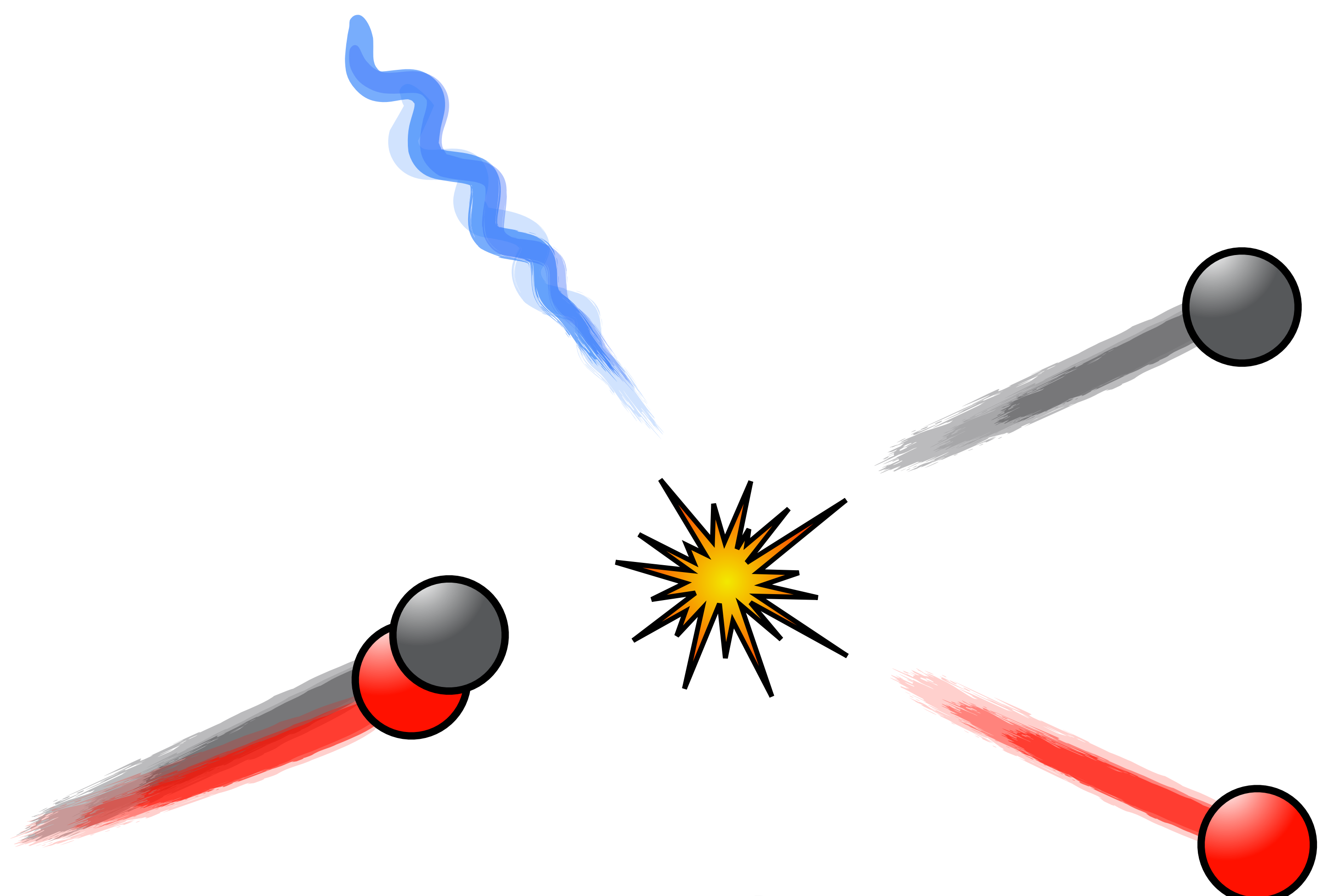
Connecting Matrix Elements to Multi-Hadron Form-Factors

Andrew W. Jackura

The 38th International Symposium on Lattice Field Theory
26th-30th July, 2021



OLD DOMINION UNIVERSITY

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Thomas Jefferson National Accelerator Facility

