



Trinity College Dublin
Coláiste na Tríonóide, Baile Átha Cliath
The University of Dublin



Isospin-1/2 $D\pi$ scattering and the lightest D_0^* resonance from lattice QCD

Based on [arXiv:2102.04973]

Lattice 2021

Speaker: Nicolas Lang^a

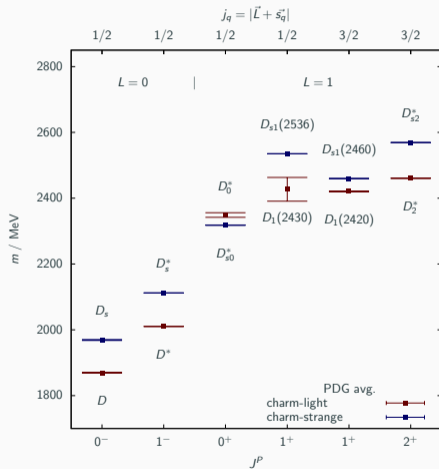
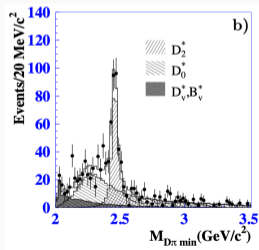
Authors: L. Gayer, N. Lang, S. M. Ryan, D. Tims, C. E. Thomas, D. J. Wilson
for the Hadron Spectrum Collaboration

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^aTrinity College Dublin

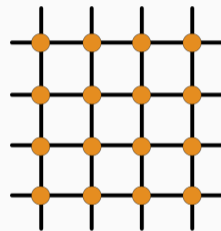
D_0^* - the experimental puzzle

- D_0^* lightest scalar charm-light resonance (Belle 2004: $m_{D_0^*} \approx 2300$ MeV, broad enhancement)
- Quark model construction: $q\bar{q}$ in relative P -wave
- PDG average: D_{s0}^* below D_0^* in mass (quark model: predicted above)



Enter: Lattice QCD

- Lattice QCD \rightarrow first principles approach to understand QCD dynamics
- On Lattice: D_0^* as part of $D\pi \rightarrow D\pi$ scattering
- Other HadSpec lattice studies of these systems:
 - D_0^* in $D\pi \rightarrow D\pi$ at $m_\pi = 391$ MeV¹
 - D_{s0}^* in $DK \rightarrow DK$ at both $m_\pi = 391$ MeV and $m_\pi = 239$ MeV²
- Existing lattice study of D_0^* in $D\pi$ -scattering found resonance close to PDG average³
- Objective here: determine pole position; understand mass ordering and light-quark mass dependence of these two states



¹G. Moir et al. [arXiv:1607.07093]

²G. K. C. Cheung et al. [arXiv:2008.06432]

³D. Mohler et al. [arXiv:1208.4059]

- Basis of interpolating operators (quark bilinears and meson-meson) with $C = 1$, $I = 1/2$ projected to irreducible representations (*irreps*) of the lattice
- Contractions make use of distillation framework¹ with 256 vectors
- Principal correlators computed using GEV method:
 $C_{ij}(t)v_j^{(n)} = \lambda_n(t, t_0)C_{ij}(t_0)v_j^{(n)}$
- Correlator fits (sum of exponentials) → Finite volume spectrum

a_s	0.11 fm
a_t^{-1}	6.079 GeV
$(L/a_s)^3 \times (T/a_t)$	$32^3 \times 256$
m_π	239 MeV
N_f	2 + 1
N_{cfg}	484

¹Hadron Spectrum collaboration [arXiv:0905.2160]

Spectrum \rightarrow Infinite volume amplitudes

- Fit of spectrum to solutions of Lüscher condition¹:
 $\det [1 + i\rho(s) \cdot \mathbf{t}(s) \cdot (1 + i\mathcal{M}(s, L))] = 0$
- Parametric form of \mathbf{t} -matrix undetermined by Lüscher condition
- **Unitarity, causality and analyticity** provide constraints
- Using a single parametrisation could introduce bias
- We use a range of different parametrisations:
 - K -matrix: $(\mathbf{t}^{(\ell)})^{-1}(s) = \frac{1}{(2k)^\ell} K^{-1}(s) \frac{1}{(2k)^\ell} + I(s)$
 - Effective range
 - Breit Wigner
 - Unitarized chiral amplitude²

