Three-hadron s- and d-wave interactions from lattice QCD

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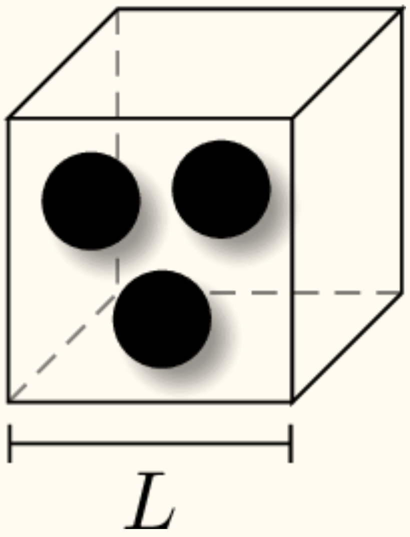
Outline

1. Motivation
    - Test and push the limits of three-particle quantization condition
2. Setup and technical details
    - Ensembles, code, analysis
    - Three-particle quantization condition
3. Results for pions and kaons
    - Three pion masses: 200, 280, 340 MeV
    - d-wave interactions constrained



Motivation

- Most QCD resonance decays involve three or more particles
  - $\omega(782) \to \pi\pi\pi$, $a_1(1260) \to \pi\pi\pi$, $N(1440) \to N\pi\pi$
- Many recent developments on the theoretical side (and their applications)
  - See plenary by Ben Hörz and other talks in this session
- Three competing formalisms to interpret three-particle finite-volume energies
  - Relativistic Field Theory (RFT) approach [Hansen, Sharpe, ...]
  - Non-relativistic effective field theory (NREFT) [Mai, Döring, ...]
  - Finite-volume unitarity (FVU) approach [Hammer, Pang, Rusetsky, ...]
- Provide real lattice data to test and push the limits of various three-particle formalisms

Lattice setup

- $N_F = 2 + 1$ *O(a)*-improved Wilson-clover fermions generated by CLS
- Three pion masses allow study of chiral dependence
  - Trace of bare quark masses held fixed
- One lattice spacing, $a = 0.06426(76)$ fm
- Consider constituent momenta up to and including $\boldsymbol{d}^2 = L^2/(2\pi)^2\, \boldsymbol{P}^2 = 9$

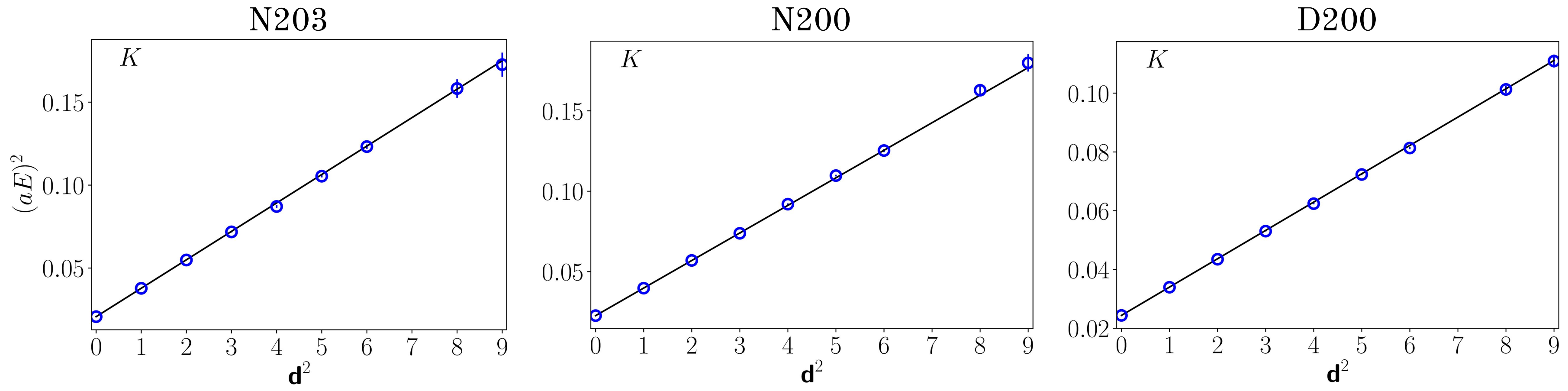
	$(L/a)^3 \times (T/a)$	M_π [MeV]	M_K [MeV]	N_{cfg}	t_{src}	N_{ev}
N203	$48^3 \times 128$	340	440	771	32, 52	192
N200	$48^3 \times 128$	280	460	1712	32, 52	192
D200	$64^3 \times 128$	200	480	2000	35, 92	448