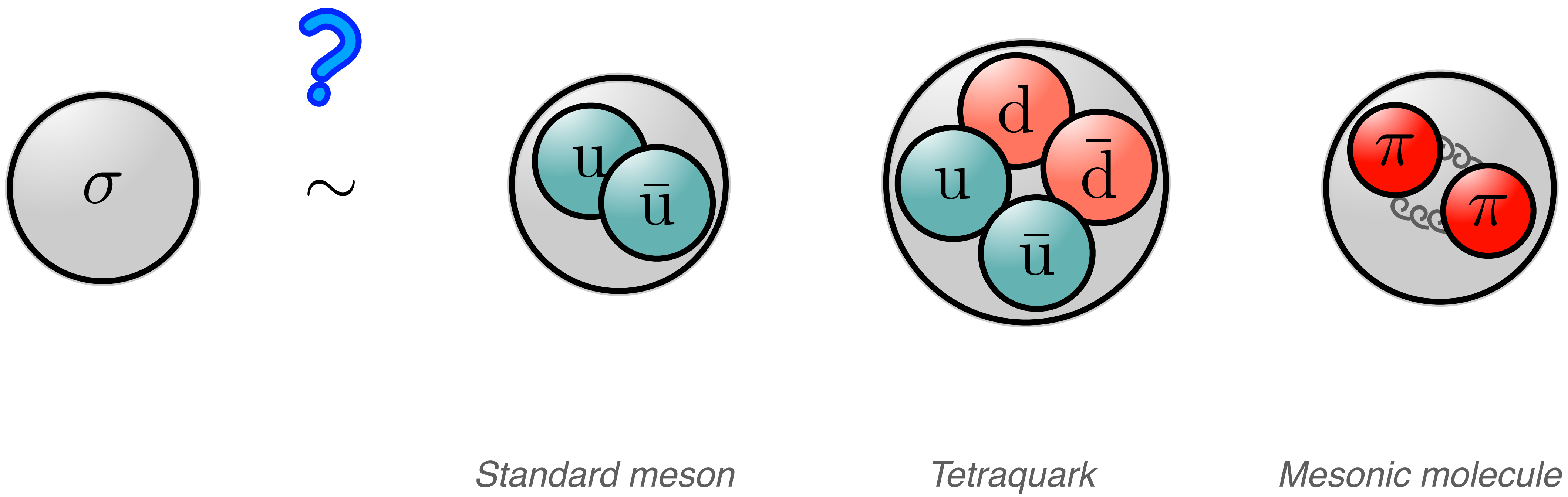
LATTICE 21
JULY 26-30 2021, ZOOM/GATHER@MIT

Structural composition of hadrons

How do we determine structure of hadrons from QCD?

- Probe stable hadrons via external currents
- Gives access to form-factors, parton distribution functions, etc.

What about unstable particles, i.e. resonances? — e.g., the σ meson



Structural composition of hadrons

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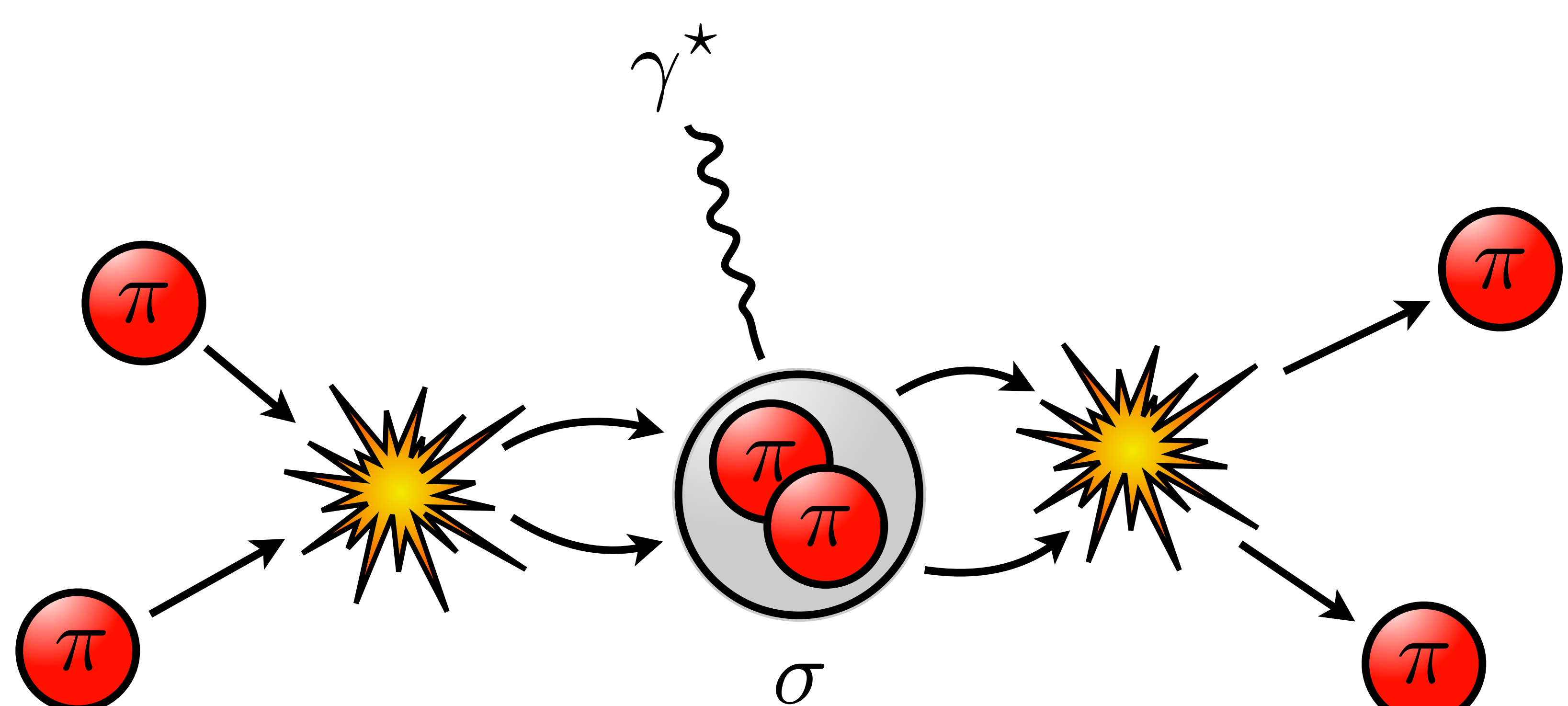
What about unstable particles, i.e. resonances?

