A light hybrid resonance from lattice QCD

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hadspec
hadspec.org

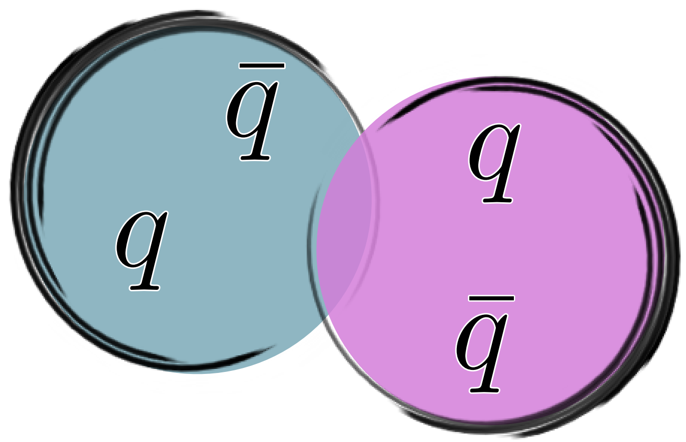
Lattice 2021
Friday 30th July

UNIVERSITY OF CAMBRIDGE

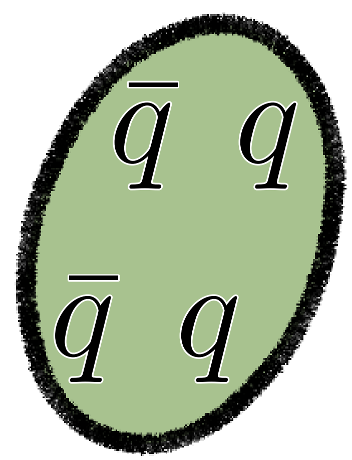
THE ROYAL SOCIETY

beyond the quark model:

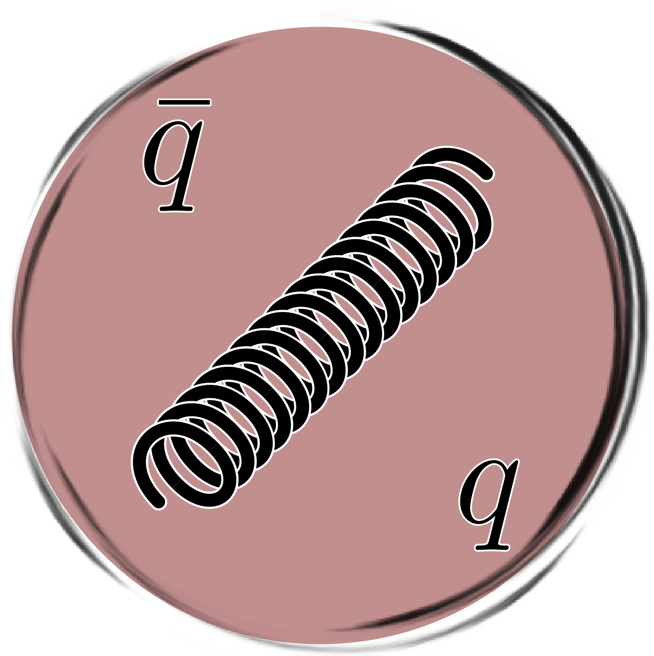
- molecules



- tetraquarks

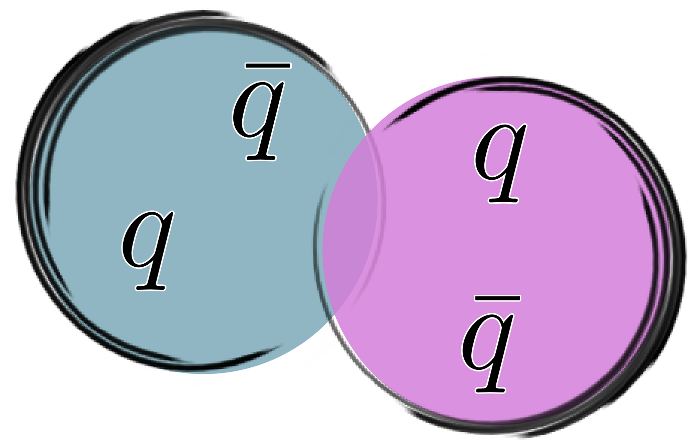


- hybrids

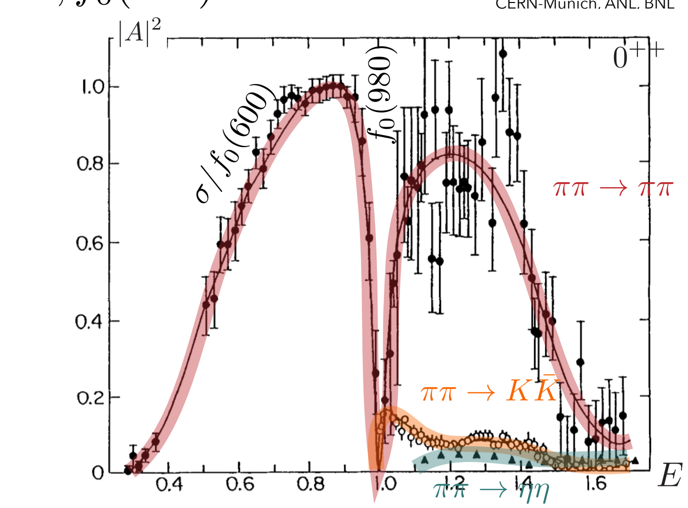


beyond the quark model:

- molecules



σ, $f_0(980)$?



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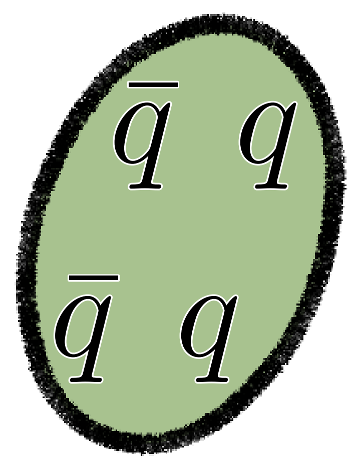
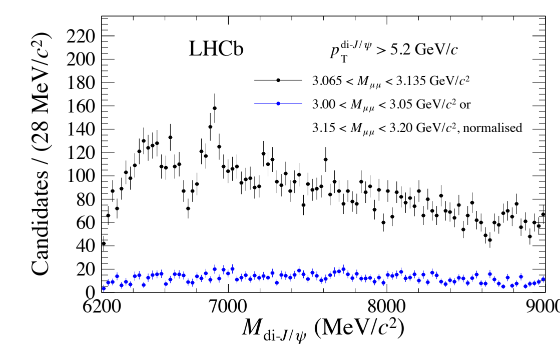
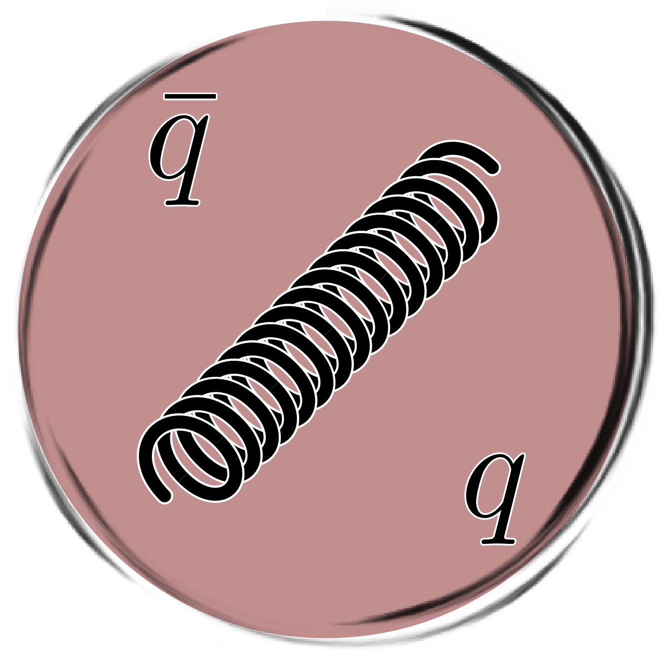


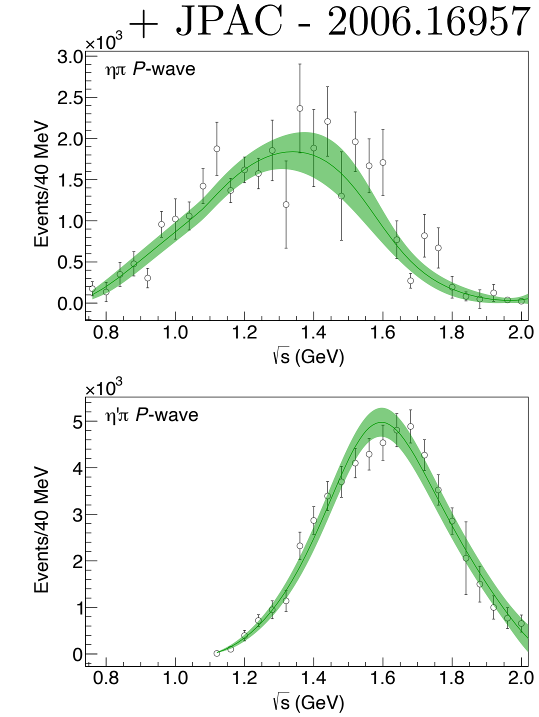
$X(6900)$?



