Isospin Breaking Effects in Octet and Decuplet Baryon Masses

Alexander Segner
Andrew Hanlon, Renwick James Hudspith, Andreas Risch, Hartmut Wittig

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Motivation

- Discrepency between theory and experiment on $a_\mu = \frac{(g-2)_\mu}{2}$
  - Theoretical uncertainty is dominated by QCD
  - CLS $N_f = 2+1$ ensembles
    $\rightarrow$ Isospin symmetric action leads to systematic uncertainty
    $\rightarrow$ Need to account for QED effects and difference in light quark masses

- Scale setting currently done using f_π and f_K[1]
  $\rightarrow$ Difficult to determine IB corrections[2] reliably
  $\rightarrow$ Use baryon masses instead

- Allows for calculations of mass splittings of isospin multiplets[3]

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[1] Bruno et al. 2017, *Phys. Rev. D* **95** no. 7, p. 074504.
[2] Carrasco et al. 2015, *Phys. Rev. D* **91** no. 7, p. 074506.
[3] Borsanyi et al. 2015, *Science* **347**, pp. 1452–1455.

QCD+QED vs. QCD_{iso}

- Method based on a procedure introduced by the *RM123 collaboration*[4][5]
- Consider a QCD+QED action S with parameters:

$$\varepsilon = \left(\beta, e^2, m_u, m_d, m_s\right)$$

- Expand around isosymmetric action $S^{(0)}$ with parameters

$$\varepsilon^{(0)} = \left(\beta^{(0)}, 0, m_{ud}^{(0)}, m_{ud}^{(0)}, m_s^{(0)}\right)$$

- Dividing S into three parts, write

$$S[U, A, \psi, \bar{\psi}] = S_g[U] + S_\gamma[A] + S_q[U, A, \psi, \bar{\psi}]$$

- QED_L action S_γ[6] in Coulomb gauge
  $\rightarrow$ IR regularisation by setting $\sum_{\mathbf{x}} A^{\mathbf{x},t} = 0 \, \forall t$

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[4] Divitiis et al. 2012, *JHEP* **04**, p. 124.
[5] Divitiis et al. 2013, *Phys. Rev. D* **87** no. 11, p. 114505.
[6] Hayakawa and Uno 2008, *Prog. Theor. Phys.* **120**, pp. 413–441.

Expectation values

$$
\begin{aligned}
\langle \mathcal{O} \rangle^{\varepsilon} &= \left\langle \langle \mathcal{O} \rangle_{q\gamma} \right\rangle_{\text{eff}}^{\varepsilon} \\
\langle \mathcal{O} \rangle_{q\gamma}[U] &= \frac{1}{Z_{q\gamma}[U]} \int \mathcal{D}A \mathcal{D}\psi \mathcal{D}\bar{\psi}\, \mathcal{O}[U, A, \psi, \bar{\psi}] e^{-S_\gamma[A] - S_q[U, A, \psi, \bar{\psi}]} \\
\langle \mathcal{O}[U] \rangle_{\text{eff}}^{\varepsilon} &= \frac{1}{Z} \int \mathcal{D}U\, \mathcal{O}[U] Z_{q\gamma}[U] e^{-S_g[U]} \\
&= \frac{1}{Z} \int \mathcal{D}U\, \mathcal{O}[U] e^{-S_{\text{eff}}[U]}
\end{aligned}
$$

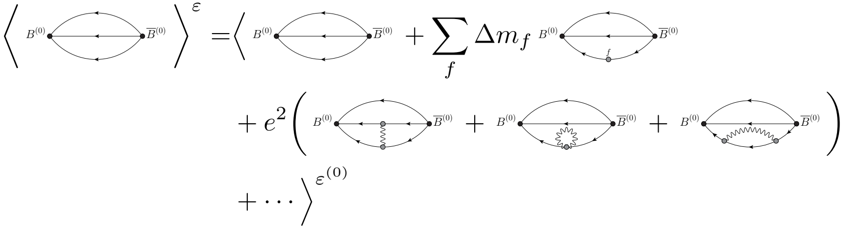
Can relate $\langle \mathcal{O} \rangle^{\varepsilon}$ to $\langle \mathcal{O} \rangle^{\varepsilon^{(0)}}$ via reweighting:

$$
\langle \mathcal{O} \rangle^{\varepsilon} = \frac{\left\langle R \langle \mathcal{O} \rangle_{q\gamma} \right\rangle_{\text{eff}}^{\varepsilon^{(0)}}}{\langle R \rangle_{\text{eff}}^{\varepsilon^{(0)}}}, \quad R = \frac{\exp(-S_{\text{eff}})}{\exp\left(-S_{\text{eff}}^{(0)}\right)} = \frac{\exp(-S_g) Z_{q\gamma}}{\exp\left(-S_g^{(0)}\right) Z_q^{(0)}}
$$

Expand expectation values

$$\langle \mathcal{O} \rangle^{\varepsilon} = \langle \mathcal{O} \rangle^{\varepsilon^{(0)}} + \sum_{\varepsilon_i \in \varepsilon} \underbrace{\left(\varepsilon_i - \varepsilon_i^{(0)} \right)}_{=:\Delta\varepsilon_i} \frac{\partial \langle \mathcal{O} \rangle^{\varepsilon}}{\partial \varepsilon_i} \bigg|_{\varepsilon = \varepsilon^{(0)}} + O\left(\Delta\varepsilon^2\right)$$

Baryon correlation function:



Perturbative Expansion – Spectroscopy

- Correlation functions asymptotically behave like

$$C(t_2, t_1) = ce^{-m(t_2-t_1)}$$

- Expansion in isospin breaking parameters ($\Delta\varepsilon_i := \varepsilon_i - \varepsilon_i^{(0)}$)

$$\begin{aligned}\rightarrow C(t_2, t_1) =& c^{(0)} e^{-m^{(0)}(t_2-t_1)} \\ &+ \sum_i \Delta\varepsilon_i \Big(c_i^{(1)} - c^{(0)} m_i^{(1)} (t_2 - t_1) \Big) e^{-m^{(0)}(t_2-t_1)} \\ &+ \mathcal{O}(\Delta\varepsilon^2)\end{aligned}$$

- $\Delta\varepsilon_i$ can be set by matching different average multiplet masses and mass splittings

Operator Basis

- General structure of baryonic operator:

$$O = \sum_{a,b,c,f_i,\mu_j} \varepsilon_{abc} \lambda^{\mu_1,\mu_2,\mu_3}_{f_1,f_2,f_3} q^{f_1,a}_{\mu_1} q^{f_2,b}_{\mu_2} q^{f_3,c}_{\mu_3}$$

  a, b, c: color, f_i: flavor, μ_j: spin

- λ must be symmetric under simultaneous exchange of μ_i and f_i to fulfill anticommutation relations

- Goal: Find sets of coefficients $\lambda^{\mu_1,\mu_2,\mu_3}_{f_1,f_2,f_3}$ to match symmetries of octet and decuplet baryons

Operator Basis

- Construction based on isospin-symmetric QCD following a procedure introduced by the *Lattice Hadron Physics Collaboration*[7]
- Classification by symmetries in flavor and spin (and parity eigenvalues)
- Example for symmetric spin and flavor indices (e.g. $\Delta^{++}_{ijk} = u_i u_j u_k$, $\Delta^{+}_{ijk} = \frac{1}{\sqrt{3}}(u_i u_j d_k + u_i d_j u_k + d_i u_j u_k)$, etc.)
  (Operators in Dirac-Pauli basis)

Embedding	S_z	gerade (even)	ungerade (odd)
1	$\frac{3}{2}$	Δ_{111}	$\sqrt{3}\Delta_{113}$
1	$\frac{1}{2}$	$\sqrt{3}\Delta_{112}$	$\Delta_{114} + 2\Delta_{123}$
1	$-\frac{1}{2}$	$\sqrt{3}\Delta_{122}$	$2\Delta_{124} + \Delta_{223}$
1	$-\frac{3}{2}$	Δ_{222}	$\sqrt{3}\Delta_{224}$
2	$\frac{3}{2}$	$\sqrt{3}\Delta_{133}$	Δ_{333}
2	$\frac{1}{2}$	$2\Delta_{134} + \Delta_{233}$	$\sqrt{3}\Delta_{334}$
2	$-\frac{1}{2}$	$\Delta_{144} + 2\Delta_{234}$	$\sqrt{3}\Delta_{344}$
2	$-\frac{3}{2}$	$\sqrt{3}\Delta_{244}$	Δ_{444}

H irrep for symmetric spin and flavor indices

[7] Basak et al. 2005, *Phys. Rev. D* **72**, p. 074501.