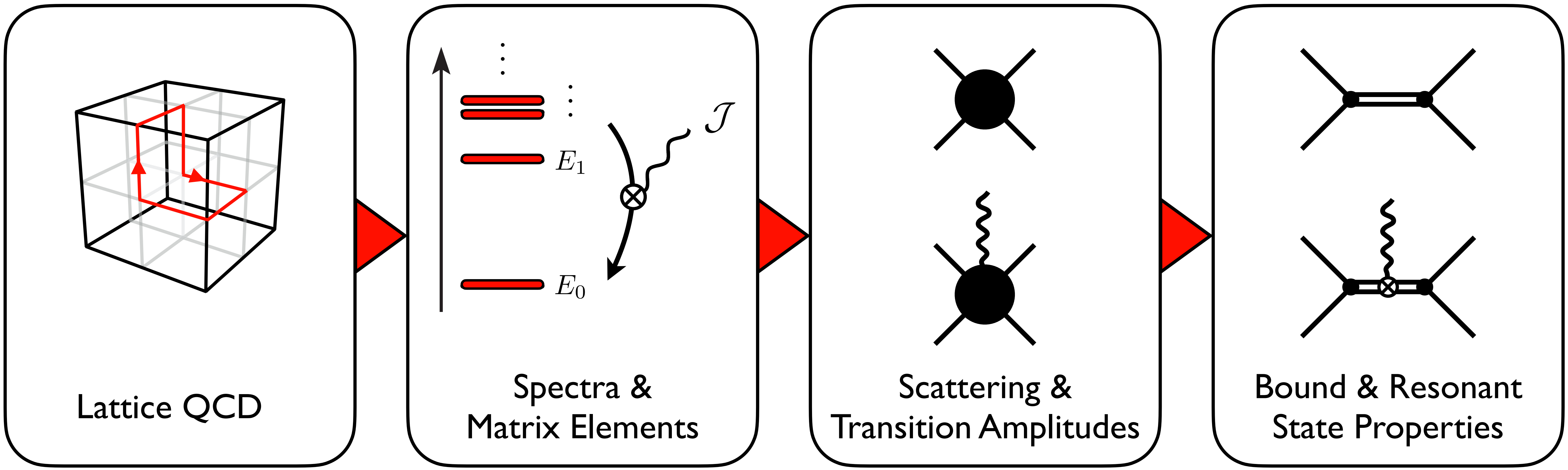
- Couple to multi-hadron states — need transition amplitudes evaluated at poles



Path to Hadronic Properties from QCD

Lattice QCD offers a systematic avenue to compute multi-hadron transition amplitudes

- Convert spectra and matrix elements to transition amplitudes via Lüscher methodology



Spectral Analysis → Finite-Volume Mappings → Analytic Continuation

Finite volume mappings - Two hadron matrix elements

Mapping between matrix elements and $\mathbf{2} + \mathcal{J} \to \mathbf{2}$ amplitudes