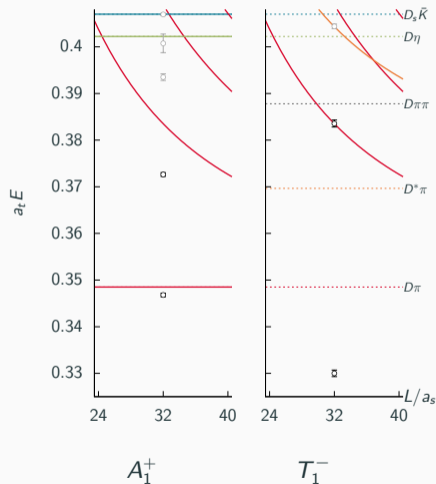
¹Arbitrary two-particle QC from QFT: R. A. Briceño [arXiv:1401.3312];

Extension to three particles: M. T. Hansen et al. [arXiv:1901.00483]

²Z.-H. Guo et al. [arXiv:1811.05585]

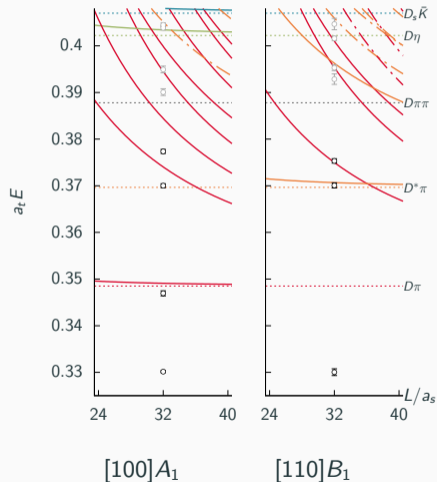
Spectra at rest

- Irreps are labelled $[d\vec{d}]\Lambda^{(P)}$ - with parity P and lattice momentum $\vec{P} = 2\pi\vec{d}/L$
- At rest: neat separation of lowest partial waves
 - A_1^+ : S -wave
 - T_1^- : P -wave
 - E^+ : D -wave
- A_1^+ : additional level around $a_t E_{\text{cm}} = 0.37$; levels above and below shifted up and down respectively \rightarrow suggestive of non-trivial interactions
- T_1^- : level far below threshold; little interaction above threshold
- E^+ : negligible interactions



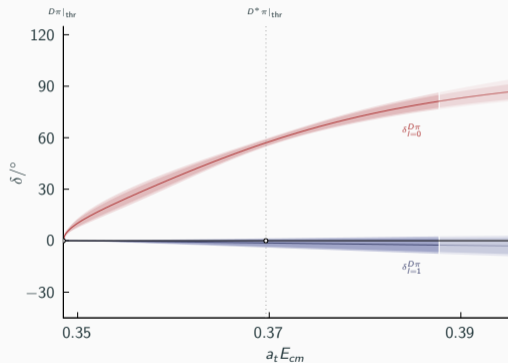
Spectra at non-zero momentum

- Moving-frame \rightarrow rotational symmetry further broken \rightarrow further mixing of partial waves
- A_1 irreps have contributions from S - and P -wave
- $[110]B_1/B_2$ and $[100]E_2$ irreps have a contribution from $D^*\pi$ S -wave



$D\pi$ S- and P-wave

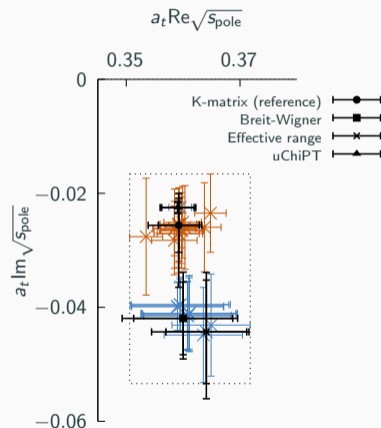
- Fit of energy levels below $D\pi\pi$ threshold in A_1^+ , T_1^- & moving-frame A_1 irreps
- Excluding irreps that have $D^*\pi$ contribution
- Deeply bound level in all irreps with P -wave contribution; "extra" level in irreps with S -wave contribution
- Parametrisation: K -matrix for 2 partial waves, both containing a pole term



- Cluster of poles from 30 different parametrisations; all above threshold
→ Resonance
- Amplitudes similar at real energies but differ in complex plane; pole common feature
- Scatter of poles: single parametrisation might underestimate uncertainties
- Mass and coupling considering all parametrisations:

