S-wave $\pi\pi$ I=0 and I=2 scattering with physical pion mass

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Why shall we perform this calculation?

- Important component in $K \to \pi\pi$ calculation
  - Energy and amplitude are needed in determining lattice matrix element
  - Phase shift and its derivative are necessary in LL factor calculation
  - Information about the final $\pi\pi$ state in the decay process
- 2015 results gives $\pi\pi$ energy which is 3σ(7σ with more statistics) higher than the phenomenological prediction(Pi-Pi puzzle)

$$\begin{aligned}\delta_0 &= 23.8(4.9)(1.2)^\circ(PRL, 2015)\\ &= 19.1(2.5)^\circ(1386 confs)\\ &\approx 36^\circ(Dispersion)\end{aligned}$$

- First physical pion mass with disconnected diagram
- A calculation on GPBC lattice

Calculation details and techniques

General techniques:

- G-parity boundary condition
- All to all propagator
- Time separated pipi operator

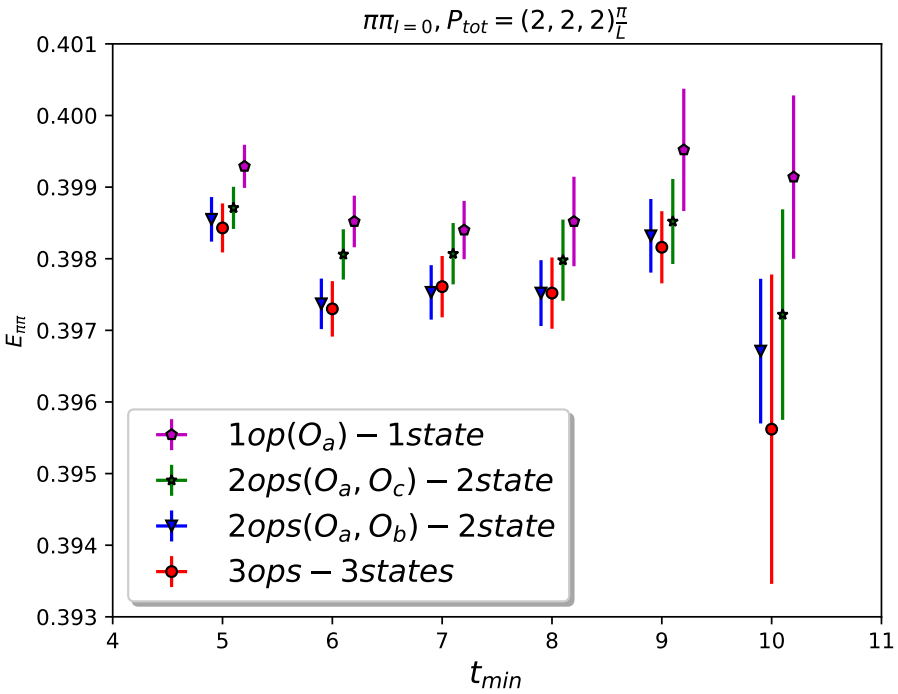
Improvements:

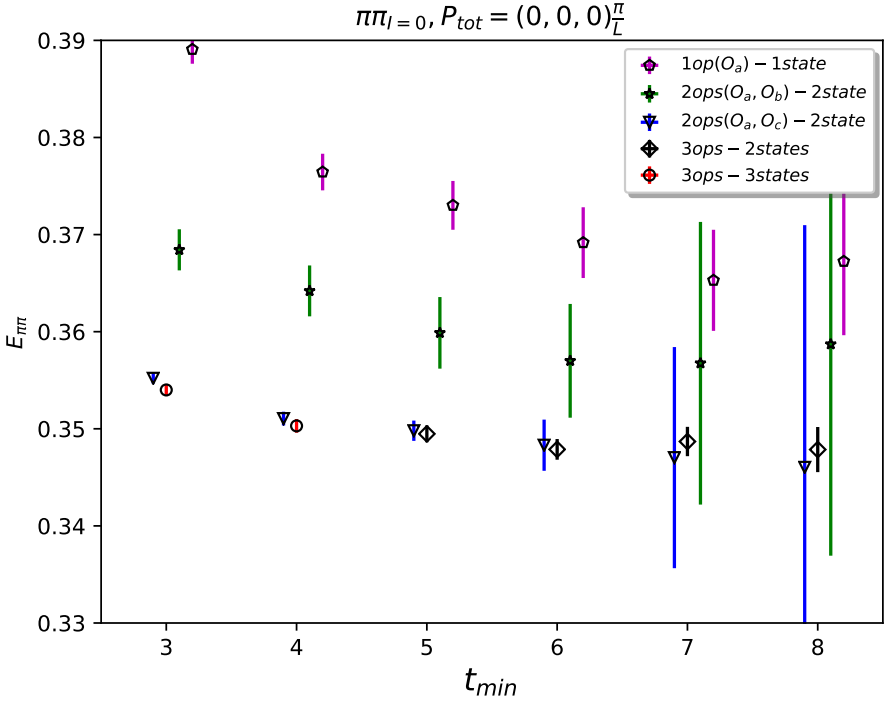
- Adding more operators
  $I = 0$: New σ operators $(\bar{u}u + \bar{d}d)$ with zero momentum.
  $I = 0$ and $I = 2$: $\pi(311)$ operator with momentum $(\pm\frac{3\pi}{L}, \pm\frac{\pi}{L}, \pm\frac{\pi}{L})$
- Using Non-overlapping blocked bootstrap to obtain a more reliable p-value in order to search for the optimal fitting setup
- Use Luscher's formula (single channel) generalized to moving frame and GPBC lattice to obtain scattering phase shift at corresponding finite volume energy