Features

- *Relativistic*
- *Model independent*
- *Arbitrary current structure*

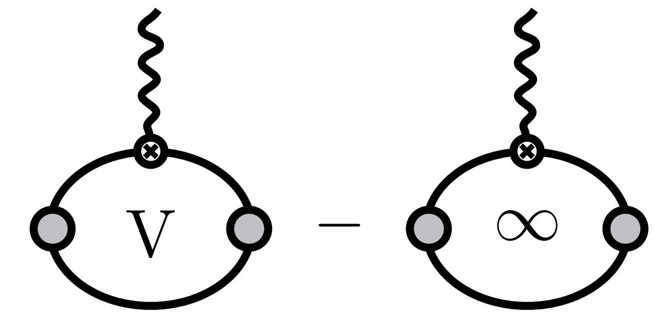
Assumptions

- *Spinless particles*
- *Below three-particle thresholds*

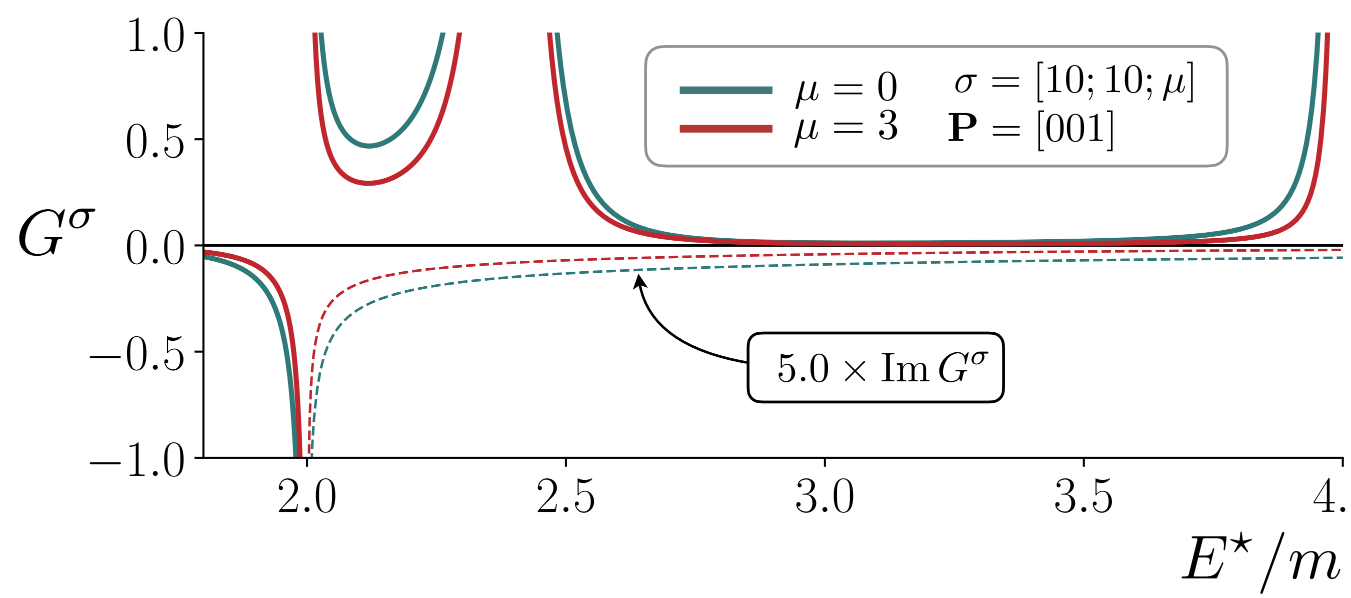
$$\langle \mathfrak{m} | \mathcal{J} | \mathfrak{n} \rangle_L = \frac{1}{L^3} \mathcal{W}_{L,\mathrm{df}} \cdot \sqrt{\mathcal{R}_{L,\mathfrak{m}} \cdot \mathcal{R}_{L,\mathfrak{n}}}$$

$$\mathcal{W}_{L,\mathrm{df}} = \mathcal{W}_{\mathrm{df}} + \mathcal{M} \cdot f \cdot G_L \cdot \mathcal{M}$$

Finite volume geometric function $G_L =$



$mL = 4$



R. Briceño, M. Hansen,
Phys. Rev. D **94** 13008 (2016)

A. Baroni, R. Briceño, M. Hansen, F. Ortega-Gama,
Phys. Rev. D **100** 034511 (2019)

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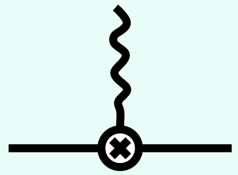
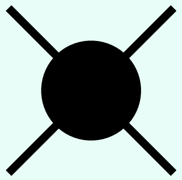
Previously determined functions

Single hadron form-factors $f =$ 

Hadronic scattering amplitude $\mathcal{M} =$ 

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Consistency Checks

Need to ensure formalism recovers known results in various limits

- Volume scaling of charge associated with conserved vector current
- Exponential suppression for two-body bound states
- Series expansion for perturbative systems

AJ, R. Briceño, M. Hansen,
Phys. Rev. D **100** 114505 (2019)