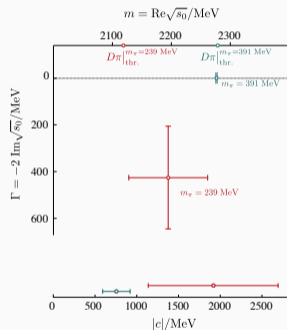
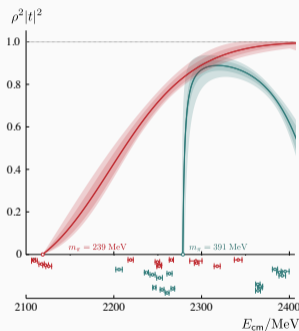
$$\sqrt{s_0}/\text{MeV} = (2196 \pm 64) - \frac{i}{2}(425 \pm 224)$$

$$c/\text{MeV} = (1916 \pm 776) \exp i\pi(-0.59 \pm 0.41)$$

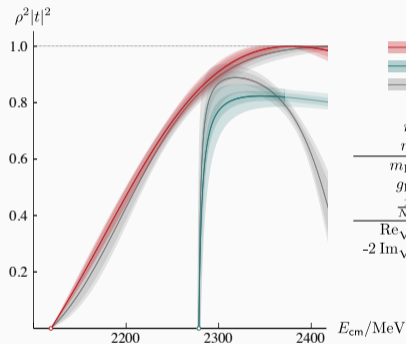


$D\pi$ at different light-quark masses

- Earlier study of $D\pi \rightarrow D\pi$ at $m_\pi = 391$ MeV: shallow bound-state ($\approx 2 \pm 1$ MeV below threshold)
- At 239 MeV: pole migrates into complex plane ($\approx 77 \pm 64$ MeV above threshold)
- Mass **below reported experimental value** (despite heavier-than-physical light quarks)
- Strong coupling of poles to $D\pi$ channel in both cases



Study: parametrisations of the $D\pi$ S -wave amplitude



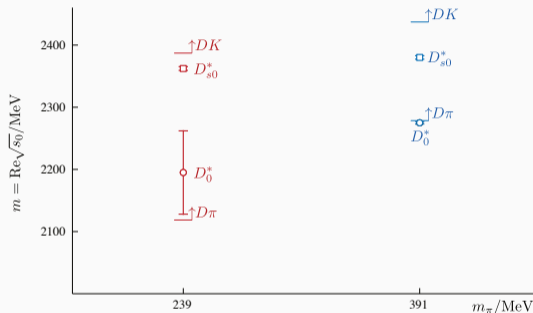
- Breit-Wigner $m_\pi = 239$ MeV
- Breit-Wigner $m_\pi = 391$ MeV
- K-matrix

m_π/MeV	239	391
m_D/MeV	1880	1887
m_{BW}/MeV	2380(36)	2206(32)
g_{BW}	5.39(56)	7.62(75)
$\frac{\chi^2}{N_{\text{dof}}}$	$\frac{14.6}{20-4}$	$\frac{36.0}{29-5}$
$\text{Re}\sqrt{s_0}/\text{MeV}$	2189(72)	2275(1)
$-2 \text{Im}\sqrt{s_0}/\text{MeV}$	510(97)	-
$ c /\text{MeV}$	2391(411)	826(133)

- Comparison: K -matrix and Breit-Wigner
- Real parts of the poles are compatible between both parametrisations
- Breit-Wigner mass parameter incompatible with pole location

D_0^* and D_{s0}^* : mass hierarchy

- Locations of poles follow expectations from SU(3) symmetry
- D_0^* shallow bound state at $m_\pi = 391$ MeV¹ - resonance at $m_\pi = 239$ MeV
- Pole mass decreases with pion mass \rightarrow extrapolation to physical pion mass would suggest D_0^* well below D_{s0}^*
- D_{s0}^* bound at both masses²
- Recent study³ performing UChPT amplitude fit to LHCb data also suggests a lower mass of the D_0^*



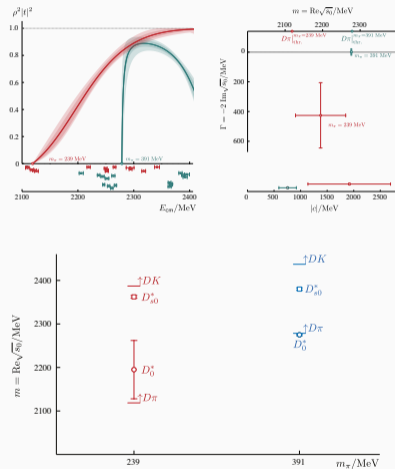
¹G. Moir et al. [arXiv:1607.07093]