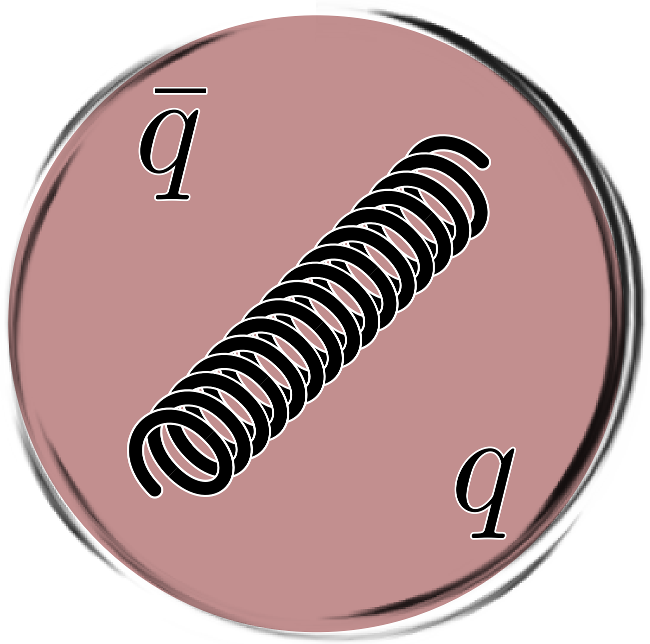
- hybrids



$\pi_1(1564)$ COMPASS + JPAC - 2006.16957



$\pi_1 \quad J^{PC} = 1^{-+}$



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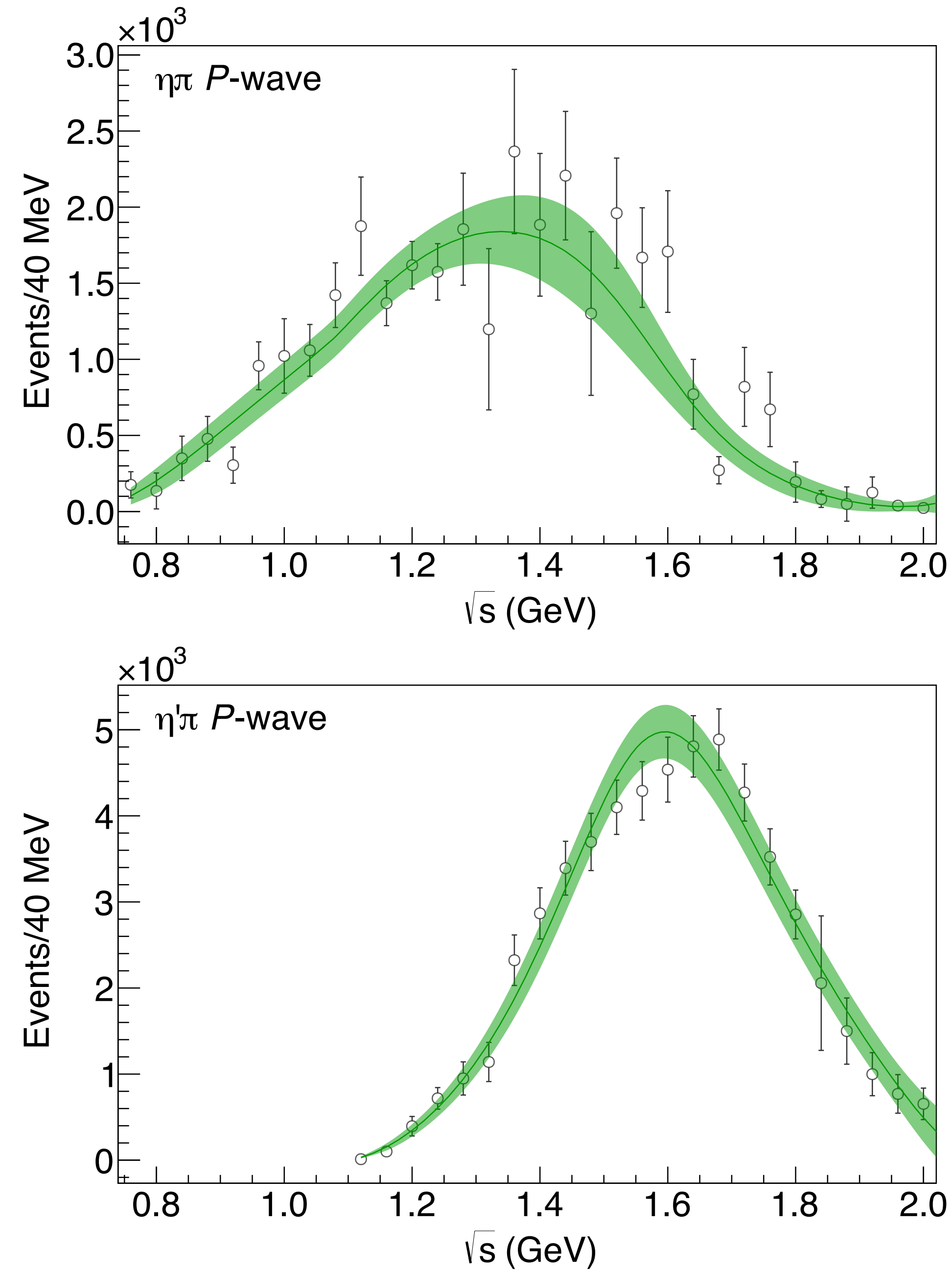


peaks at different masses, two resonances or one?

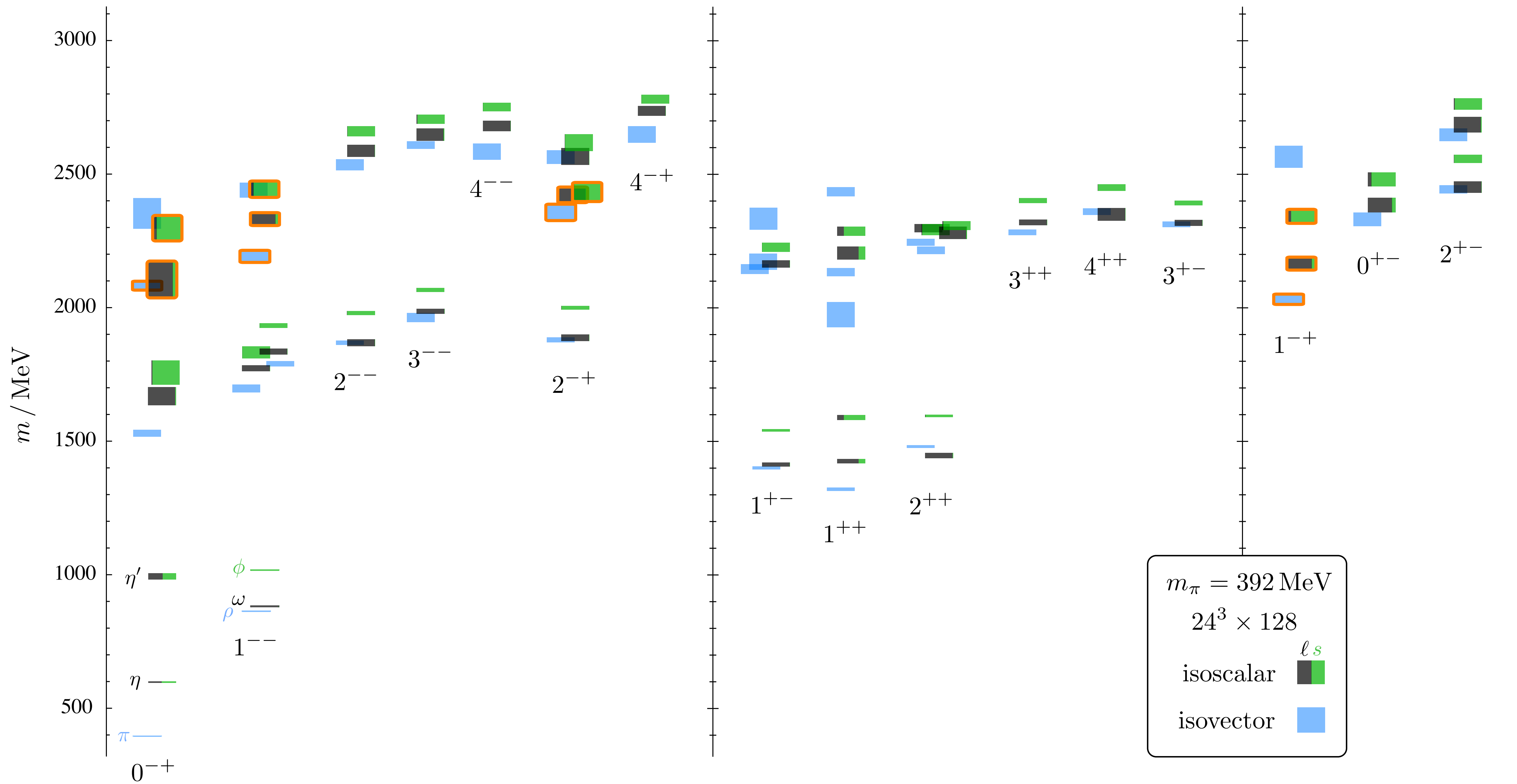
JPAC: COMPASS data can be described by a single resonance pole m~1564 MeV, Γ~500 MeV

similar result: COMPASS+Crystal Barrel data, B. Kopf et al - 2008.11566, Γ~400 MeV

Hadron Spectrum Collaboration: spectra from local $q\bar{q}$ constructions

- hybrids found at ***all*** masses from **light** to bottom

Dudek, Edwards, Guo, Thomas - 1309.2608



& similar states can be identified for half-integer spin

- see https://arxiv.org/abs/1201.2349

$$m_\pi = 391 \text{ MeV}$$

$$\pi_1 \to \pi b_1 \to \pi\pi\omega$$
$$\to \pi\pi\phi$$
$$\to \pi\pi\pi\eta$$
$$\to \pi K\bar{K}$$

a problem for another day!