Procedure

1. Calculate matrices of two-point correlation functions
    a. Use stochastic LapH for quark propagation
    b. Construct operators to transform in irreps of little group
    c. Optimize contractions (https://github.com/laphnn/contraction_optimizer)
2. Extract finite-volume energies from correlation matrices
    a. Solve Generalized EigenValue Problem (GEVP) for correlator matrices
    b. Fit ratio of rotated correlators to single-exponential to extract shifts from non-interacting
    c. Reconstruct energies and boost to center-of-momentum frame
3. Obtain K-matrices from spectrum
    a. Adjust K-matrix parameters until lattice energies match predictions from quantization condition

Single-Meson Energies

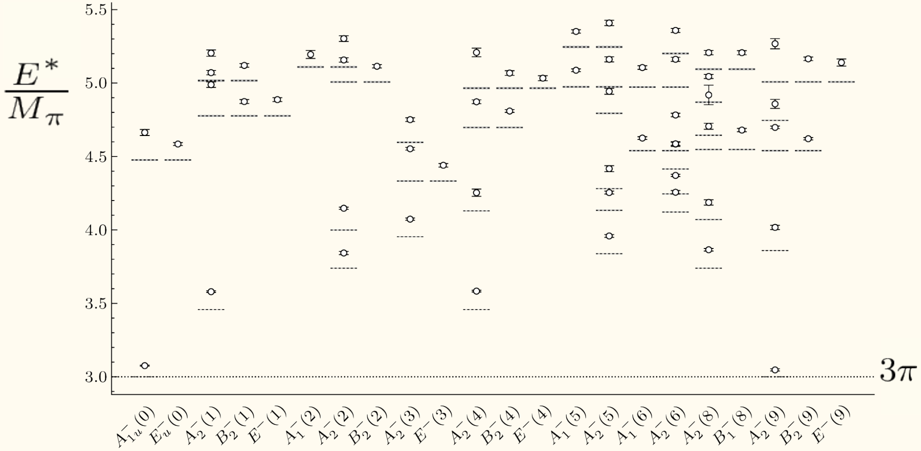
- Single-exponential fits to correlators of momentum-projected kaon operators
- Continuum dispersion relation works well up to $\boldsymbol{d}^2 = 9$
- No sign of cutoff effects here (see talk from Jeremy Green [Tue. 13:30])
- Similar situation for pions

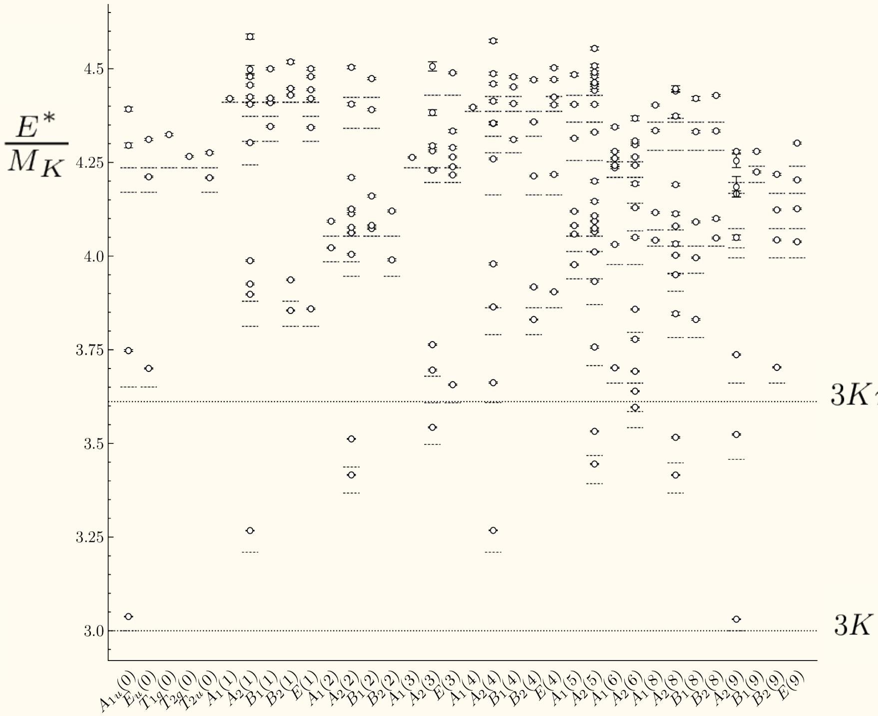


Spectrum Results on N200

Single-exponential fits to

$$R_n(t) \equiv \frac{v_n^{\dagger}(\tau_0, \tau_D) C(t) v_n(\tau_0, \tau_D)}{\prod_i C^{(\mathrm{sh})}(\mathbf{p}_i^2, t)}$$