²G. K. C. Cheung et al. [arXiv:2008.06432]

³M.-L. Du et al. [arXiv:2012.04599]

Summary

- Found D_0^* resonance pole with
 - $m = (2194 \pm 64)$ MeV
(77 ± 64) MeV above $D\pi$ threshold
 - $\Gamma = (425 \pm 224)$ MeVfrom first principles
(no external inputs after fixing quark masses)
- Range of parametrisations
(major contribution to uncertainty)
- Strongly coupled to $D\pi$ channel;
compatible with $D_{s0}^* \rightarrow DK$
- Slight decrease in pole mass with decreasing pion mass
- Significantly lower than current PDG value
 \rightarrow puzzling D_0^* heavier than D_{s0}^* not reproduced by Lattice



Questions?

Correlators on the lattice

- Compute matrix of (euclidean) correlators:

$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) | 0 \rangle ,$$

- $\mathcal{O}_i(t)$ have quantum numbers of $I = 1/2 D\pi$
- Find "optimal" interpolators by solving *Generalised Eigenvalue* (GEV) problem

$$C_{ij}(t) v_j^{(n)} = \lambda_n(t, t_0) C_{ij}(t_0) v_j^{(n)} ,$$

- Fit Principal correlators (eigenvalues):

$$\lambda_n(t, t_0) = (1 - A_n) e^{-E_n(t-t_0)} + A_n e^{-E'_n(t-t_0)} .$$

Operator Table (S-wave)

$A_1^+[000]$	$A_1[100]$	$A_1[110]$	$A_1[111]$	$A_1[200]$
$D_{[000]} \pi_{[000]}$	$D_{[000]} \pi_{[100]}$	$D_{[000]} \pi_{[110]}$	$D_{[000]} \pi_{[111]}$	$D_{[100]} \pi_{[100]}$
$D_{[100]} \pi_{[100]}$	$D_{[100]} \pi_{[000]}$	$D_{[100]} \pi_{[100]}$	$D_{[100]} \pi_{[110]}$	$D_{[110]} \pi_{[110]}$
$D_{[110]} \pi_{[110]}$	$D_{[100]} \pi_{[110]}$	$D_{[110]} \pi_{[000]}$	$D_{[110]} \pi_{[100]}$	$D_{[200]} \pi_{[000]}$
$D_{[111]} \pi_{[111]}$	$D_{[100]} \pi_{[200]}$	$D_{[110]} \pi_{[110]}$	$D_{[111]} \pi_{[000]}$	$D_{[210]} \pi_{[100]}$
$D_{[000]} \eta_{[000]}$	$D_{[110]} \pi_{[100]}$	$D_{[111]} \pi_{[100]}$	$D_{[211]} \pi_{[100]}$	$D_{[200]} \eta_{[000]}$
$D_{[100]} \eta_{[100]}$	$D_{[110]} \pi_{[111]}$	$D_{[210]} \pi_{[100]}$	$D_{[110]}^* \pi_{[100]}$	
$D_{s[000]} \bar{K}_{[000]}$	$D_{[111]} \pi_{[110]}$	$D_{[100]}^* \pi_{[100]}$	$D_{[111]} \eta_{[000]}$	
	$D_{[200]} \pi_{[100]}$	$D_{[111]}^* \pi_{[100]}$	$D_{s[111]} \bar{K}_{[000]}$	
	$D_{[210]} \pi_{[110]}$	$D_{[110]} \eta_{[000]}$		
	$D_{[000]} \eta_{[100]}$	$D_{s[110]} \bar{K}_{[000]}$		
	$D_{[100]} \eta_{[000]}$			
	$D_{s[000]} \bar{K}_{[100]}$			
	$D_{s[100]} \bar{K}_{[000]}$			
$8 \times \bar{\psi} \Gamma \psi$	$18 \times \bar{\psi} \Gamma \psi$	$18 \times \bar{\psi} \Gamma \psi$	$9 \times \bar{\psi} \Gamma \psi$	$16 \times \bar{\psi} \Gamma \psi$

Operators used in the S-wave fits. Subscripts indicate momentum types. Γ represents some monomial of γ matrices and derivatives.