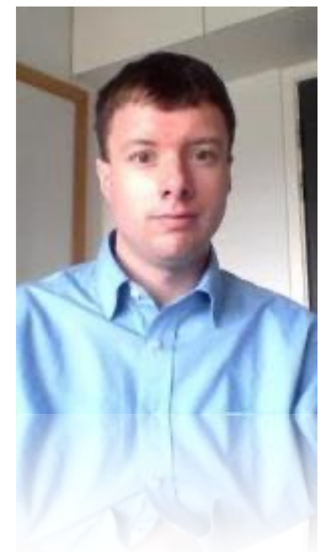
$$m_\pi = 688 \text{ MeV}$$

$$m_u = m_d = m_s$$
$$m_\pi = m_K = m_{\eta^8}$$

much simpler
fewer channels for a first attempt

simple counting 3*700 MeV = 2100 MeV
- 3 body is pushed off to higher energies

Woss Dudek Edwards Thomas



JSA thesis prize
PANDA thesis prize

Decays of an exotic 1^{-+} hybrid meson resonance in QCD

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(for the Hadron Spectrum Collaboration)
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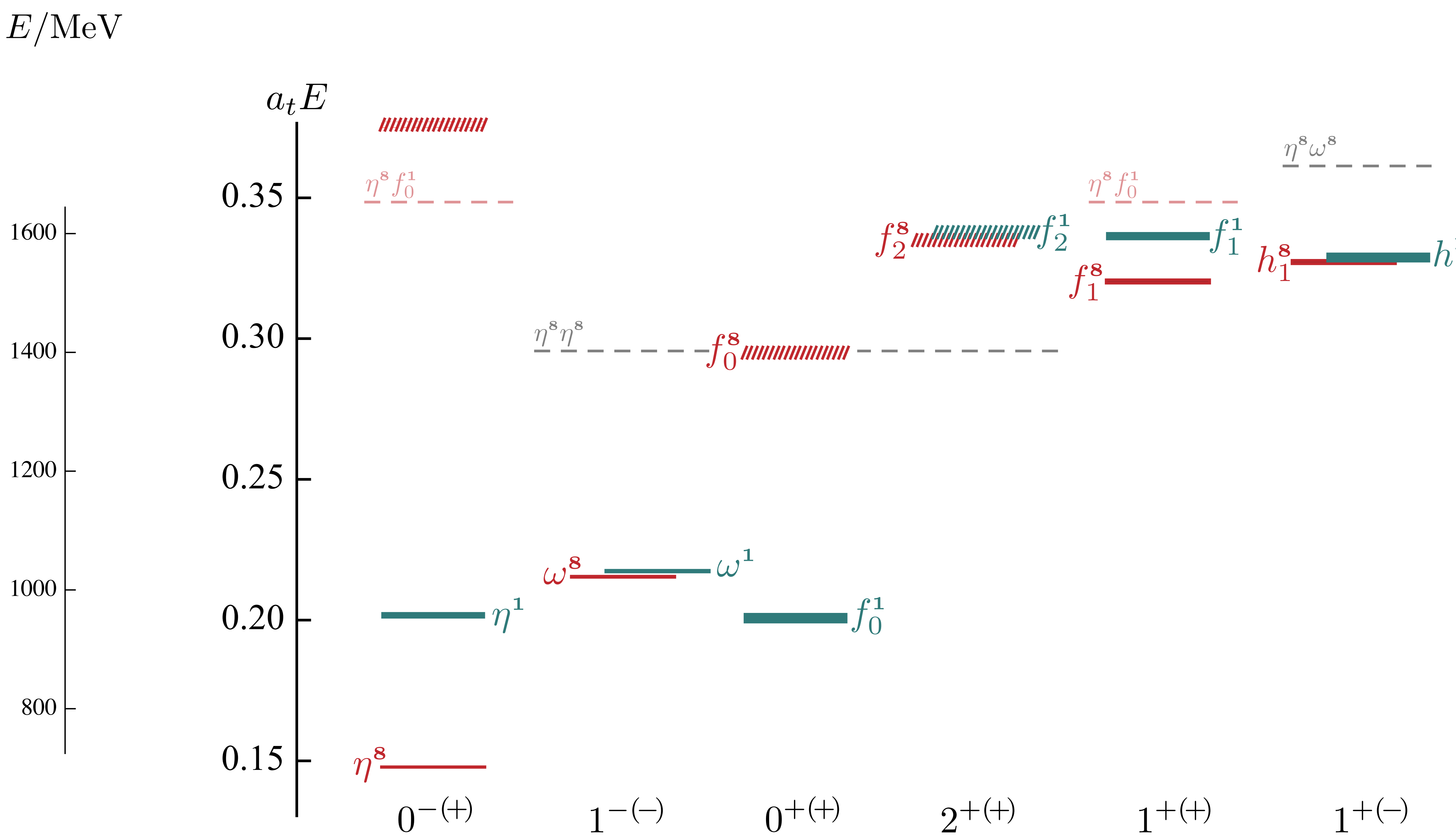
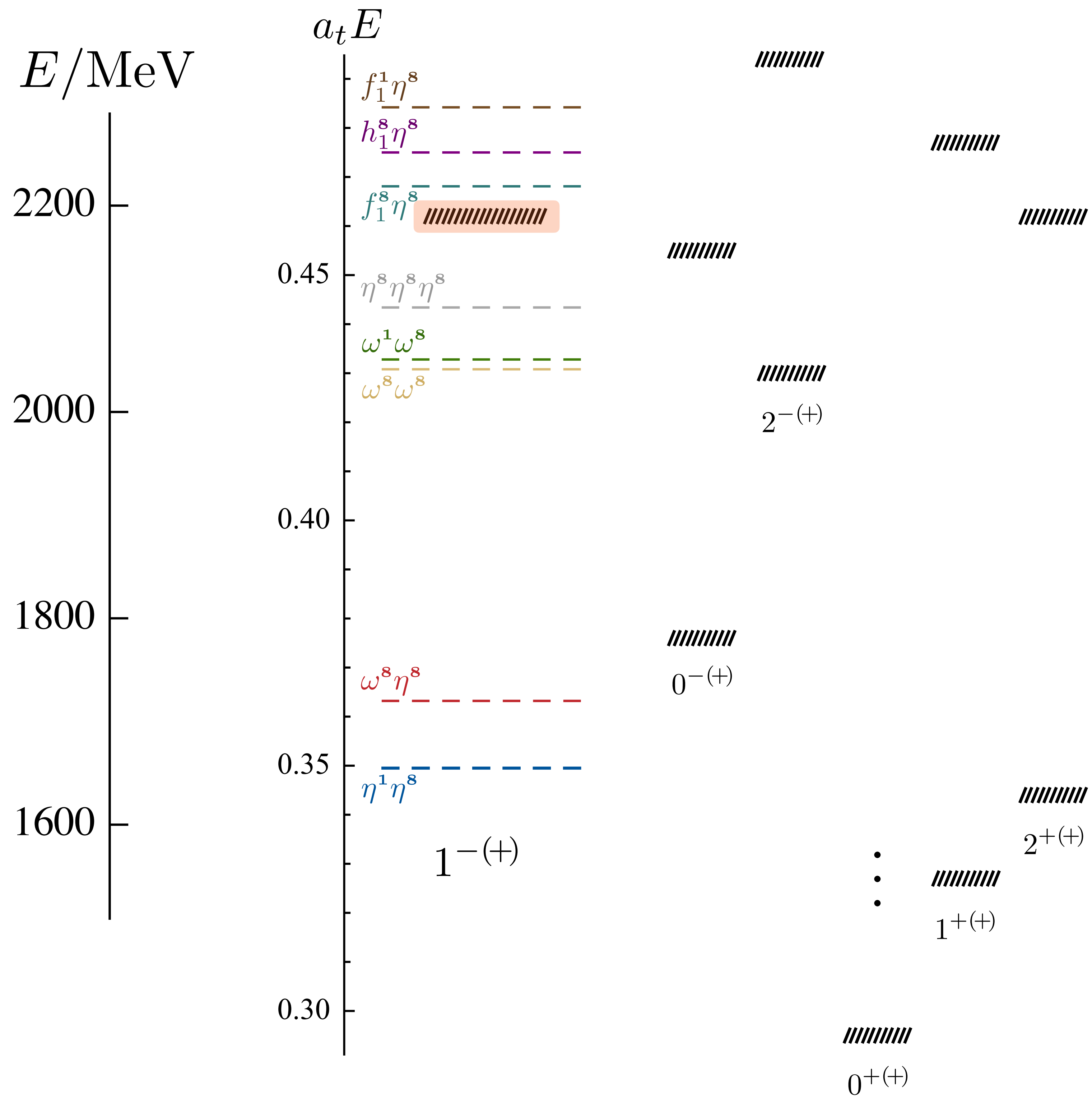
[2]*Thomas Jefferson National Accelerator Facility, 12000 Jefferson Avenue, Newport News, VA 23606, USA*
[3]*Department of Physics, College of William and Mary, Williamsburg, VA 23187, USA*
(Dated: 21 September 2020)