Two- and Three-particle Quantization Conditions

Two-particle QC

$$\det\left[F(E_2,\mathbf{P},L)^{-1}+\mathcal{K}_2(E_2^*)\right]=0$$

- F is a purely kinematic known finite-volume function
- $\mathcal{K}_2(E_2^*)_{\ell' m';\ell m}=\delta_{\ell'\ell}\delta_{m'm}\mathcal{K}_2^{(\ell)}(E_2^*)$ is an infinite-volume quantity with algebraic relation to two-particle scattering amplitude

Three-particle QC

$$\det\left[F_3(E,\mathbf{P},L)^{-1}+\mathcal{K}_{\mathrm{df},3}(E^*)\right]=0$$

- F_3 contains both kinematic functions and the two-particle K-matrix
- $\mathcal{K}_{\mathrm{df},3}$ is an infinite-volume quantity but is scheme-dependent
- Must solve integral equation to obtain three-particle scattering amplitude

Fitting the Spectrum

- Parameterization of two-particle K-matrix
  - For s-wave, use the effective range expansion or a form that explicitly includes the Adler zero
  - Use the d-wave scattering length
- Parameterization of $\mathcal{K}_{\text{df},3}$ given by threshold expansion to quadratic order

$$\mathcal{K}_{\text{df},3} = \mathcal{K}_{\text{df},3}^{\text{iso},0} + \mathcal{K}_{\text{df},3}^{\text{iso},1}\Delta + \mathcal{K}_{\text{df},3}^{\text{iso},2}\Delta^2 + \underbrace{\mathcal{K}_A\Delta_A + \mathcal{K}_B\Delta_B}_{\text{Two-particle d-wave contributions}}$$

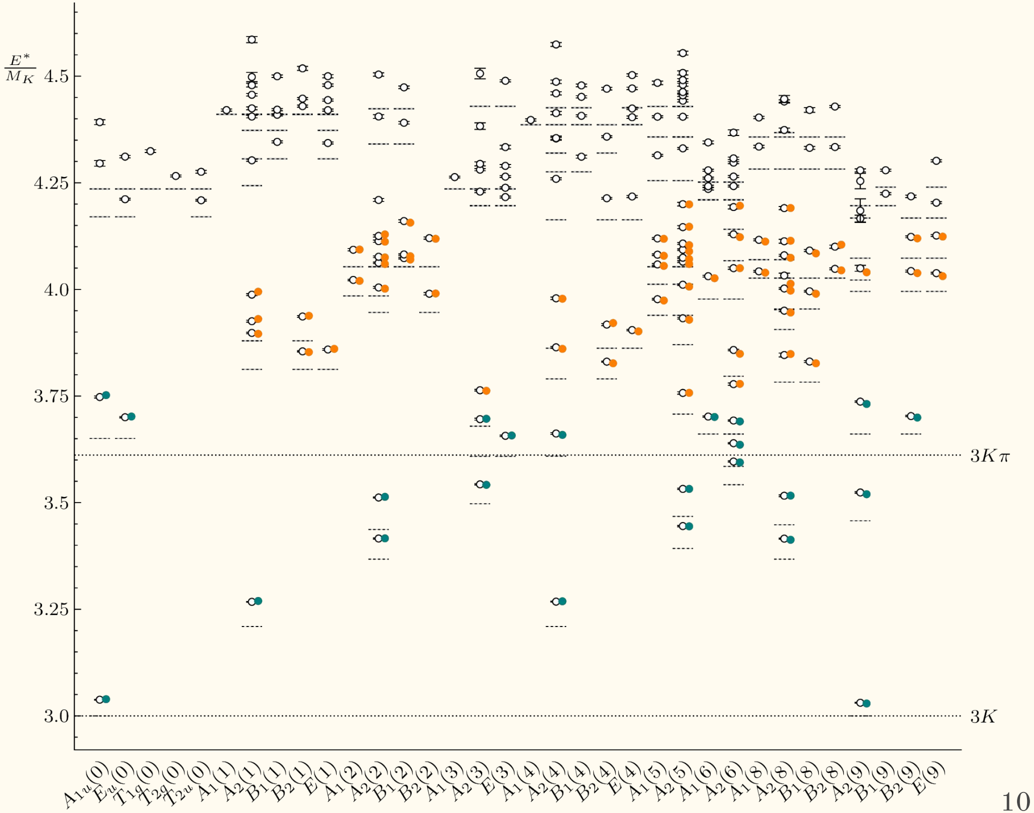
(see arXiv:1901.07095 for details)