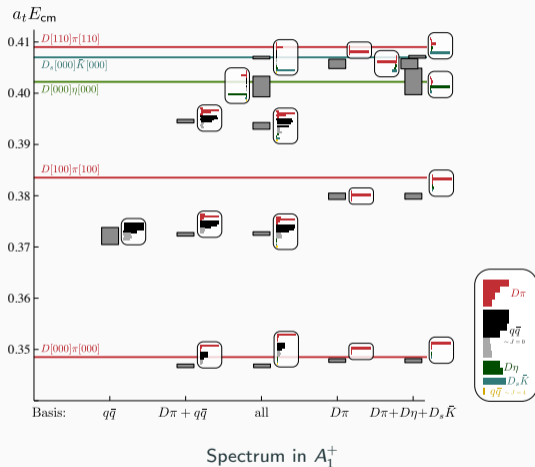
Operator Table (P -wave)

$T_1^- [000]$	$E_2 [100]$	$B_1 [110]$	$B_2 [110]$
$D_{[100]} \pi_{[100]}$	$D_{[100]} \pi_{[110]}$	$D_{[100]} \pi_{[100]}$	$D_{[100]} \pi_{[111]}$
$D_{[110]} \pi_{[110]}$	$D_{[110]} \pi_{[100]}$	$D_{[110]} \pi_{[110]}$	$D_{[110]} \pi_{[110]}$
$D^*_{[100]} \pi_{[100]}$	$D^*_{[000]} \pi_{[100]}$	$D_{[210]} \pi_{[100]}$	$D_{[111]} \pi_{[100]}$
	$D^*_{[100]} \pi_{[000]}$	$D^*_{[100]} \pi_{[100]}$	$D^*_{[000]} \pi_{[110]}$
		$D^*_{[110]} \pi_{[000]}$	$D^*_{[100]} \pi_{[100]} \{2\}$
			$D^*_{[110]} \pi_{[000]}$
			$D^*_{[111]} \pi_{[100]}$
$6 \times \bar{\psi} \Gamma \psi$	$18 \times \bar{\psi} \Gamma \psi$	$18 \times \bar{\psi} \Gamma \psi$	$20 \times \bar{\psi} \Gamma \psi$

Operators used in the P -wave fits. Subscripts indicate momentum types. Γ represents some monomial of γ matrices and derivatives. The number in curly parentheses indicates the number of operators of this momentum combination.

Operator basis variations

- Varying the basis affects the spectrum
- $I = 1/2$ allows both meson-meson and $q\bar{q}$ -like operator constructions
- Interpolating the complete spectrum requires both types of operator
- Other meson-meson operators do not play a significant role below coupled-channel threshold



How are operators constructed?

- Two types of interpolating operator:
 - quark bilinears: $\bar{\psi}\Gamma D\dots\psi$
 - meson-meson like operators: $\sum_{\vec{p}_1+\vec{p}_2=\vec{p}} \mathcal{C}(\vec{p}_1, \vec{p}_2)\Omega_{M_1}^\dagger(\vec{p}_1)\Omega_{M_2}^\dagger(\vec{p}_2)$
- Rotational symmetry broken \Rightarrow eigenstates labelled by irreducible representations of O_h or $LG(\vec{P})$ (*irreps*)
- Continuum spins *subduce* into one or more finite volume irreps; operators are projected into irreps
- Correlators are computed using distillation with 256 vectors

Subduction Table

\vec{P}	Irrep Λ	J^P ($\vec{P} = \vec{0}$) $ \lambda ^{(\vec{\eta})}$ ($\vec{P} \neq \vec{0}$)	$D\pi J_{[M]}^P$	$D^*\pi J_{[M]}^P$
[000]	A_1^+	$0^+, 4^+$	$0^+, \dots$	\dots
	T_1^-	$1^-, 3^-$	$1^-, \dots$	\dots
	E^+	$2^+, 4^+$	$2^+, \dots$	\dots
[n00]	A_1	$0^{(+)}, 4$	$0^+, 1^-, 2^+, \dots$	\dots
	E_2	$1, 3$	$1^-, 2^+, \dots$	$1^+, \dots$
[nn0]	A_1	$0^{(+)}, 2, 4$	$0^+, 1^-, 2_{[2]}^+, \dots$	\dots
	B_2, B_2	$1, 3$	$1^-, 2^+, \dots$	$1^+, \dots$
[nnn]	A_1	$0^{(+)}, 3$	$0^+, 1^-, 2^+, \dots$	\dots