Fitting strategies and results

$$C_{ij}(t_{snk}, t = t_{snk} - t_{src} - \Delta_j) = \langle O_i^\dagger(t_{snk}) O_j(t_{src})\rangle - \langle 0|O_i(t_{snk})|0\rangle\langle 0|O_j(t_{src})|0\rangle \times \delta_{I,0} \quad (1)$$

$$C_{ij}(t) = \sum_{x=1}^{N} A_{ix} A_{jx} \left(e^{-E_x t} + e^{-E_x(T-t-\Delta_i-\Delta_j)}\right) + B_{ij} \times \delta_{I,2} \quad (2)$$





- Some of the additional operators significantly improve the result
- We decide to include all operators

Current result

P_{tot}	l	$E_{\pi\pi}$	k	$\sqrt{s}$	δ
$(0,0,0)\frac{\pi}{L}$	0	0.3479(11)	193.0(9)	471.0(1.5)	**32.3(1.0)°**
$(2,0,0)\frac{\pi}{L}$	0	0.3774(23)	170.6(2.4)	435.1(3.8)	24.0(3.4)°
$(2,2,0)\frac{\pi}{L}$	0	0.3895(17)	123.2(2.6)	365.6(3.4)	18.0(4.5)°
$(0,0,0)\frac{\pi}{L}$	2	0.4153(4)	248.4(3)	565.4(5)	-11.0(2)°
$(2,0,0)\frac{\pi}{L}$	2	0.4045(3)	197.9(2)	479.1(3)	-7.96(23)°
$(2,2,0)\frac{\pi}{L}$	2	0.4001(3)	138.4(3)	386.7(4)	-4.48(40)°
$(2,2,2)\frac{\pi}{L}$	2	0.3984(3)	14.4(2.1)	271.5(4)	-0.32(20)°

$$\begin{aligned}\delta_0 &= 23.8(4.9)(1.2)^\circ(PRL, 2015)\\ &= 19.1(2.5)^\circ(1386 confs)\\ &\approx 36^\circ(Dispersion)\\ &= \mathbf{32.3(1.0)^\circ}(741 confs)\end{aligned}$$

- Better control over excited state error
- Better statistical error with smaller number of confs

Excited state error

- Simply look into plateau region is less reliable due to rapid increase of error as a function of $t_{\min}$
- Including additional state in fitting function makes the result much noisier, even if we freeze the energy of that state
- For $I = 0$ channel, the overlap matrix in moving frames are highly diagonal, and different overlaps have similar size in stationary frame
- As CM momentum goes up, $\sqrt{s}$ decrease, phase shift decrease, interaction decrease

$P_{CM} = (2,2,2)$	state_0	state_1	state_2
$\pi\pi(111,111)$	1.0(0.0)	-0.07(1)	-0.035(8)
$\pi\pi(111,311)$	-0.013(6)	1.0(0.0)	-0.19(5)
$\pi\pi(311,311)$	-0.015(2)	0.05(2)	1.0(0.0)

