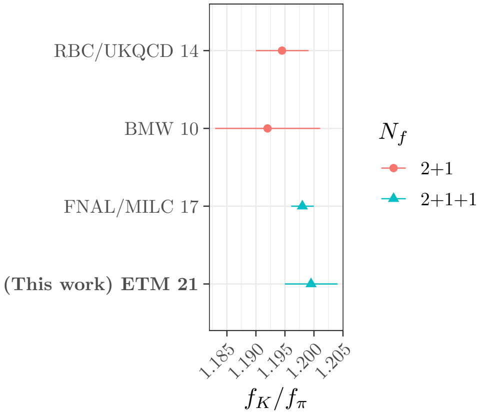
- $\chi^2/\text{d.o.f.} \sim 0.5$
- systematics
  - R_2
  - $\tilde{D}_1$
  - $M_{\rm PS}$ range

$$\left(\frac{f_K}{f_\pi}\right)^{\rm iso} = 1.1995(44)(7)$$

error of ~ 3.8 per-mille

Comparison to other lattice results and outlook





total error reduced by factor > 3
compared to old ETM result (not shown)

Outlook

- fourth lattice spacing & larger volumes at phys. point
- QED & strong IB breaking effects, M_Ω scale setting

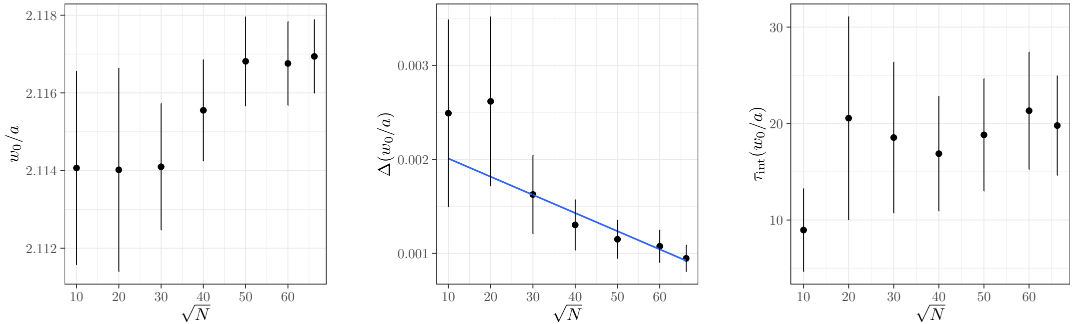
Thanks a lot for your attention!

ETMC Wilson-clover twisted-mass talks at this conference

- Jacob Finkenrath, *"Twisted mass gauge ensembles at physical values of the light, strange and charm quark masses"*, Algorithms, Friday, July 30th, 07:15 local time
- Gustavo Remirez-Hidalgo, *"Coarsest-Level Improvements of Multigrid for Lattice QCD on Large-Scale Computers"*, Algorithms, Friday July 30th, 07:30 local time
- Shuhei Yamamoto, *"Implementation of Simultaneous Inversion of a Multi-shifted Dirac Matrix for Twisted-Mass Fermions within DDαAMG"*, Algorithms, Friday July 30th, 07:45 local time
- Petros Dimopoulos, *"K- and $D(s)$-meson leptonic decay constants with physical light, strange and charm quarks by ETMC"*, Standard Model Parameters, Thursday, July 29th, 05:30 local time
- Constantia Alexandrou, *"Determination of the light, strange and charm quark masses using twisted mass fermions"*, Standard Model Parameters, Wednesday, July 28th, 13:00 local time
- Matteo Di Carlo, *"Renormalization constants of quark bilinear operators in QCD with dynamical up, down, strange and charm quarks"*, Standard Model Parameters, Friday 30th, 06:15 local time
- Bartosz Kostrzewa, *"Gradient-flow scale setting with $N_f = 2 + 1 + 1$ Wilson-clover twisted-mass fermions"*, Hadron Spectroscopy, Friday 30th July, 06:00 local time
- Sebastian Andreas Burri, *"Pion Pole Contribution to HLbL from Twisted Mass Lattice QCD at the physical point"*, QCD Searches for BSM Physics, Tuesday, July 27th, 06:45 local time
- Giannis Koutsou, *"Nucleon form factors from N_f=2+1+1 twisted mass QCD at the physical point"*, Hadron Structure, Tuesday 27th July, 07:30 local time
- Martha Constantinou, *"First study of twist-3 PDFs and GPDs for the proton from Lattice QCD"*, Hadron Structure, Thursday 29th July, 14:00 local time
- Kyriakos Hadjiyiannakou, *"Decomposition of the proton spin from lattice QCD"*, Hadron Structure, Thursday 29th July, 13:00 local time
- Aniket Sen, *"Nuclear Parity Violation from 4-quark Interactions"*, Hadron Spectroscopy, Thursday 29th July, 07:45 local time
- Aurora Scapellato, *"Generalized parton distributions of the proton from lattice QCD"*, Hadron Structure, Thursday 29th July, 13:30 local time
- Colin Lauer, *"x-dependence reconstruction of pion and kaon PDFs from Mellin moments"*, Hadron Structure, Wednesday July 28th, 21:00 local time
- Ferenc Pittler, *"Elastic $\pi - N$ scattering in the $I = 3/2$ channel"*, Hadron Spectroscopy, Tuesday 27th July, 06:30 local time
- Antonino Todaro, *"Neutron electric dipole moment using lattice QCD simulations at the physical point"*, QCD Searches for BSM Physics, Wednesday 28th July, 07:15 local time
- Floriano Manigrasso, *"Flavor decomposition for the proton unpolarized, helicity and transversity parton distribution functions"*, Hadron Structure, Tuesday 27th July, 14:30 local time