We present the first determination of the hadronic decays of the lightest exotic $J^{PC} = 1^{-+}$ resonance in lattice QCD. Working with SU(3) flavor symmetry, where the up, down and strange quark masses approximately match the physical strange-quark mass giving $m_\pi \sim 700$ MeV, we compute finite-volume spectra on six lattice volumes which constrain a scattering system featuring eight coupled channels. Analytically continuing the scattering amplitudes into the complex energy plane, we find a pole singularity corresponding to a narrow resonance which shows relatively weak coupling to the open pseudoscalar–pseudoscalar, vector–pseudoscalar and vector–vector decay channels, but large couplings to at least one kinematically-closed axial-vector–pseudoscalar channel. Attempting a simple extrapolation of the couplings to physical light-quark mass suggests a broad π_1 resonance decaying dominantly through the $b_1\pi$ mode with much smaller decays into $f_1\pi$, $\rho\pi$, $\eta'\pi$ and $\eta\pi$. A large total width is potentially in agreement with the experimental $\pi_1(1564)$ candidate state, observed in $\eta\pi$, $\eta'\pi$, which we suggest may be heavily suppressed decay channels.

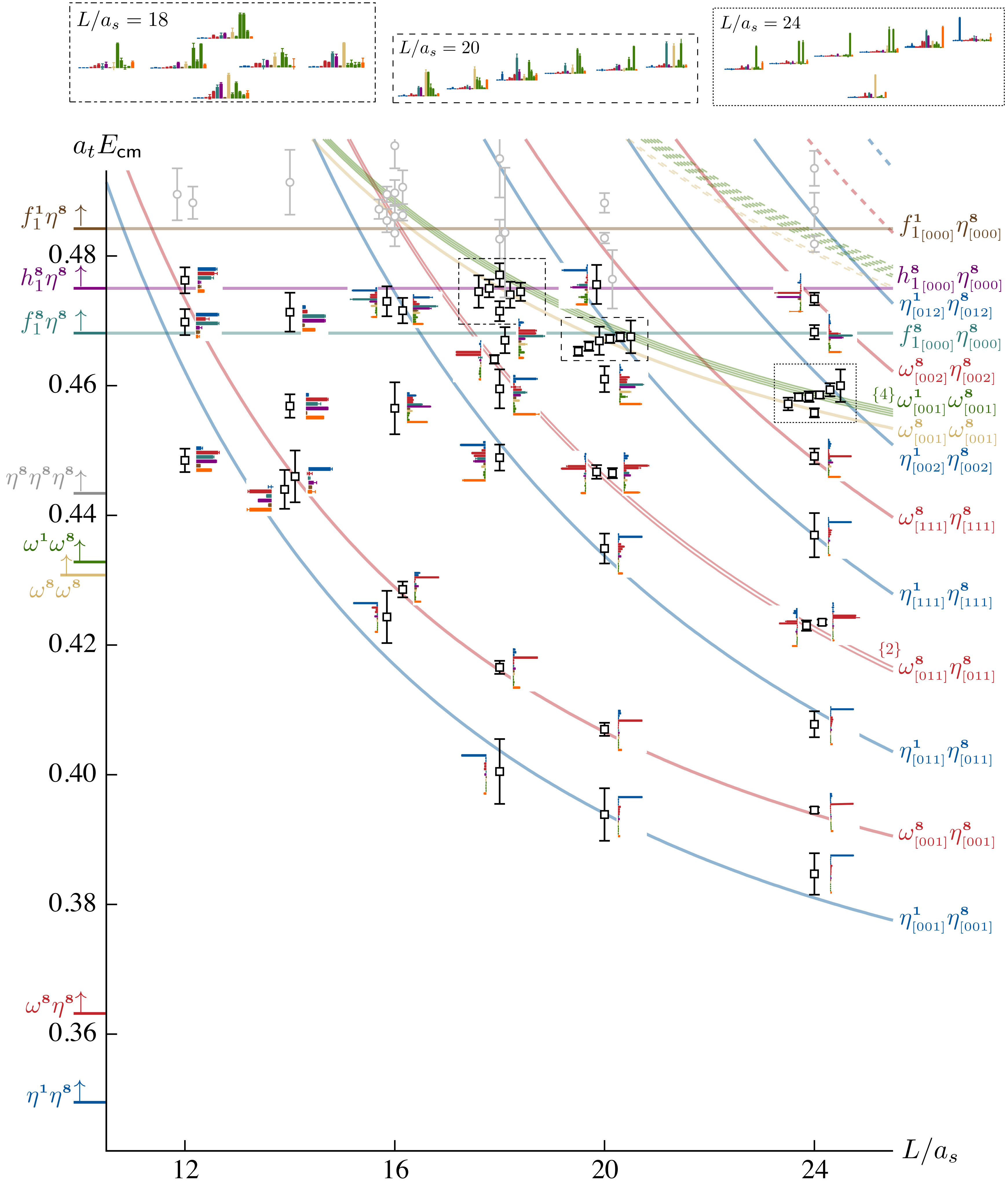
PRD 103 (2021) 5, 054502 (arXiv:2009.10034)


E/MeV
1600
1400
1200
1000
800
$a_t E$
0.35
0.30
0.25
0.20
0.15
$\eta^8 f_0^1$
$\eta^8 \eta^8$
$\eta^8 \omega^8$
f_2^8
f_2^1
f_1^1
f_1^8
h_1^8
h_1^1
f_0^8
ω^8
ω^1
η^1
f_0^1
η^8
$0^{-(+)}$
$1^{-(-)}$
$0^{+(+)}$
$2^{+(+)}$
$1^{+(+)}$
$1^{+(-)}$