$P_{CM} = (0,0,0)$	state_0	state_1	state_2
$\pi\pi(111,111)$	1.0(0.0)	0.47(2)	0.31(7)
$\pi\pi(311,311)$	0.053(9)	-0.84(12)	1.0(0.0)
σ	1.0(0.0)	-0.83(3)	-0.87(22)

- Use different method on data with different pattern

Excited state error

- Use optimal fit with one extra state
- Energy is obtained from dispersive model
- Small intercoupling between states and operators $\Rightarrow$ obtain overlap factors from **single operator fit**
- Large intercoupling between states and operators $\Rightarrow$ obtain overlap factors from **multiple operator fit with smaller $t_{\mathbf{min}}$**
- Calculate the maximum energy difference between two fits

Final result

P_{tot}	I	$\sqrt{s}$(MeV)	δ	$\Delta\delta_{\rm dis}$	$\Delta\delta_{\rm FV}$	$\Delta\delta_{\rm unphy}$	$\Delta\delta_{\rm exc}$
$(0,0,0)\frac{\pi}{L}$	0	471.0	32.3(1.0)(1.4)	0.64	0.32	0.83	**0.90**
$(2,0,0)\frac{\pi}{L}$	0	435.1	24.0(3.4)(7.6)	0.46	0.23	0.71	**7.6**
$(2,2,0)\frac{\pi}{L}$	0	365.6	18.0(4.5)(4.9)	0.36	0.18	0.47	**4.9**
$(0,0,0)\frac{\pi}{L}$	2	565.4	-10.98(22)(44)	0.20	0.10	0.18	**0.34**
$(2,0,0)\frac{\pi}{L}$	2	479.1	-7.96(23)(29)	0.16	0.08	0.03	**0.23**
$(2,2,0)\frac{\pi}{L}$	2	386.7	-4.48(40)(77)	0.09	0.04	0.06	**0.76**
$(2,2,2)\frac{\pi}{L}$	2	271.5	-0.32(20)(63)	0.01	0.00	0.02	**0.63**

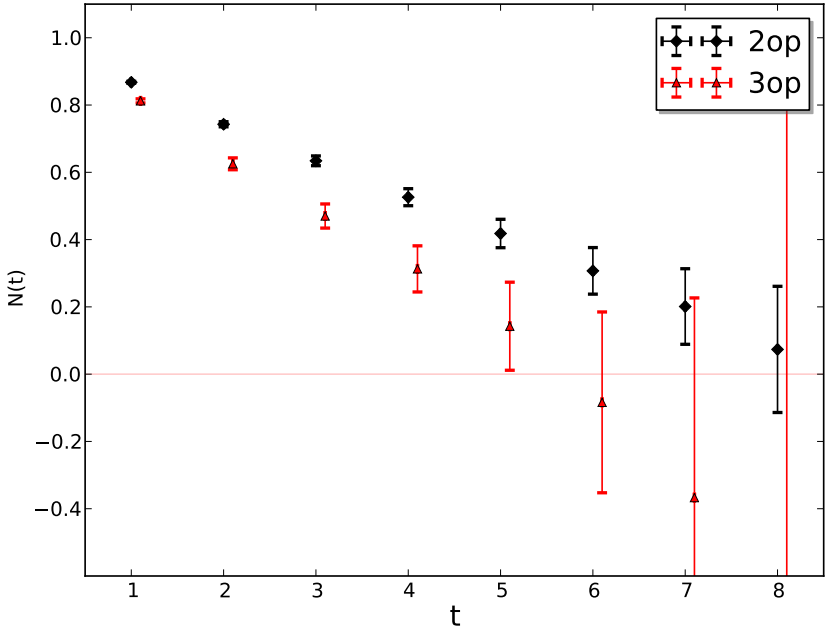
- Small error with stationary $I = 0$
- Huge error with moving $I = 0$