Lowest $D\pi$ and $D^*\pi$ continuum J^P and helicity λ subductions by irrep

Masses and thresholds

	$a_t m$
π	0.03928(18)
K	0.08344(7)
η	0.09299(56)
D	0.30923(11)
D_s	0.32356(12)
D^*	0.33058(24)

	$a_t E_{\text{threshold}}$
$D\pi$	0.34851(21)
$D\pi\pi$	0.38779(27)
$D\eta$	0.40222(57)
$D_s\bar{K}$	0.40700(14)
$D^*\pi\pi$	0.40914(35)

Left: A summary of the stable hadron masses relevant for this calculation. Right: kinematic thresholds relevant for $I = 1/2$ $D\pi$ scattering.

From the spectrum to scattering amplitudes

- Need a mapping between finite-volume spectrum and infinite volume scattering amplitudes
→ Lüscher quantisation condition

$$\det [1 + i\rho(s) \cdot \mathbf{t}(s) \cdot (1 + i\mathcal{M}(s, L))] = 0$$

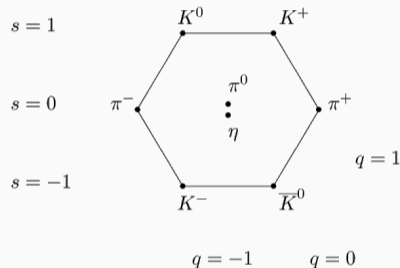
- $\rho(s) = 2k(s)/\sqrt{s}$ with $k(s)$ the COM-momentum function
- $\mathbf{t}(s)$ = infinite volume t-matrix
- $\mathcal{M}(s, L)$ encodes finite-volume effects (dense in partial waves)
- Procedure
 - solve equation (20) for a given parametrisation of $\mathbf{t}(s)$ to obtain a spectrum
 - vary the parameters in $\mathbf{t}(s)$ in a χ^2 -minimisation to best match the spectrum obtained from the lattice

SU(3) flavour symmetry

- When $m_u = m_d = m_s$ π and K are rows of the same SU(3) octet
→ $D\pi$ and DK scattering related by SU(3) flavour symmetry

$$\bar{\mathbf{3}} \otimes \mathbf{8} \rightarrow \bar{\mathbf{3}} \oplus \mathbf{6} \oplus \bar{\mathbf{15}}$$

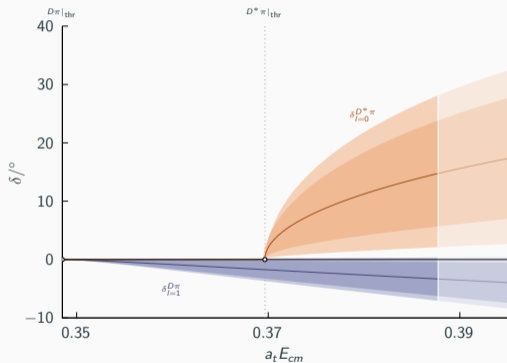
- Symmetry is less broken at heavier light-quark masses
- We expect the number of poles to stay the same as function of quark mass



[https://en.wikipedia.org/wiki/Eightfold_way_\(physics\)](https://en.wikipedia.org/wiki/Eightfold_way_(physics)) - Creative Commons

$D\pi$ P-wave and $D^*\pi$ S-wave

- Determined from spectrum fits in $[000]T_1^-$, $[100]E_2$, $[110]B_1$ and $[110]B_2$
- Deeply bound level in all irreps $\rightarrow J^P = 1^-$ D^* bound state
- $D^*\pi$ S-wave \rightarrow contribution in moving frames
- Parametrisation: K -matrix with 2 channels with a pole term in $D\pi$ P-wave
- Phase shift indicates very weak effect of P-wave above threshold



Combined $D\pi$ $S + P$ -wave and $D^*\pi$ S -wave

- Sanity check: Fit of all relevant partial waves below three-body threshold
- Fit of energy levels below $D\pi\pi$ threshold in all irreps we computed
- Parametrisation: K -matrix with 2 channels / 3 partial waves
- Pole term in $D\pi$ S - and P -wave
- Constant in $D^*\pi$ S -wave
- Results compatible with fit excluding $D^*\pi$