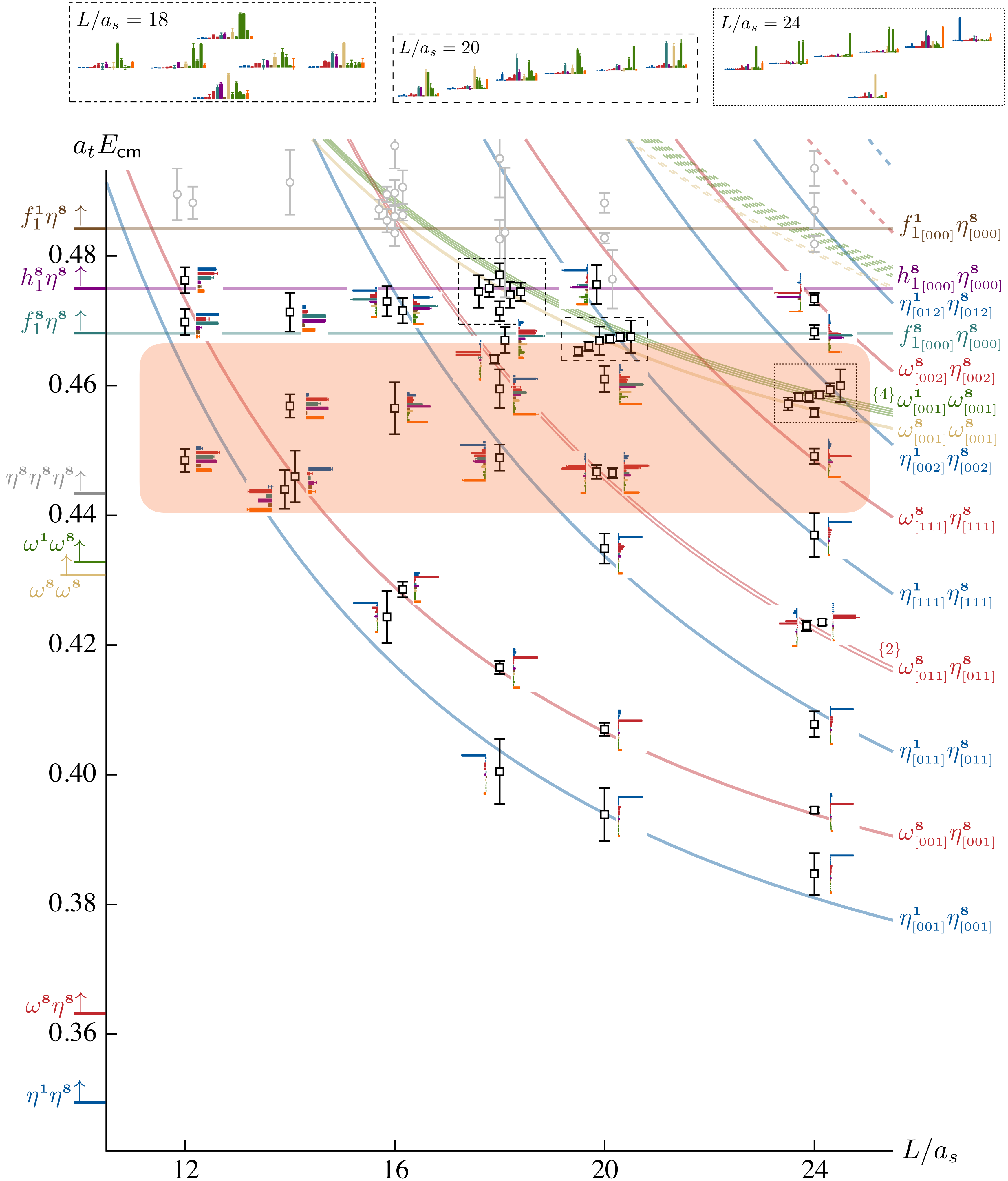


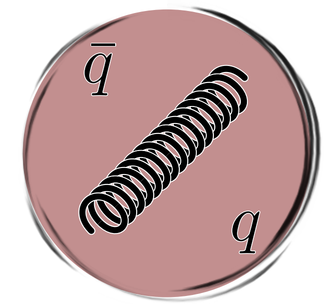
in moving frames

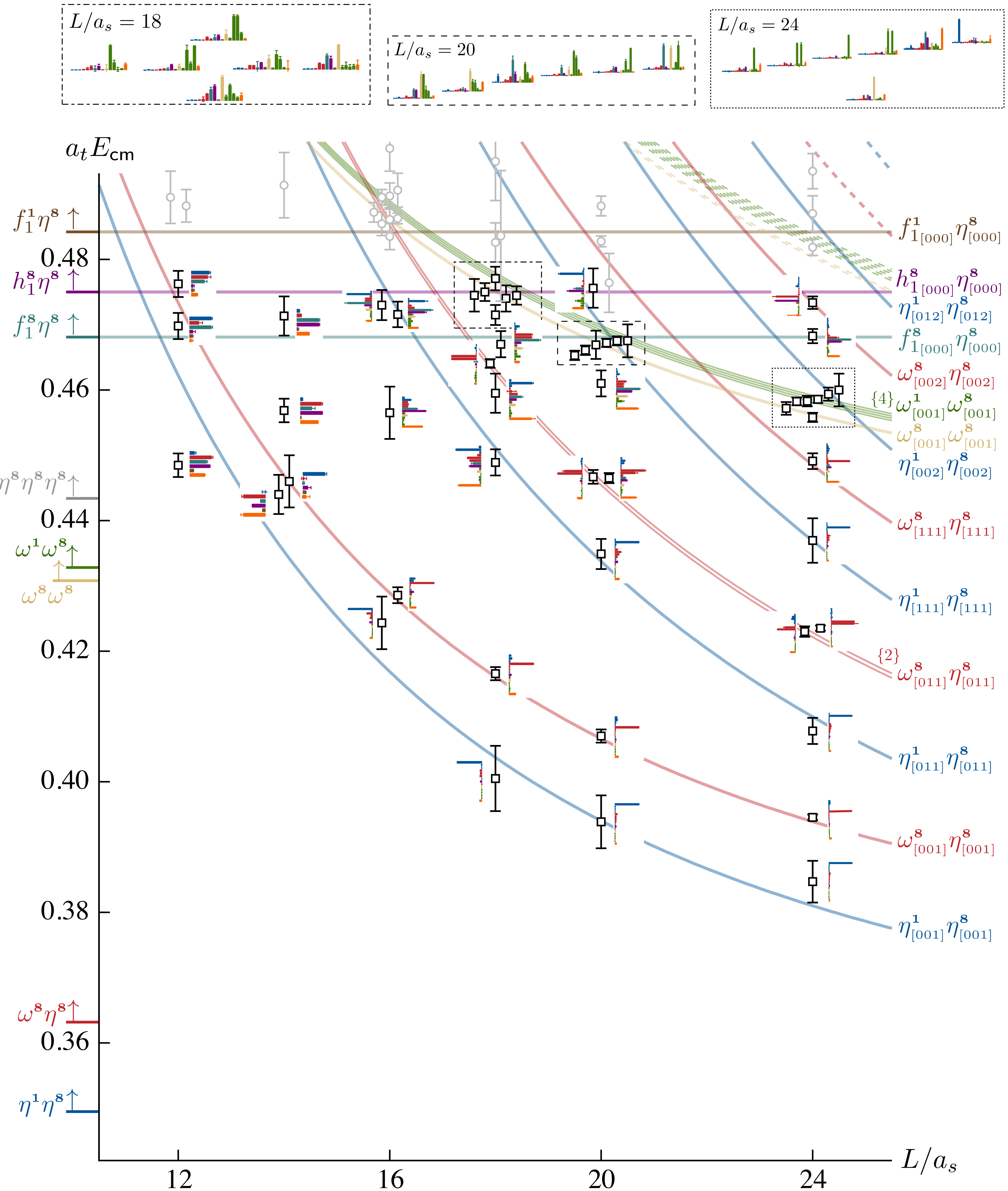
- many other resonances to consider



$L/a_s = 18$
$L/a_s = 20$
$L/a_s = 24$
$a_t E_{\mathsf{cm}}$
$f_1^1\eta^8$
$h_1^8\eta^8$
$f_1^8\eta^8$
$\eta^8\eta^8\eta^8$
$\omega^1\omega^8$
$\omega^8\omega^8$
$\omega^8\eta^8$
$\eta^1\eta^8$
0.48
0.46
0.44
0.42
0.40
0.38
0.36
12
16
20
24
L/a_s
$f^1_{1[000]}\eta^8_{[000]}$
$h^8_{1[000]}\eta^8_{[000]}$
$\eta^1_{[012]}\eta^8_{[012]}$
$f^8_{1[000]}\eta^8_{[000]}$
$\omega^8_{[002]}\eta^8_{[002]}$
{4}
$\omega^1_{[001]}\omega^8_{[001]}$
$\omega^8_{[001]}\omega^8_{[001]}$
$\eta^1_{[002]}\eta^8_{[002]}$
$\omega^8_{[111]}\eta^8_{[111]}$
$\eta^1_{[111]}\eta^8_{[111]}$
{2}
$\omega^8_{[011]}\eta^8_{[011]}$
$\eta^1_{[011]}\eta^8_{[011]}$
$\omega^8_{[001]}\eta^8_{[001]}$
$\eta^1_{[001]}\eta^8_{[001]}$


$L/a_s = 18$
$L/a_s = 20$
$L/a_s = 24$
$a_t E_{\mathsf{cm}}$
$f_1^1 \eta^8$
0.48
$h_1^8 \eta^8$
$f_1^8 \eta^8$
0.46
$\eta^8 \eta^8 \eta^8$
0.44
$\omega^1 \omega^8$
$\omega^8 \omega^8$
0.42
0.40
0.38
$\omega^8 \eta^8$
0.36
$\eta^1 \eta^8$
12
16
20
24
L/a_s
$f^1_{1\,[000]} \eta^8_{[000]}$
$h^8_{1\,[000]} \eta^8_{[000]}$
$\eta^1_{[012]} \eta^8_{[012]}$
$f^8_{1\,[000]} \eta^8_{[000]}$
$\omega^8_{[002]} \eta^8_{[002]}$
{4}
$\omega^1_{[001]} \omega^8_{[001]}$
$\omega^8_{[001]} \omega^8_{[001]}$
$\eta^1_{[002]} \eta^8_{[002]}$
$\omega^8_{[111]} \eta^8_{[111]}$
$\eta^1_{[111]} \eta^8_{[111]}$
{2}
$\omega^8_{[011]} \eta^8_{[011]}$
$\eta^1_{[011]} \eta^8_{[011]}$
$\omega^8_{[001]} \eta^8_{[001]}$
$\eta^1_{[001]} \eta^8_{[001]}$