AJ, R. Briceño, M. Hansen,
Phys. Rev. D **101** 094508 (2020)

Consistency Checks (I)

Expect conserved vector charge to remain independent of finite volume corrections

$$\langle \mathfrak{n} | \, \mathcal{Q} \, | \mathfrak{n} \rangle_L = L^3 \langle \mathfrak{n} | \, \mathcal{J}^0 \, | \mathfrak{n} \rangle_L$$

$$= \mathcal{W}_{L,\mathrm{df}} \cdot \mathcal{R}_L$$

$$= \left(\mathcal{W}^0_{\mathrm{df}} + \mathrm{Q}_0 \, \mathcal{M} \cdot G^0_L \cdot \mathcal{M} \right) \left(\frac{\partial}{\partial E} \mathcal{M} + \mathcal{M} \cdot G^0_L \cdot \mathcal{M} \right)^{-1}$$

Ward-Takahashi identity

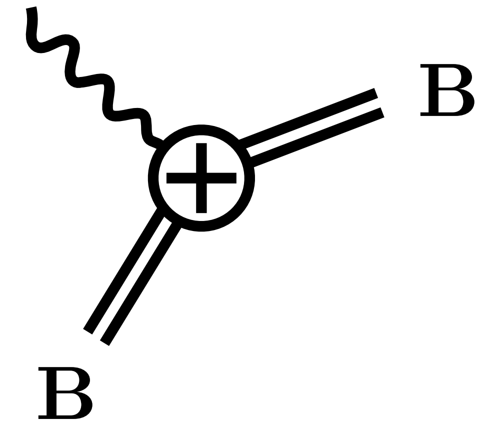
$$\lim_{P_i \to P_f} \mathcal{W}^{\mu}_{\mathrm{df}} = \mathrm{Q}_0 \, \frac{\partial}{\partial P_\mu} \mathcal{M}$$

$$\langle \mathfrak{n} | \, \mathcal{Q} \, | \mathfrak{n} \rangle_L = \mathrm{Q}_0$$

AJ, R. Briceño, M. Hansen,
Phys. Rev. D **100** 114505 (2019)

Generalization with partial wave mixing
AJ, F. Ortega-Gama, in preparation

Consistency Checks (II)



Bound state limit — corrections exponentially suppressed

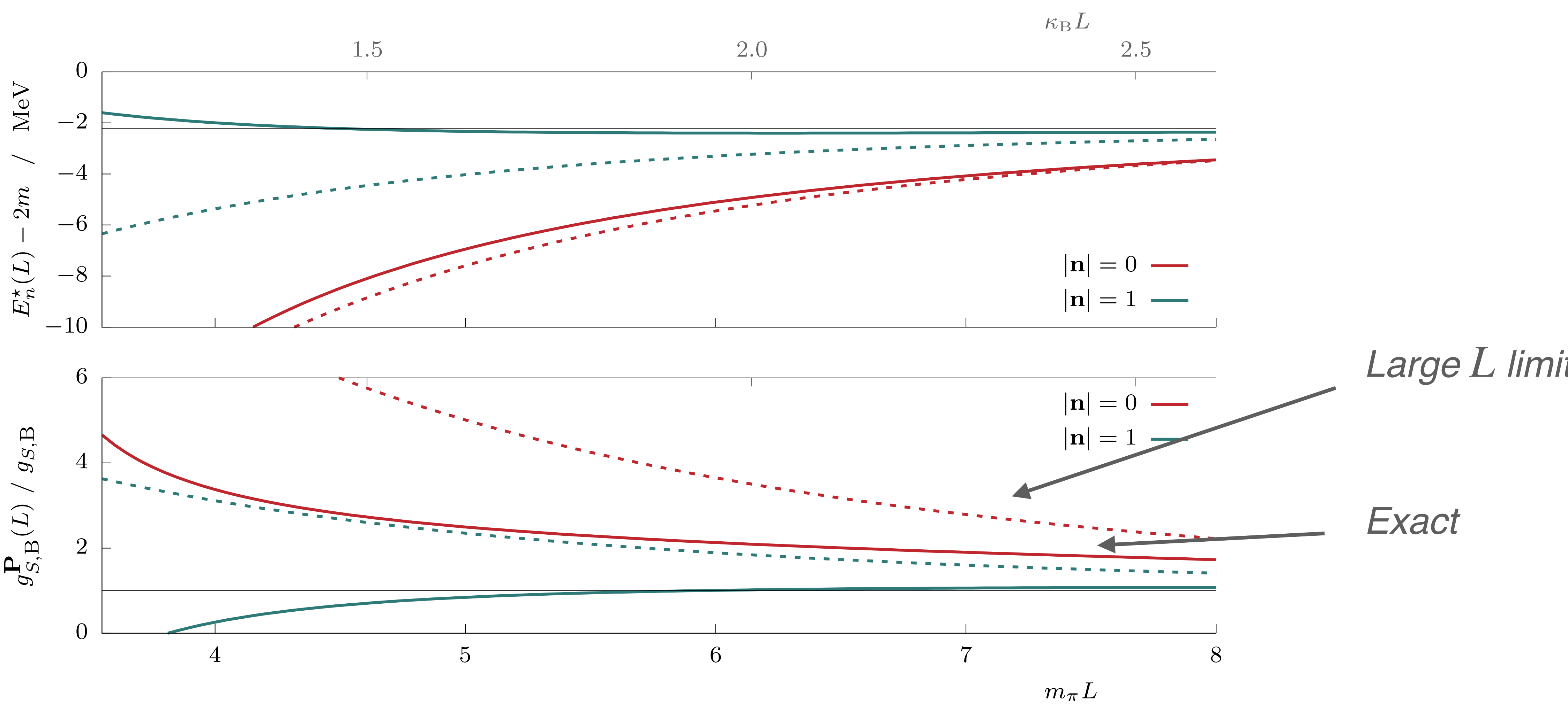
$$m_B^2(L) = m_B^2 + \mathcal{O}(e^{-\kappa L})$$