Normalized determinant

A good test for whether the residual excited state error is small

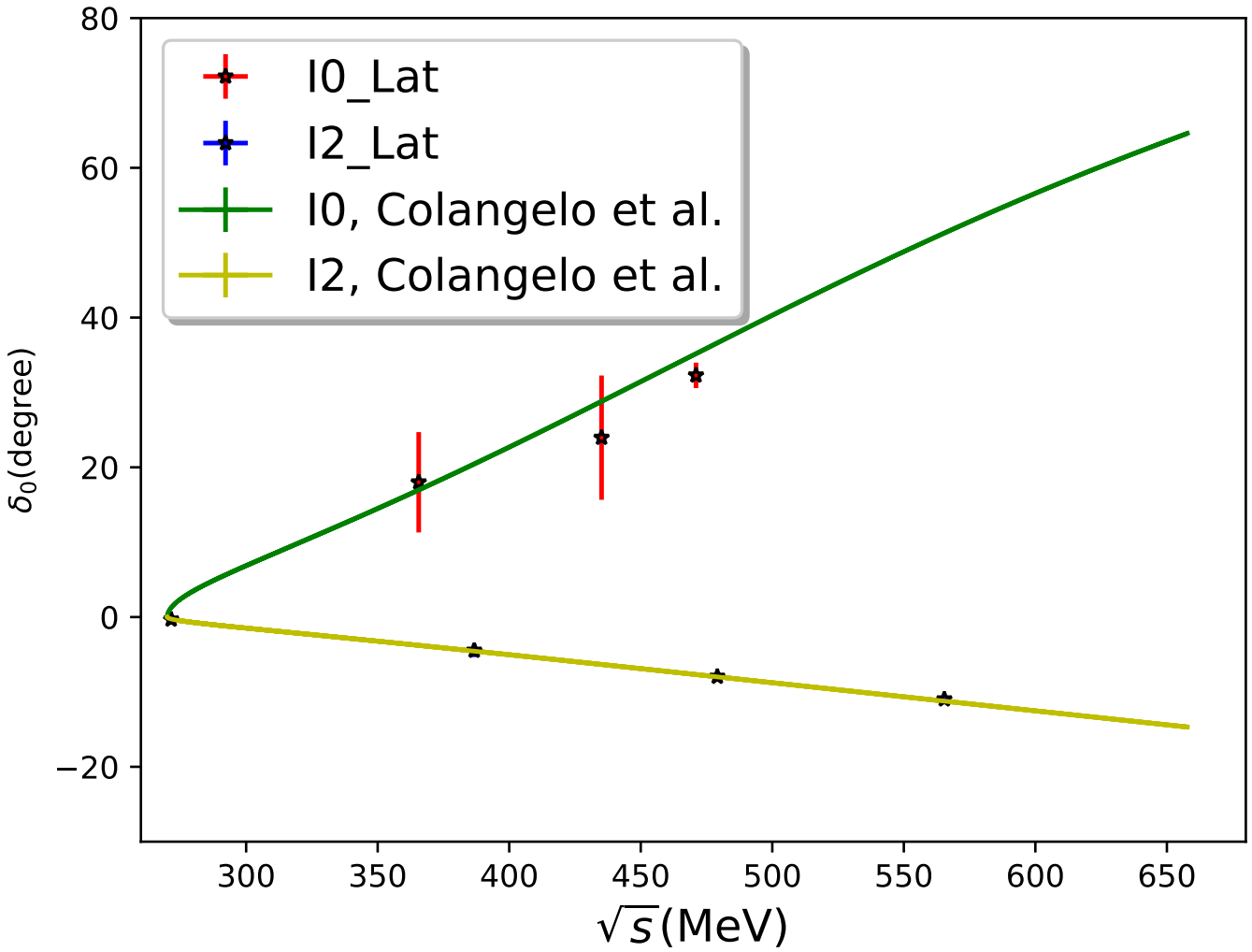
$$\mathcal{N}(t) = \frac{\mathrm{Det}(C(t))}{\prod_{i=1}^{N} C_{ii}(t)} \tag{3}$$

- Scale invariant
- #states that contribute to $C(t)$ is smaller than $N \Rightarrow \mathcal{N}(t) = 0$
- $\mathcal{N}(t) \neq 0 \Rightarrow$ At least N distinct states contribute to $C(t)$
- Allow us to examine the excited state error on a single time slice



In our 2015 calculation with single operator and fit range of $6 - 25$, we have a huge residual excited state error

Final plot


I0_Lat
I2_Lat
I0, Colangelo et al.
I2, Colangelo et al.
80
60
40
20
0
−20
δ_0(degree)
300
350
400
450
500
550
600
650
$\sqrt{s}$(MeV)

Conclusion and future steps

Conclusions:

- Calculate $\pi\pi$ scattering with $I = 0$ and $I = 2$ at seven different energies
- Estimate all major systematic error, including the previously underestimated excited state error carefully

Future steps:

- Add moving σ operator in moving frame $I = 0$ calculation
- Using ensemble different lattice spacing to evaluate at continuum limit
- Combine this result with other lattice results to improve the dispersive prediction