$\bar{q}$
q



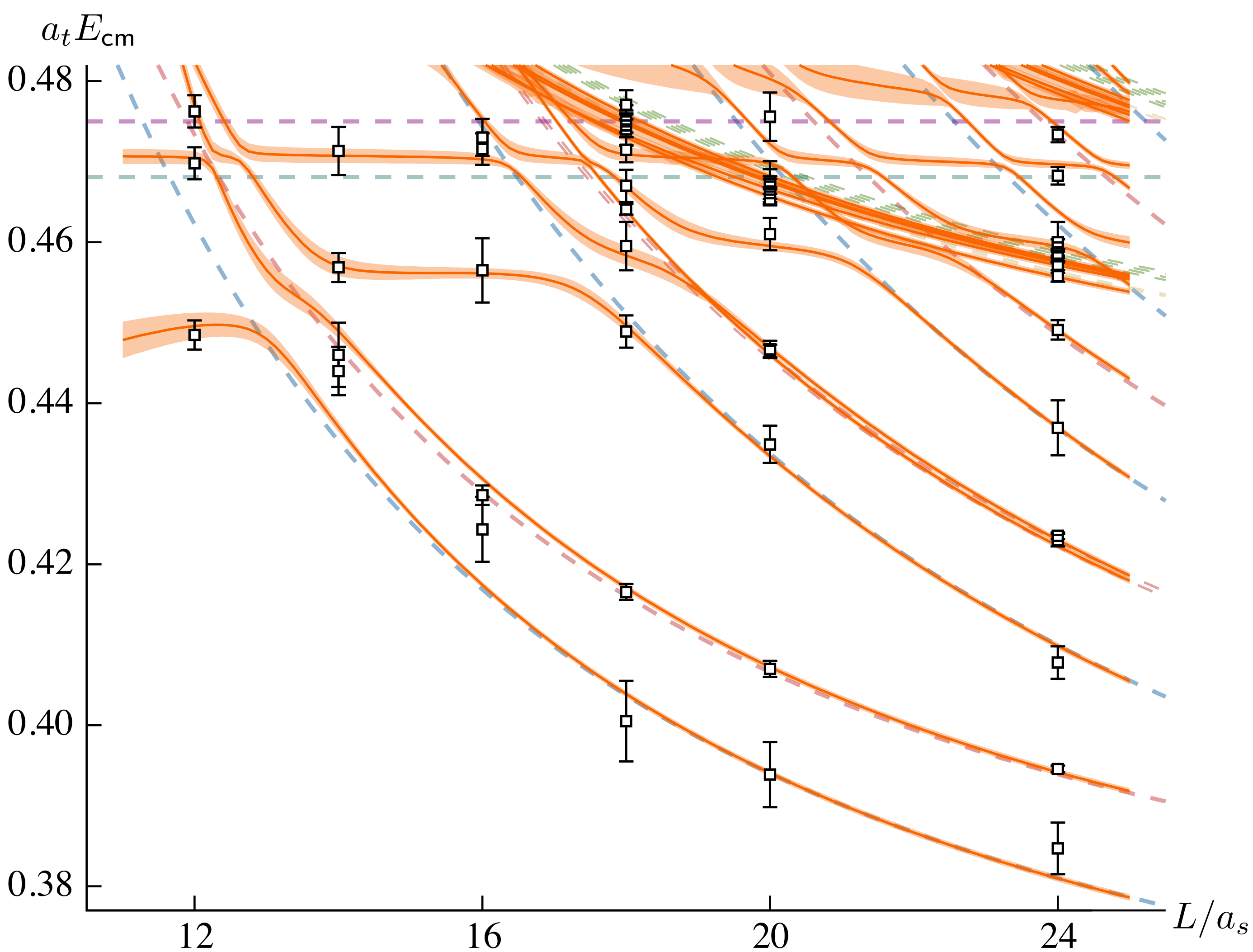
partial waves

$1^{-(+)}$	$\eta^1\eta^8\{^1P_1\}$ $\omega^8\eta^8\{^3P_1\}$ $\omega^8\omega^8\{^3P_1\}$, $\omega^1\omega^8\{^1P_1, {}^3P_1, {}^5P_1\}$ $f_1^8\eta^8\{^3S_1\}$, $h_1^8\eta^8\{^3S_1\}$
$3^{-(+)}$	$\eta^1\eta^8\{^1F_3\}$ $\omega^8\eta^8\{^3F_3\}$ $\omega^1\omega^8\{^5P_3\}$

K-matrix parametrisation

$$t^{-1} = K^{-1} + I$$

- pole coupled in 1^{-+}
- various constants



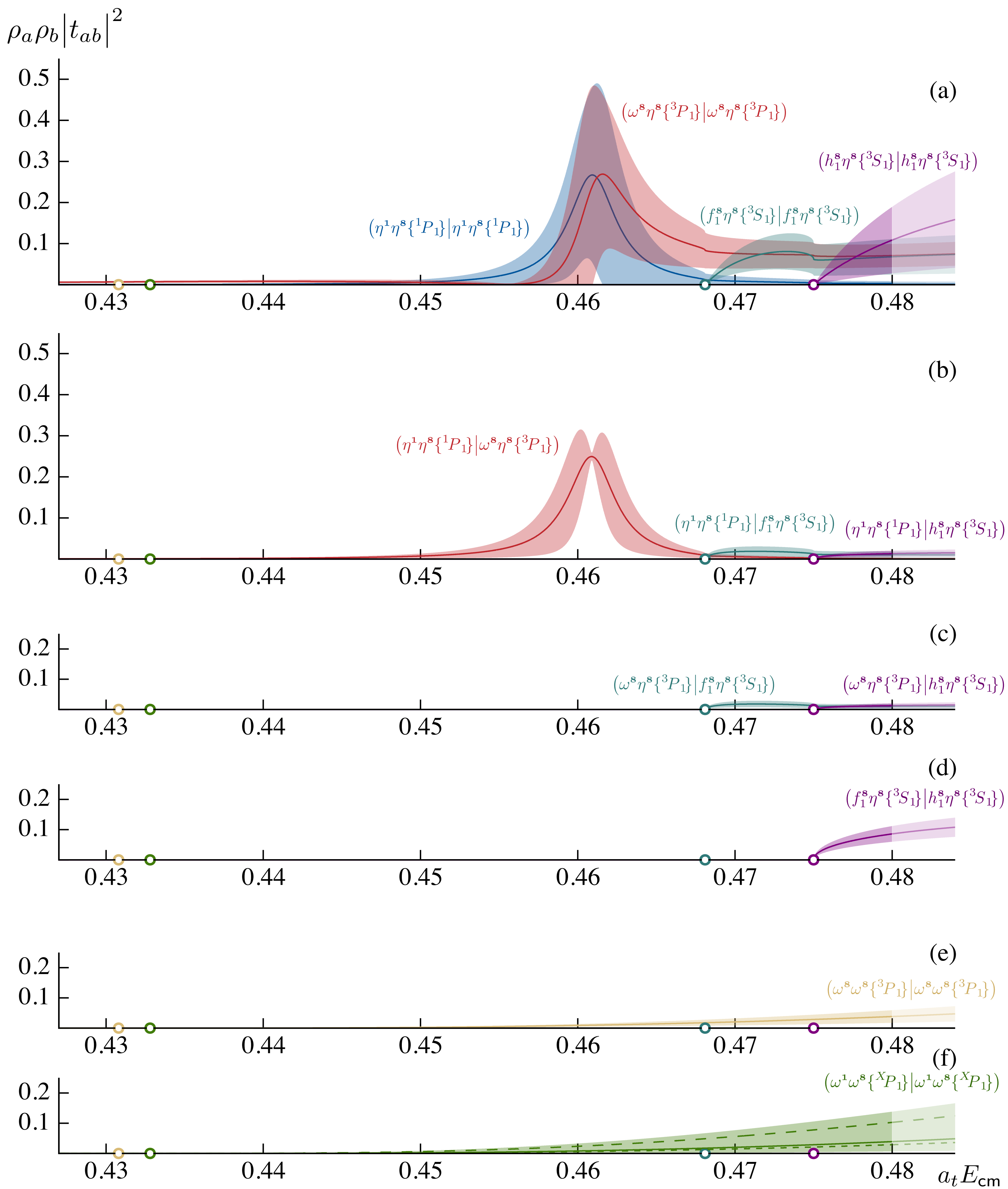
partial waves

$1^{-(+)}$	$\eta^{1}\eta^{8}\{^{1}P_1\}$
	$\omega^{8}\eta^{8}\{^{3}P_1\}$
	$\omega^{8}\omega^{8}\{^{3}P_1\}$, $\omega^{1}\omega^{8}\{^{1}P_1, {}^{3}P_1, {}^{5}P_1\}$
	$f_1^{8}\eta^{8}\{^{3}S_1\}$, $h_1^{8}\eta^{8}\{^{3}S_1\}$
$3^{-(+)}$	$\eta^{1}\eta^{8}\{^{1}F_3\}$
	$\omega^{8}\eta^{8}\{^{3}F_3\}$
	$\omega^{1}\omega^{8}\{^{5}P_3\}$

K-matrix parametrisation

$$t^{-1} = K^{-1} + I$$

- pole coupled in 1^{-+}
- various constants


$\rho_a \rho_b |t_{ab}|^2$
(a)
$(\omega^8\eta^8\{^3P_1\}|\omega^8\eta^8\{^3P_1\})$
$(h_1^8\eta^8\{^3S_1\}|h_1^8\eta^8\{^3S_1\})$
$(f_1^8\eta^8\{^3S_1\}|f_1^8\eta^8\{^3S_1\})$
$(\eta^1\eta^8\{^1P_1\}|\eta^1\eta^8\{^1P_1\})$
(b)
$(\eta^1\eta^8\{^1P_1\}|\omega^8\eta^8\{^3P_1\})$
$(\eta^1\eta^8\{^1P_1\}|f_1^8\eta^8\{^3S_1\})$
$(\eta^1\eta^8\{^1P_1\}|h_1^8\eta^8\{^3S_1\})$
(c)
$(\omega^8\eta^8\{^3P_1\}|f_1^8\eta^8\{^3S_1\})$
$(\omega^8\eta^8\{^3P_1\}|h_1^8\eta^8\{^3S_1\})$
(d)
$(f_1^8\eta^8\{^3S_1\}|h_1^8\eta^8\{^3S_1\})$
(e)
$(\omega^8\omega^8\{^3P_1\}|\omega^8\omega^8\{^3P_1\})$
(f)
$(\omega^1\omega^8\{^XP_1\}|\omega^1\omega^8\{^XP_1\})$
$a_t E_{\mathsf{cm}}$