Z. Davoudi, M. Savage,
Phys. Rev. D **84** 114502 (20111)

$$\frac{g_B(L)}{g_B} = 1 + \mathcal{O}(e^{-\kappa L})$$

AJ, R. Briceño, M. Hansen,
Phys. Rev. D **100** 114505 (2019)

Low-order large L behavior can have significant deviations



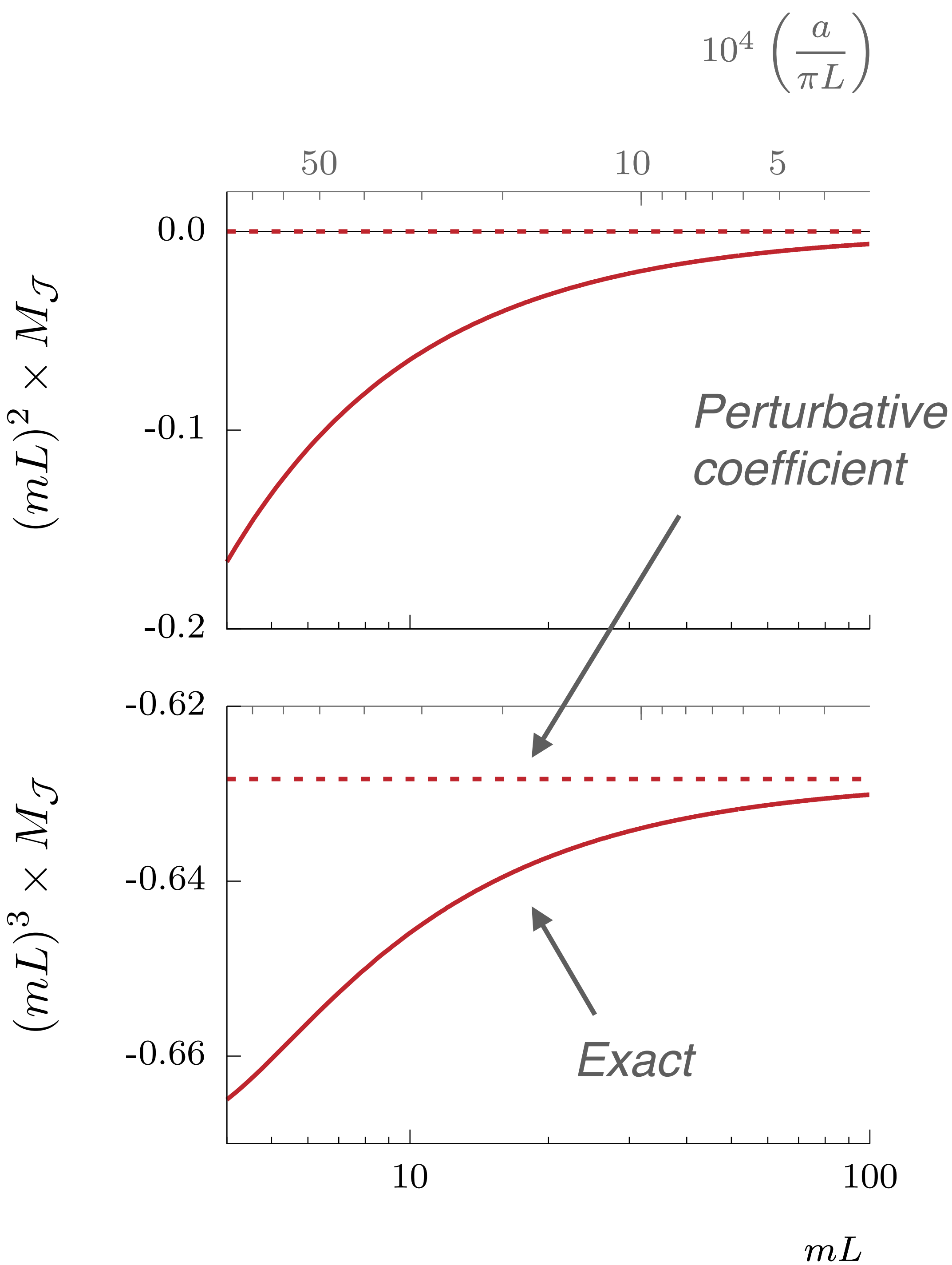
Consistency Checks (III)