- Parameters $\{p_\text{n}\}$ determined from minimum of

$$\chi^2(\{p_n\}) = \sum_{ij}\left(E_i - E_i^{\text{QC}}(\{p_n\})\right) C_{ij}^{-1}\left(E_j - E_j^{\text{QC}}(\{p_n\})\right)$$

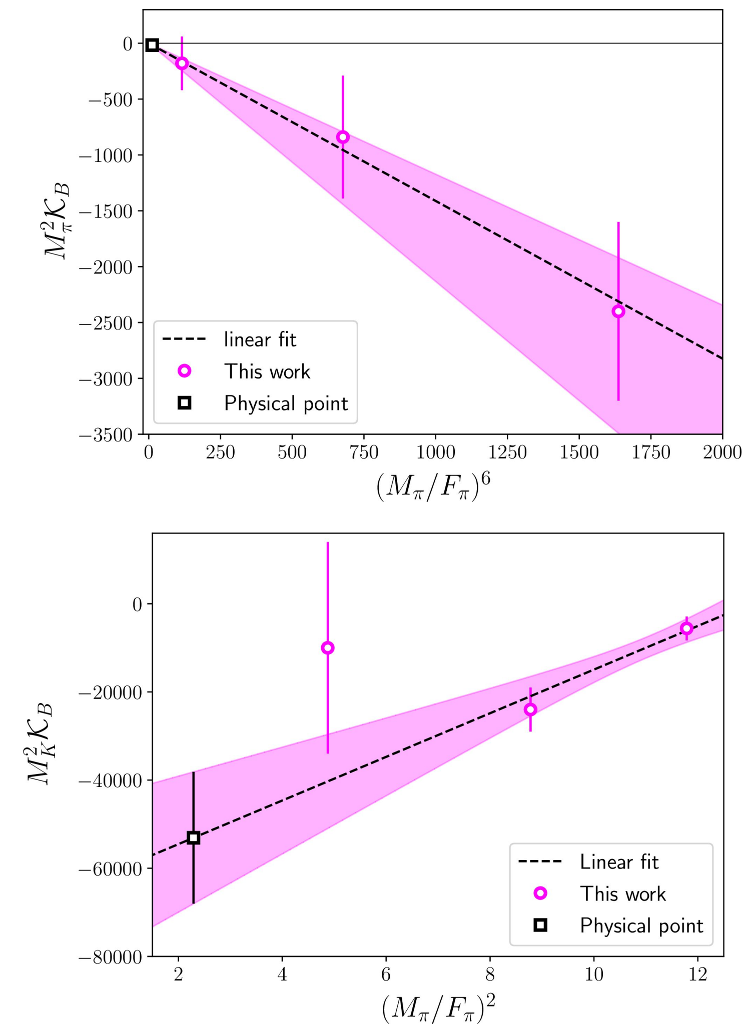
Testing the Limits of the Formalism

- QC is valid up to first threshold with more than three particles (depending on allowed transitions)
- Transition to 3Kπ expected to be NNLO in ChPT, leading to suppressed coupling near threshold
- Fits describe data well above rigorous applicability of QC