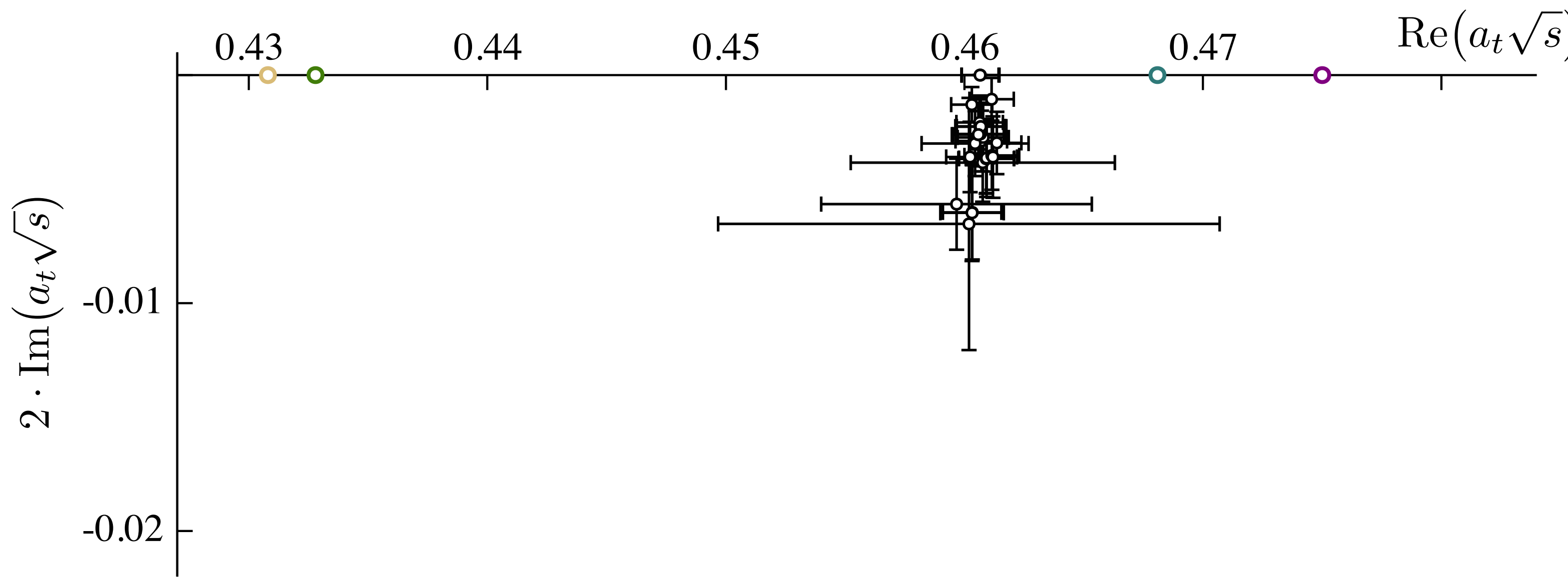
in the region of a pole

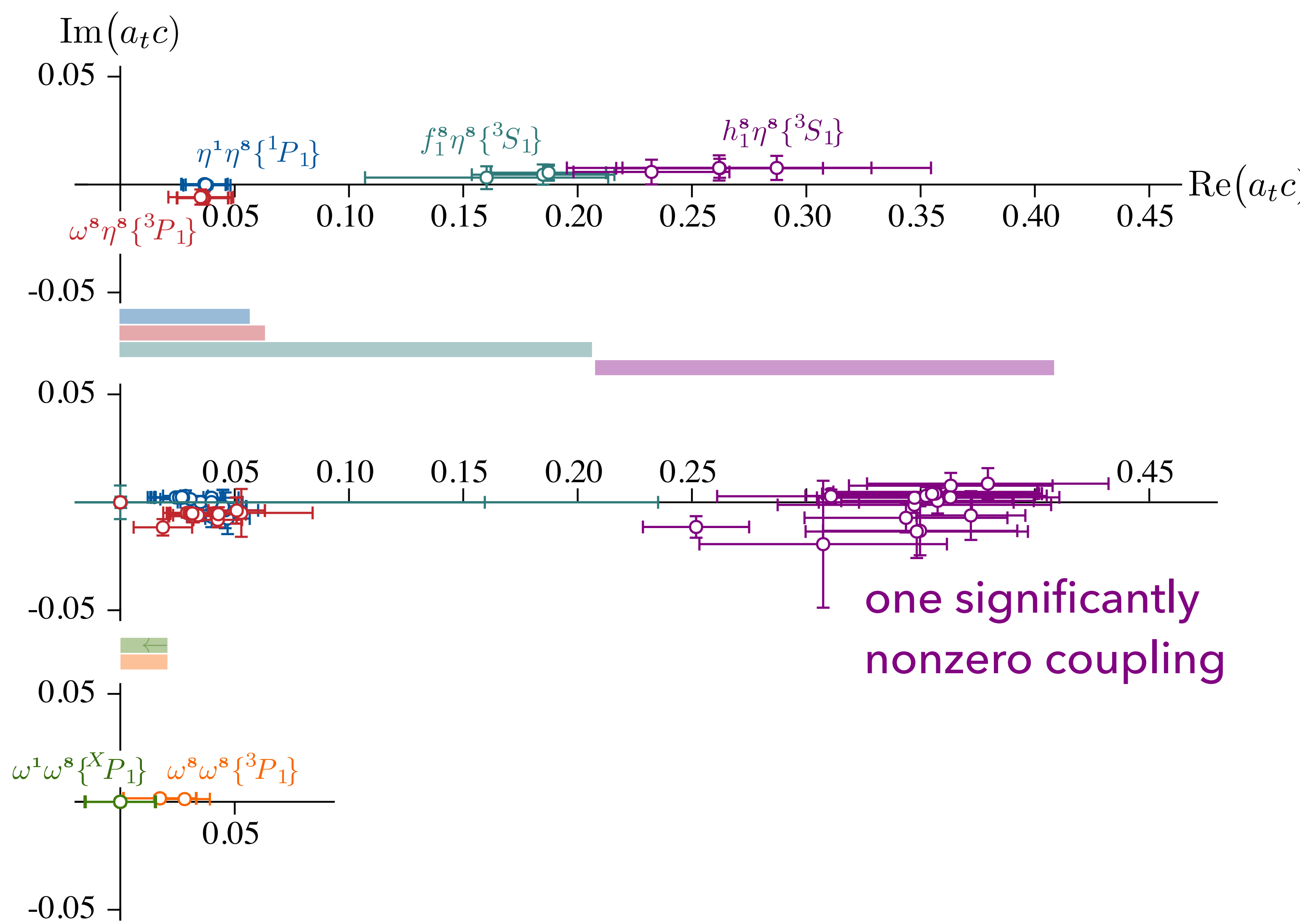
$$t \sim \frac{c^2}{s_0 - s}$$

narrow resonance

$$a_t\sqrt{s} = 0.4606(26) \pm \tfrac{i}{2}0.0039(39)$$
$$\sqrt{s} = \left(2144(12) \pm \tfrac{i}{2}18(18)\right) \text{ MeV}$$

small imaginary part, consistent with zero in some parameterisations





Flavour decomposition

- break apart SU(3) multiplets
- use CGs from de Swart (Rev. Mod. Phys. 35, 916 (1963))
- mixing angles needed for singlets taken from PDG

$$\mathbf{8} \otimes \mathbf{8} \to \mathbf{1} \oplus \mathbf{8_1} \oplus \mathbf{8_2} \oplus \mathbf{10} \oplus \overline{\mathbf{10}} \oplus \mathbf{27}$$

$$\pi_1^{\mathbf{8}} \to \omega^{\mathbf{8}} \eta^{\mathbf{8}}$$

$$\text{eg} : \pi_1^+ \to \frac{1}{\sqrt{3}} \left(\pi^+ \rho^0 - \pi^0 \rho^+\right) + \frac{1}{\sqrt{6}} \left(K^+ \bar{K}^{*0} - \bar{K}^0 K^{*+}\right)$$

$$\left|c(\pi_1 \to \rho\pi)\right| = \sqrt{\tfrac{2}{3}} \left|c_{\omega^{\mathbf{8}}\eta^{\mathbf{8}}}\right|$$

$$\left|c(\pi_1 \to K^* \overline{K})\right| = \sqrt{\tfrac{1}{3}} \left|c_{\omega^{\mathbf{8}}\eta^{\mathbf{8}}}\right|$$

very heavy quarks

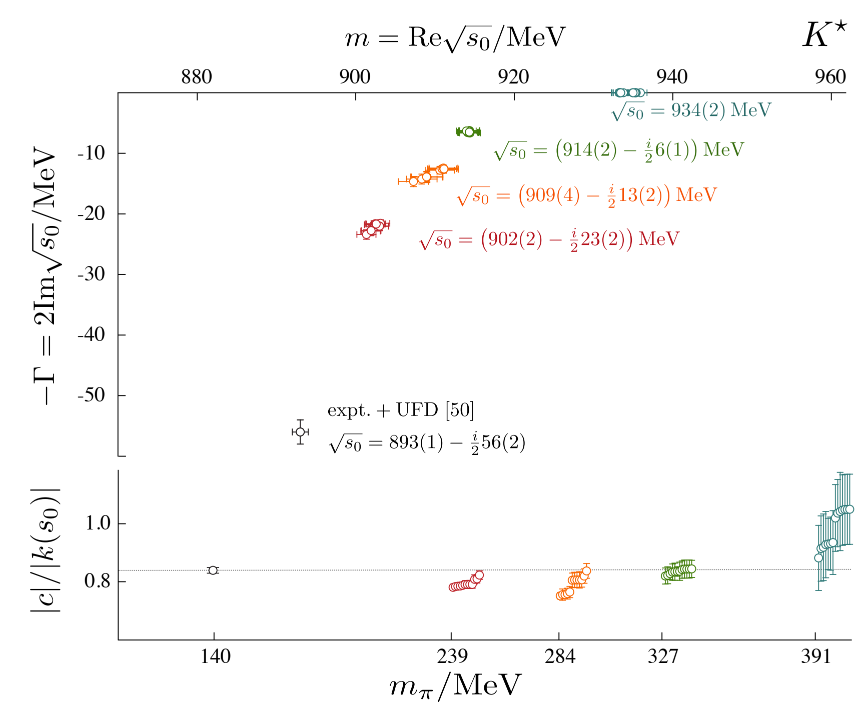
- crudely extrapolate to physical pions

scale couplings:

$$|c|^{\text{phys}} = \left| \frac{k^{\text{phys}}(m_R^{\text{phys}})}{k(m_R)} \right|^{\ell} |c|$$

choose m_R=1563 MeV

well-motivated:
eg: K* example from PRL 123 (2019) 4, 042002 (arXiv:1904.03188)



largest decay modes:

$f_1^{\mathbf{8}}\eta^{\mathbf{8}}\{^3S_1\}$

$h_1^{\mathbf{8}}\eta^{\mathbf{8}}\{^3S_1\}$

$$f_1^{\mathbf{8}}\eta^{\mathbf{8}}\{^3S_1\}$$

$$\mathbf{8}\otimes\mathbf{8}\to\mathbf{1}\oplus\mathbf{8_1}\oplus\mathbf{8_2}\oplus\mathbf{10}\oplus\overline{\mathbf{10}}\oplus\mathbf{27}$$

$$h_1^{\mathbf{8}}\eta^{\mathbf{8}}\{^3S_1\}$$

$$-\sqrt{\tfrac{3}{10}}\left(K_{1A}^+\,\overline{K}^0+\overline{K}_{1A}^0\,K^+\right)+\tfrac{1}{\sqrt{5}}\left(a_1^+\eta_8+(f_1)_8\pi^+\right)$$

$$\begin{aligned}
\left|c(\pi_1\to a_1\eta)\right| &= \tfrac{1}{\sqrt{5}}\left|c_{f_1^{\mathbf{8}}\eta^{\mathbf{8}}}\cos\theta_P\right|\\
\left|c(\pi_1\to a_1\eta')\right| &= \tfrac{1}{\sqrt{5}}\left|c_{f_1^{\mathbf{8}}\eta^{\mathbf{8}}}\sin\theta_P\right|\\
\left|c(\pi_1\to f_1(1285)\pi)\right| &= \tfrac{1}{\sqrt{5}}\left|c_{f_1^{\mathbf{8}}\eta^{\mathbf{8}}}\cos\theta_A\right|\\
\left|c(\pi_1\to f_1(1420)\pi)\right| &= \tfrac{1}{\sqrt{5}}\left|c_{f_1^{\mathbf{8}}\eta^{\mathbf{8}}}\sin\theta_A\right|.
\end{aligned}$$

$$\begin{pmatrix}\eta_8\\ \eta_1\end{pmatrix}=\begin{pmatrix}\cos\theta_P & \sin\theta_P\\ -\sin\theta_P & \cos\theta_P\end{pmatrix}\begin{pmatrix}\eta\\ \eta'\end{pmatrix}$$

$$\theta_P\approx-10^\circ$$

$$\theta_A\approx-34^\circ$$

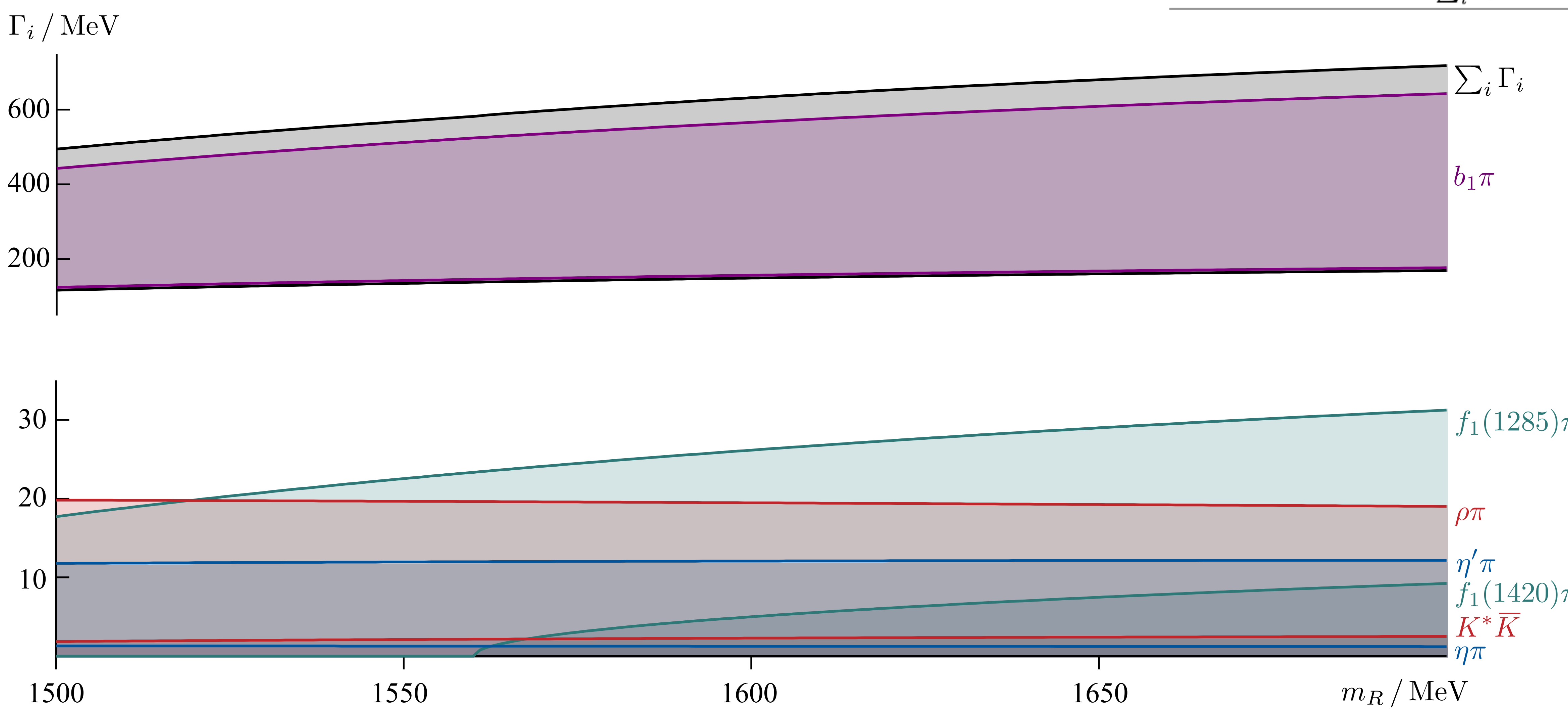
$$\tfrac{1}{\sqrt{6}}\left(K_{1B}^+\,\overline{K}^0-\overline{K}_{1B}^0\,K^+\right)+\tfrac{1}{\sqrt{3}}\left(b_1^+\pi^0-b_1^0\,\pi^+\right)$$

$$\left|c(\pi_1\to b_1\pi)\right| = \sqrt{\tfrac{2}{3}}\,\left|c_{h_1^{\mathbf{8}}\eta^{\mathbf{8}}}\right|$$

$$\Gamma(R \to i) = \frac{\left|c_i^{\text{phys}}\right|^2}{m_R^{\text{phys}}} \cdot \rho_i(m_R^{\text{phys}})$$

[PDG]

	thr./MeV	$\lvert c_i^{\text{phys}}\rvert$/MeV	Γ_i/MeV
$\eta\pi$	688	$0 \to 43$	$0 \to 1$
$\rho\pi$	910	$0 \to 203$	$0 \to 20$
$\eta'\pi$	1098	$0 \to 173$	$0 \to 12$
$b_1\pi$	1375	$799 \to 1559$	$139 \to 529$
$K^*\overline{K}$	1386	$0 \to 87$	$0 \to 2$
$f_1(1285)\pi$	1425	$0 \to 363$	$0 \to 24$
$\rho\omega\{{}^1P_1\}$	1552	$\lesssim 19$	$\lesssim 0.03$
$\rho\omega\{{}^3P_1\}$	1552	$\lesssim 32$	$\lesssim 0.09$
$\rho\omega\{{}^5P_1\}$	1552	$\lesssim 19$	$\lesssim 0.03$
$f_1(1420)\pi$	1560	$0 \to 245$	$0 \to 2$
			$\Gamma = \sum_i \Gamma_i = 139 \to 590$



For the first time, we have a first-principles QCD computation of a π_1 resonance
- a heavier than physical pion mass was used with $m_u=m_d=m_s$
- multibody decay modes are suppressed, only 2-body becomes relevant
- we find large coupling to a kinematically-closed axial-vector–pseudoscalar channel
- narrow resonance

Crudely extrapolating to the experimentally-observed mass, we find
- the dominant decay mode appears to be $b_1\pi$
- in experiment this is a 5π final state
- current analyses of $\eta\pi$ and $\eta'\pi$ channels may be quite suppressed w.r.t. $b_1\pi$
- broad resonance

This SU(3) calculation has components that apply to the other elements of the octet
- but other components are expected to also contribute
- nevertheless - there's likely to be a family of hybrids

Charmonium, bottomonium is another interesting place to look
- heavier quarks may make extrapolating to the experimental masses more straightforward