Systems near threshold — Large L expansion

$$E_0(L) = 2m + \frac{4\pi a}{mL^3} + \mathcal{O}(1/L^4)$$

Lüscher (1986), Huang and Yang (1957), *many others…*

$$L^3 \langle 0 | \mathcal{J} | 0 \rangle = \frac{g}{m}\left(1 - \frac{2\pi a}{m^2 L^3} + \mathcal{O}(1/L^4)\right)$$



Confirmation via

- *Threshold expansion*
- *Perturbation theory*
- *Numerical verification*
- *Agrees with Feynman-Hellman theory*

$$L^3 \langle 0 | \mathcal{J} | 0 \rangle = g \frac{\mathrm{d}E_0(L)}{\mathrm{d}m^2}$$

AJ, R. Briceño, M. Hansen,
Phys. Rev. D **101** 094508 (2020)

Finite-Volume Formalism

Have framework relating matrix elements to amplitudes

- Formalism is relativistic, model independent, valid for arbitrary local current
- Systematically controlled below three-hadron channels

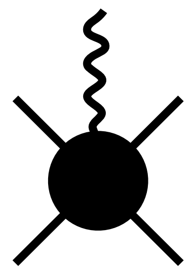
Passes cross-checks in various limits

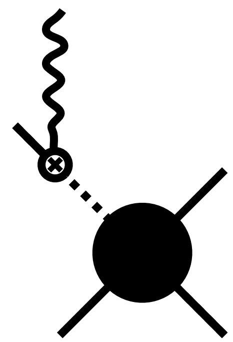
- Charge conservations ensures volume independence
- Bound states lead to exponential corrections
- Perturbative systems have consistent large L expansion

Need understanding of analytic behavior of infinite volume amplitudes

Two hadron transition amplitudes

Sum to all orders in strong interaction — interested in two-particle kinematic energies

$i\mathcal{W} =$ 

$= \mathcal{S}\Big\{$  $\Big\} + i\mathcal{W}_{\text{df}}$

Object we can determine from FV formalism

Features

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Assumptions

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AJ, R. Briceño, F. Ortega-Gama, K. Sherman,
Phys. Rev. D **103** 114512 (2021)

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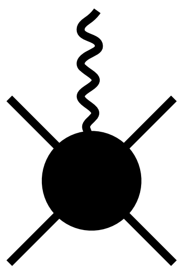
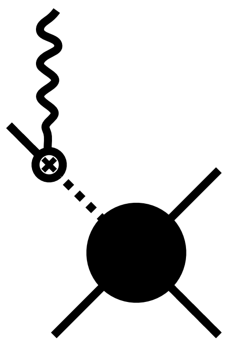
$i\mathcal{W} =$ 

$= \mathcal{S}\Big\{$  $\Big\} + i\mathcal{W}_{\rm df}$

Projecting equations to on-shell form

$$\mathcal{W}_{\rm df} = \mathcal{M} \cdot \big(\mathcal{A} + f \cdot \mathcal{G} \big) \cdot \mathcal{M}$$

AJ, R. Briceño, F. Ortega-Gama, K. Sherman,
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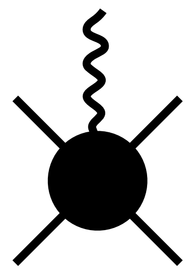
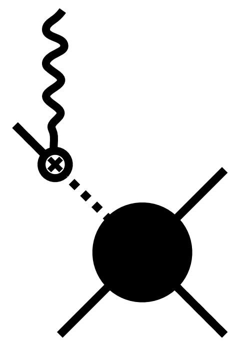
$i\mathcal{W} =$ 