Backup Slide: Do we trust our statistical errors on w_0/a?

- $V/a^4 = 64^3 \cdot 128$, $M_\pi \sim 190$ MeV, $a \sim 0.08$ fm, ~ 4400 trajectories (length $\tau = 1.5$)
- below *value*, *stat. error* and $\tau_{\rm int}$ @ $100, 400, 900, 1600, 2500, 3600, 4400$ trajectories



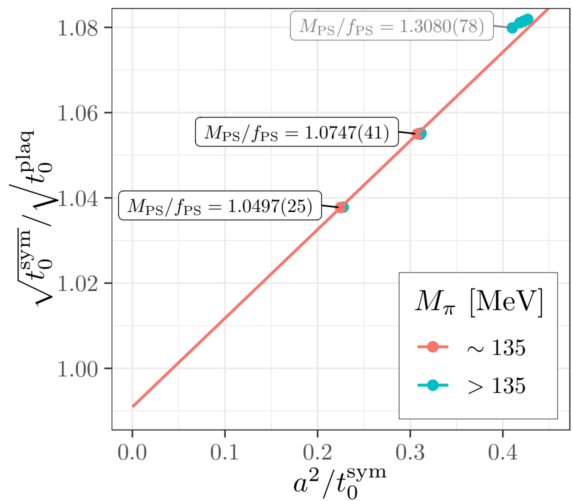
- starting at about 900 trajectories
  - reasonable error scaling
  - apparently reliable determination of $\tau_{\rm int}$
  - still, central value moves by about $1\ \sigma$ going to 4400 trajectories

$\Rightarrow$ we mostly trust our statistical errors on w_0/a on all ensembles
  - Exception: ensemble at $M_\pi \sim 170$ MeV & $a \sim 0.092$ fm (pion mass splitting too large)

Backup Slide: Do we trust our continuum-extrapolation?

- of course, we'd like to have more lattice spacings → fourth one on the way
- mild a^2-scaling violations seen in ratios of gluonic scales in the past (e.g. [Bruno et al., PoS LAT'13,321] and [JHEP02(2015)043])

⇒ let's take a (quick) look at $\sqrt{t_0^{\mathrm{sym}}}/\sqrt{t_0^{\mathrm{plaq}}}$ too



- "extrapolate" only using $M_\pi \sim 135$ MeV data (two finest a)
- in continuum limit: deviate by about 1% from 1.0
  - ! only two points used in this test
  - ! points not quite at matched $M_{\mathrm{PS}}/f_{\mathrm{PS}}$
- **on the other hand**: good relative scaling of a-determinations (slide 9)
- certainly need to study this more ...