Inclusion of d-wave terms

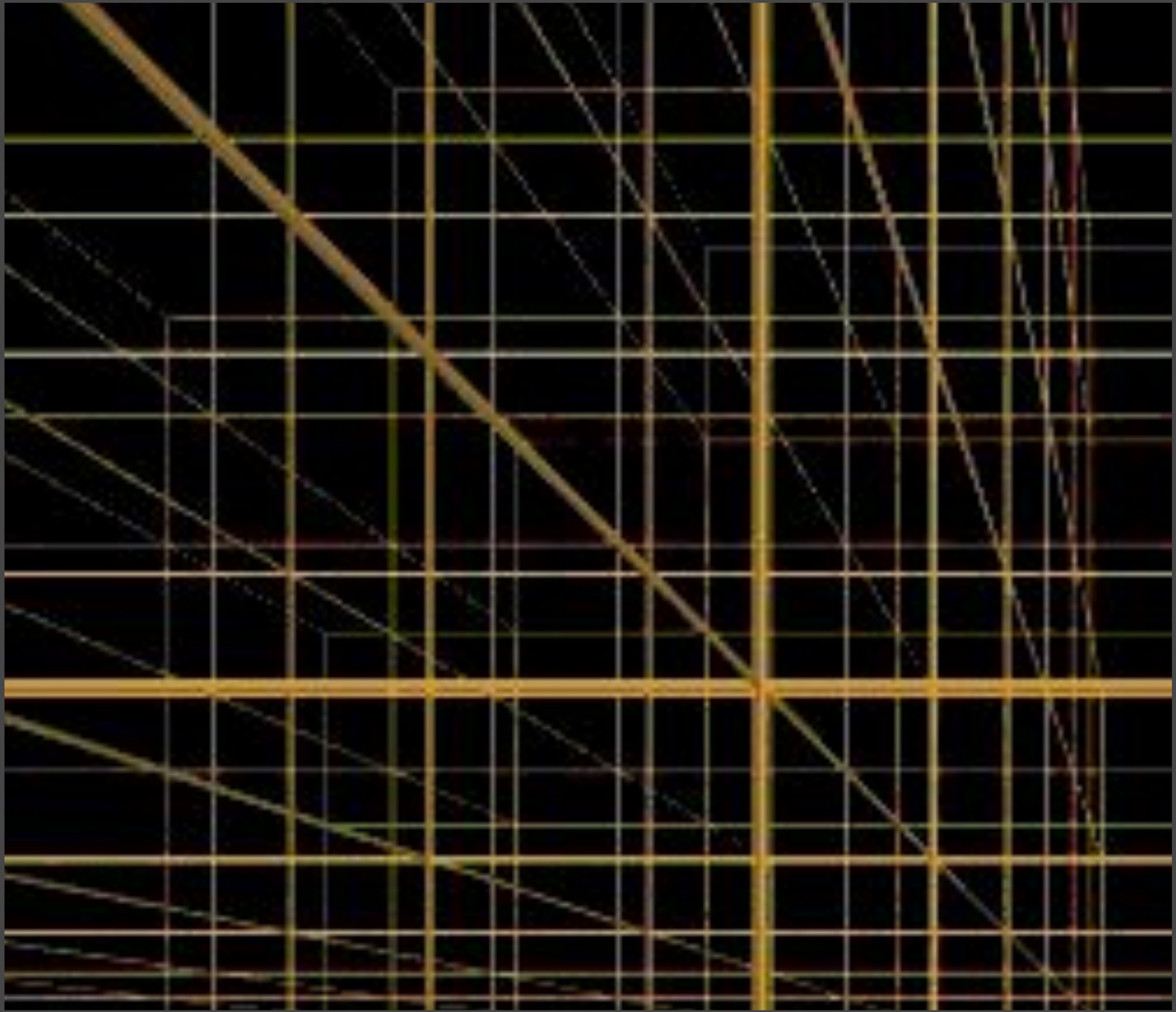
- $\mathcal{K}_B$ and d-wave in $\mathcal{K}_2$ essential for good fit quality
- Only $\mathcal{K}_B$ contributes to non-trivial irreps, making it easier to constrain
- Can only appear at NLO in ChPT
- Larger error on D200 from large $M_K L$, leading to suppression of energy shifts



Conclusions and Outlooks

- Three-particle quantization condition for simple systems
  - Hundreds of energies extracted
  - d-wave terms in two- and three-particle K-matrix improve fit quality (substantially at times)
  - First calculation showing strong indication that non-zero three-particle interactions are needed
- Future work
  - Spectra for mixed-flavor systems (e.g. $\pi\pi K$ and πKK)
    - RFT formalism worked out by Blanton and Sharpe [arXiv:2105.12094]
  - Systems with non-maximal isospin, resonances, and/or bound states
    - RFT formalism worked out by Hansen, Romero-López, Sharpe [arXiv:2003.10974]
    - Application to $a_1(1260)$ by GWU [arXiv:2107.03973]
  - Integral equations for d-wave

Thanks!



Math grid tessellation (https://gifer.com/)