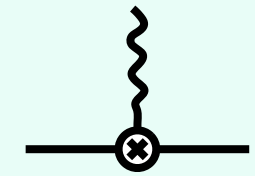
$= \mathcal{S}$ {  } $+ \; i\mathcal{W}_{\text{df}}$

Projecting equations to on-shell form

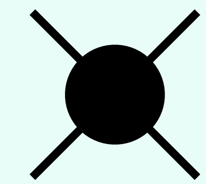
$$\mathcal{W}_{\text{df}} = \boxed{\mathcal{M}} \cdot \big(\mathcal{A} + \boxed{f \cdot \mathcal{G} \big) \cdot \mathcal{M}}$$

Known functions

Single hadron form-factors

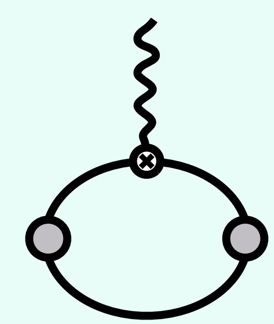
$f =$ 

Hadronic scattering amplitude

$\mathcal{M} =$ 

Triangle diagram

Contains normal and anomalous singularities from intermediate on-shell particles

$\mathcal{G} =$ 

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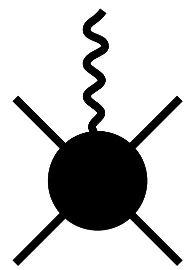
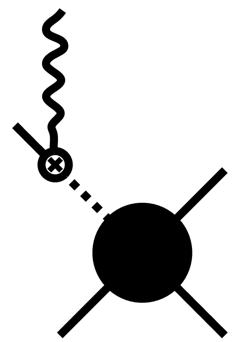
$i\mathcal{W} =$ 

$= \mathcal{S}\Big\{$  $\Big\} + i\mathcal{W}_{\mathrm{df}}$

Projecting equations to on-shell form

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Unknown function

Real and Smooth function characterizing short-distance physics

Parameterize in terms of energy-dependent form-factors

Only unknown function - determined from lattice QCD and FV formalism

Can be related to LEC's of EFT's

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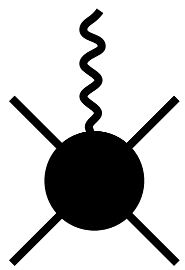
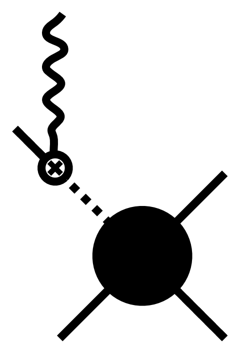
$i\mathcal{W} =$ 

$= \mathcal{S} \Big\{$  $\Big\} + i\mathcal{W}_{\text{df}}$

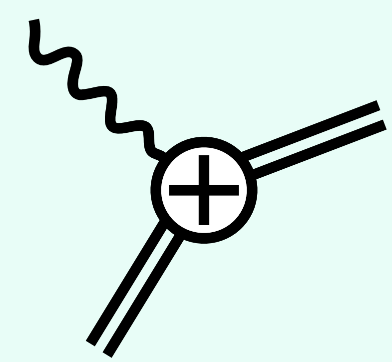
Projecting equations to on-shell form

$$\mathcal{W}_{\text{df}} = \mathcal{M} \cdot \left(\mathcal{A} + f \cdot \mathcal{G} \right) \cdot \mathcal{M}$$

Rigorous definition for resonance form factors

$$i\mathcal{W} \sim \frac{-g}{s_f - s_p} \, i f_p \, \frac{-g}{s_i - s_p}$$

$$i f_p = g^2 \left(\mathcal{A} + f \cdot \mathcal{G} \right) \Big|_{s_f = s_i = s_p}$$

$=$ 

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Phys. Rev. D **103** 114512 (2021)

Summary

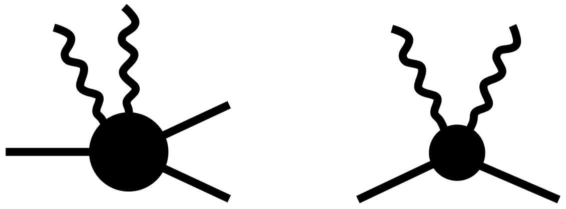
Model-independent method to determine $\mathbf{2} + \mathcal{J} \to \mathbf{2}$ processes

- Relates finite volume matrix elements to transition amplitudes
- Many cross-checks — confidence builder of formalism

Briceño, Hansen, AJ (2019 & 2020)

Global infinite volume amplitude studies

- On-shell physics fixes amplitude — single unknown real function remains
- Have forms for one current, two hadron amplitudes
- Informs structures that will appear in future FV formalisms
- Extensions to two currents



See R. Briceño, this session

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