Practical Implications

- Have many operators for each baryon
  - \+ Generally simpler than the standard operators built from Dirac bilinears
  - \+ Can find operators with maximal overlap with the ground state
  - \+ Improve precision
  - − Lots of contractions to compute
    - Mixing between operators for the same state
    - At first order, Σ^0 and Λ mix[8]

- Optimizations of the contractions:
  - Identify u and d as light quark l for non-sequential propagators
  - Identify sequential propagators including u/d and A as l interacting with A
    $\rightarrow$ Absorb charge multiplicity ($\frac{2}{3}$ for u, $-\frac{1}{3}$ for d, s) into vertex

[8] Kordov et al. 2020, *Phys. Rev. D* **101** no. 3, p. 034517.

Strategy

- Optimizations leave 101580 individual (color-contracted) terms: 8304 isosymmetric, 38316 from mass detuning, 54960 from QED of the form $\sum_{\substack{a,b,c \\ a',b',c'}} \varepsilon_{abc}\varepsilon_{a'b'c'}\lambda^{\mu_1\mu_2\mu_3}_{f_1 f_2 f_3}\lambda^{\mu_4\mu_5\mu_6}_{f_1 f_2 f_3} D^{f_1,aa'}_{\mu_1\mu_4} D^{f_2,bb'}_{\mu_2\mu_5} D^{f_3,cc'}_{\mu_3\mu_6}$
- "Only" 10104 unique terms
  $\rightarrow$ Reuse already computed terms
- 25 different flavor combinations
  $\rightarrow$ Reduces maximal number of unique terms per flavor combination to 640
  $\rightarrow$ For each flavor combination calculate all possible contractions at once
- General Procedure:
  1. Perform all inversions for a given flavor combination and keep propagators in memory for later
  2. Perform color contractions for all spin-index combinations for current flavor combination
  3. Perform spin contractions
  4. Repeat for the next flavor combination

Summary

- We perform a calculation of a complete set of octet and decuplet baryon operators
- We use a perturbative approach to include isospin breaking effects on isosymmetric ensembles
- Operator mixing is considered
- A wide array of optimizations is performed to deal with the large amount of contractions

Backup Slides

Operator Basis

- On 4D lattice, spin-group broken to O_h^D with (relevant) irreps
  - G_{1g}/G_{1u}: 2D, strongest overlap with Spin-$\frac{1}{2}$
  - H_g/H_u: 4D, strongest overlap with Spin-$\frac{3}{2}$

- In Dirac-Pauli basis, Dirac spinors composed of Weyl spinors $\psi = \chi \otimes \xi$ transform under parity and spin as

$$\begin{aligned} \mathcal{P}\psi(\mathbf{x},t) =& \gamma_4\psi(-\mathbf{x},t) = ((\sigma_3\chi)\otimes\xi)(-\mathbf{x},t) \\ S_z\psi =& -i\gamma_1\gamma_2\psi = \chi\otimes(\sigma_3\xi) \end{aligned}$$

Flavor Symmetry	Spin Symmetry	Baryon
symmetric	symmetric	Δ, Ω
mixed symmetric	symmetric	Σ^*, Ξ^*
mixed symmetric	mixed symmetric	Σ, Ξ
mixed antisymmetric	mixed antisymmetric	N, Λ
mixed antisymmetric	antisymmetric	Λ

Basak, Subhasish et al. (2005). "Clebsch-Gordan construction of lattice interpolating fields for excited baryons". In: *Phys. Rev. D* 72, p. 074501. DOI: 10.1103/PhysRevD.72.074501. arXiv: hep-lat/0508018.

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